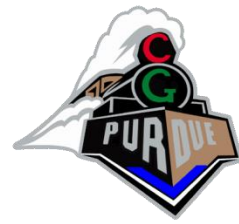




Camera Models

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Typical OpenGL Matrices

- Projection Matrix
 - Defines the projection process: perspective, orthographic, etc.
- ModelView Matrix (or View Matrix)
 - Defines where is the camera
- Model Matrices
 - Applied to geometry/model to define scene objects
- Texture Matrix
 - Is applied to the “texture” (more on this later)



Transformations

- Most popular transformations in graphics
 - Translation
 - Rotation
 - Scale
 - Projection
- In order to use a single matrix for all, we use homogeneous coordinates (we talked about this already)

3D Transformations



$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Identity

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Translation

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Mirror over X axis



3D Transformations

Rotate around Z axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 & 0 \\ \sin \Theta & \cos \Theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Rotate around Y axis:

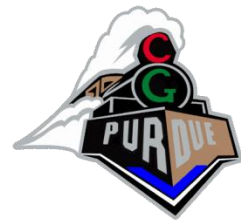
$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} \cos \Theta & 0 & -\sin \Theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin \Theta & 0 & \cos \Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Rotate around X axis:

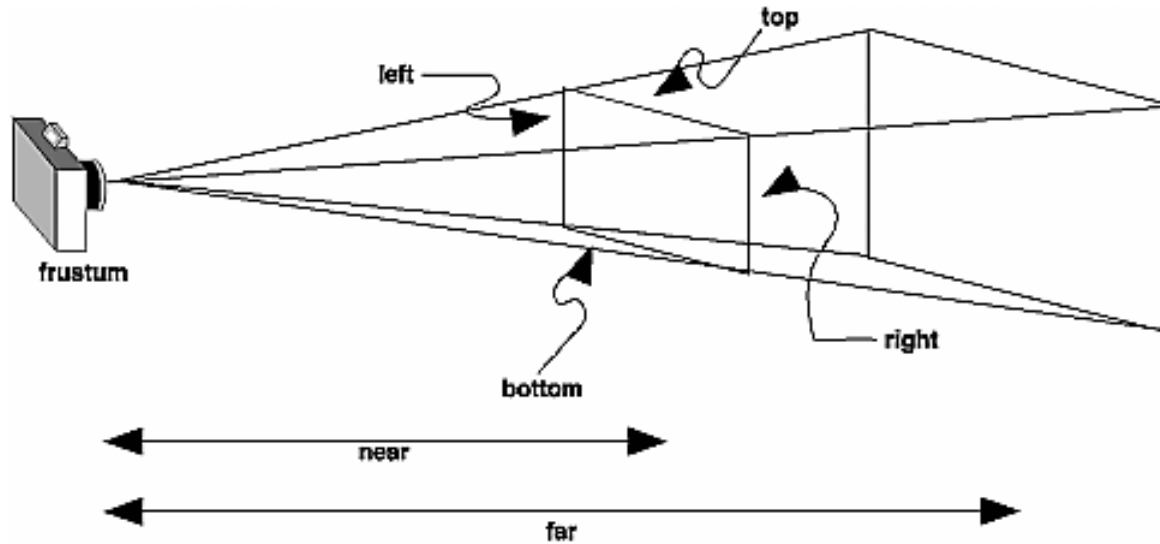
$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \Theta & -\sin \Theta & 0 \\ 0 & \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Perspective projection



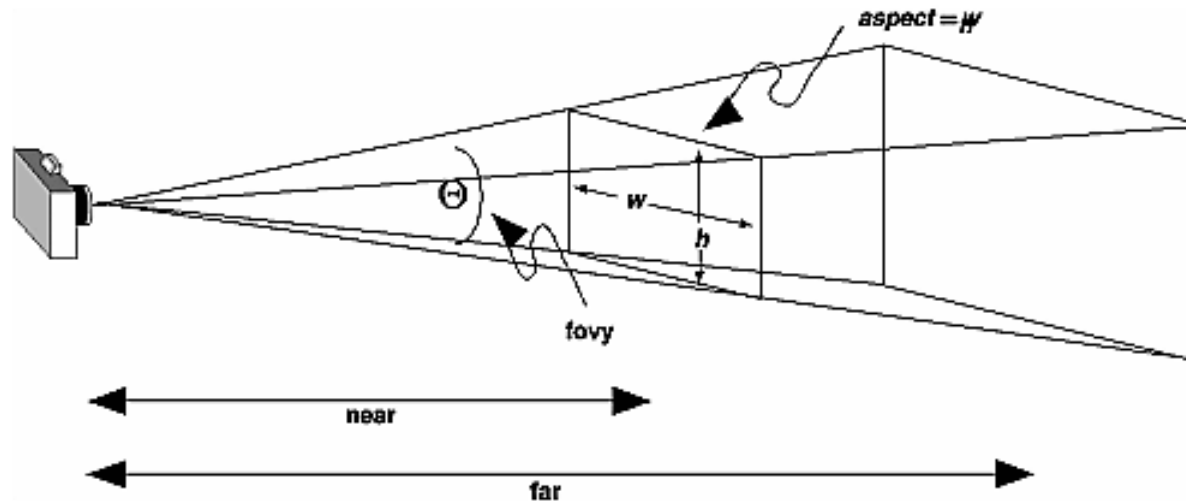
Projection Transformations



```
void glFrustum(GLdouble left, GLdouble right, GLdouble  
    bottom, GLdouble top, GLdouble near, GLdouble far);
```



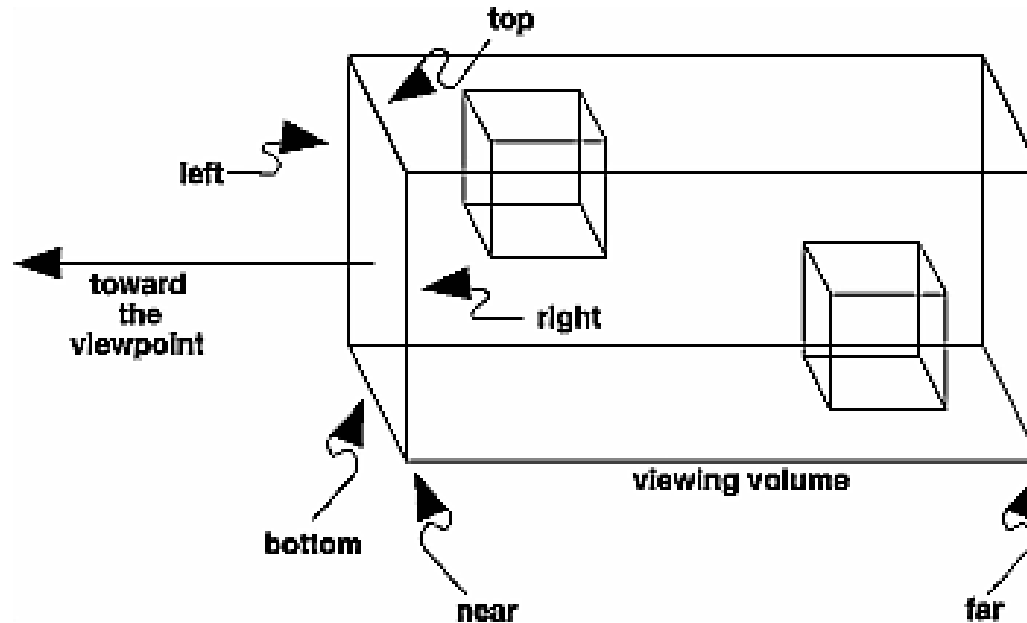
Projection Transformations



```
void gluPerspective(GLdouble fovy, GLdouble aspect, GLdouble  
near, GLdouble far);
```



Projection Transformations



```
void glOrtho(GLdouble left, GLdouble right, GLdouble  
    bottom,  
    GLdouble top, GLdouble near, GLdouble far);
```

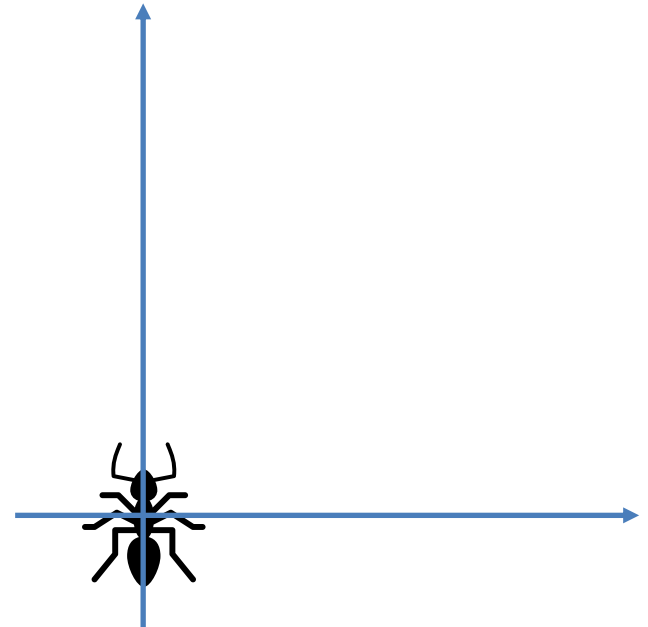
```
void gluOrtho2D(GLdouble left, GLdouble right,  
    GLdouble bottom, GLdouble top);
```


View/Model Transformations



- The order of operations matters!
- How to rotate CW 90°?
- Solution?

Rotate (90)



Ant position = a

Rotate(d): rotate CW by d degrees

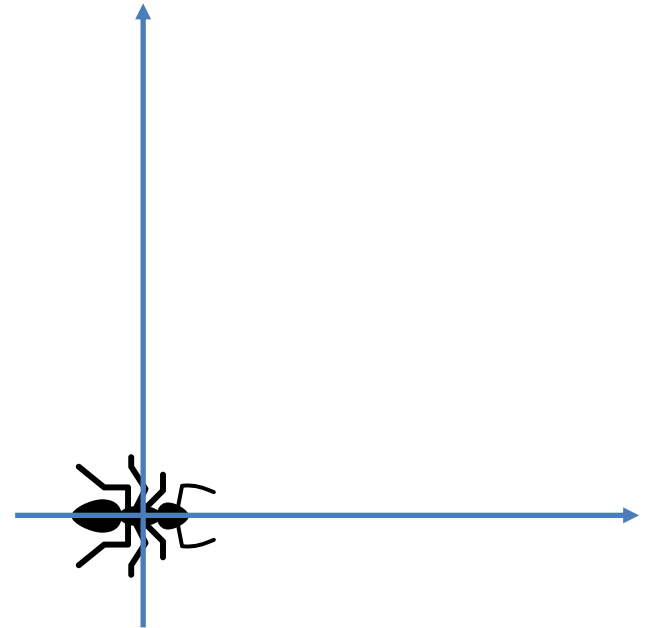
Translate(t): translate by vector t

View/Model Transformations



- The order of operations matters!
- How to rotate CW 90°?
- Solution?

Rotate (90)



Ant position = a

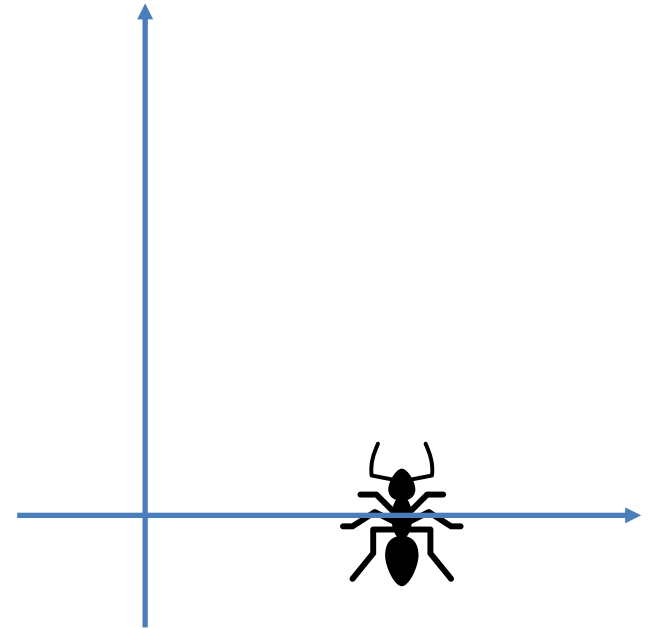
Rotate(d): rotate CW by d degrees

Translate(t): translate by vector t

View/Model Transformations



- The order of operations matters!
- How to rotate CCW 90° ?
- Solution?



Ant position = a

Rotate(d): rotate CW by d degrees

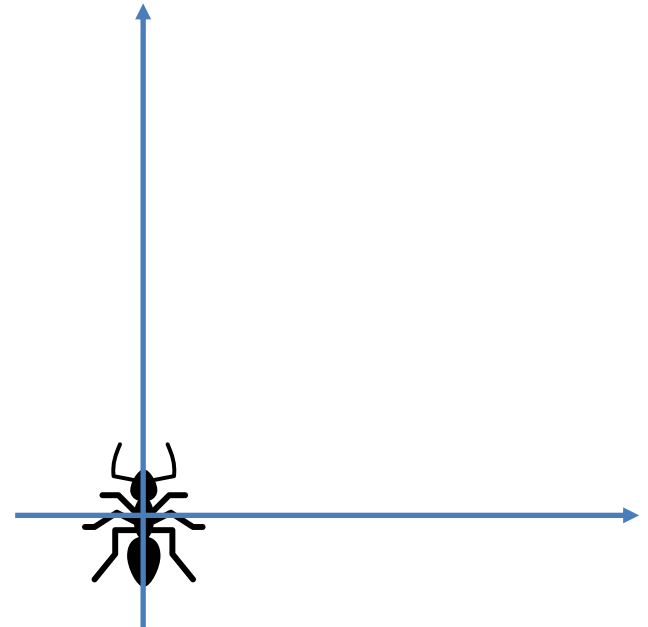
Translate(t): translate by vector t

View/Model Transformations



- The order of operations matters!
- How to rotate CCW 90° ?
- Solution?

`Translate (-a)`



`Ant position = a`

`Rotate(d): rotate CW by d degrees`

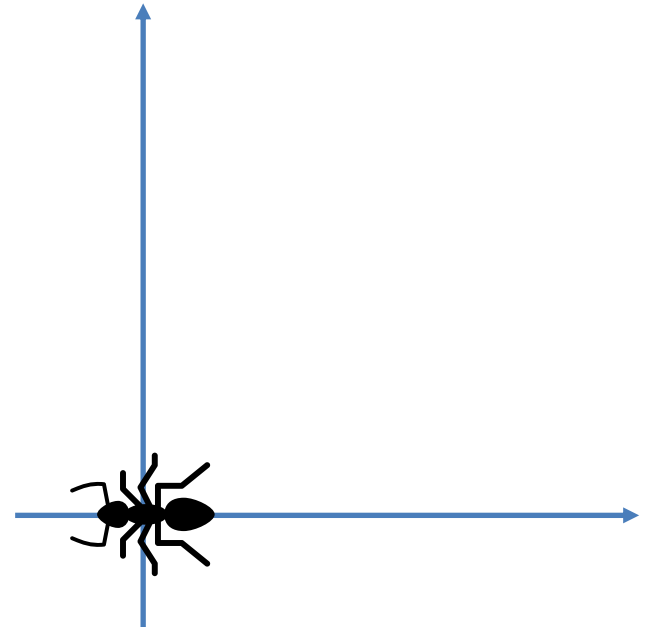
`Translate(t): translate by vector t`

View/Model Transformations



- The order of operations matters!
- How to rotate CCW 90° ?
- Solution?

```
Translate (-a)  
Rotate (-90)
```



```
Ant position = a
```

```
Rotate(d): rotate CW by d degrees
```

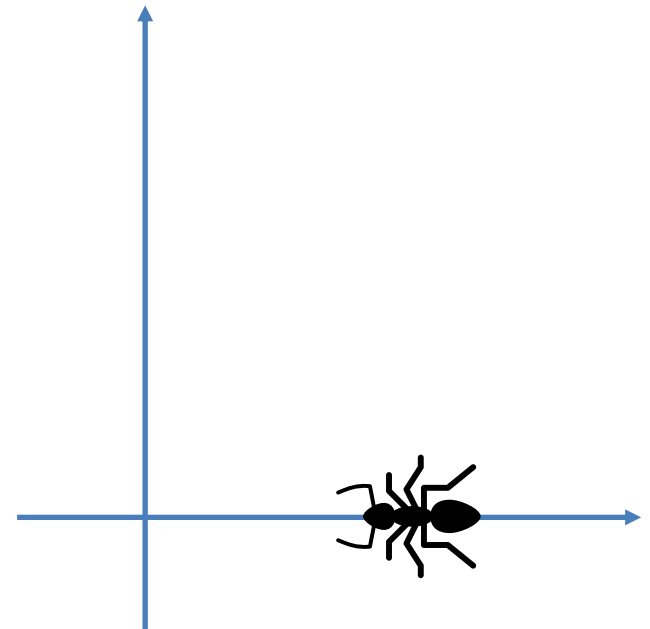
```
Translate(t): translate by vector t
```

View/Model Transformations



- The order of operations matters!
- How to rotate CCW 90° ?
- Solution?

```
Translate (-a)  
Rotate (-90)  
Translate (a)
```



Ant position = a

Rotate(d): rotate CW by d degrees

Translate(t): translate by vector t

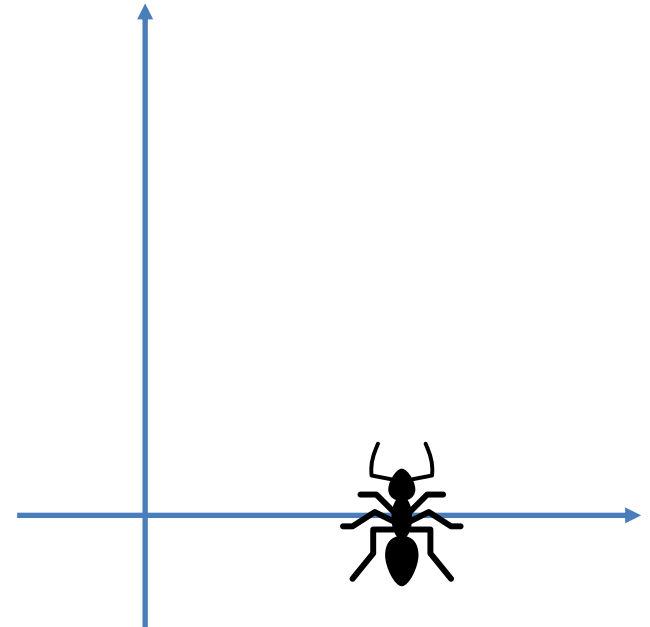
View/Model Transformations



- The order of operations matters!
- How to rotate CCW 90° ?
- What if I rotate first?

Previous solution:

```
Translate(-a)  
Rotate(-90)  
Translate(a)
```



Ant position = a

Rotate(d): rotate CW by d degrees

Translate(t): translate by vector t

View/Model Transformations

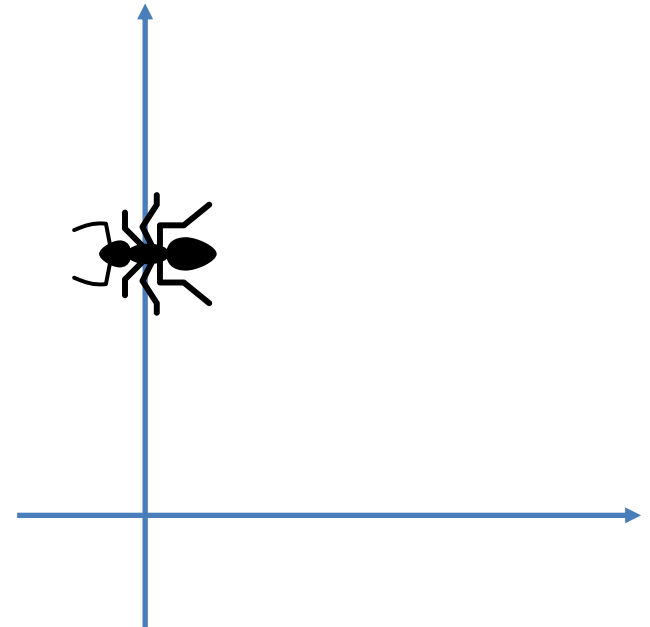


- The order of operations matters!
- How to rotate CCW 90° ?
- What if I rotate first?

Previous solution:

```
Translate(-a)  
Rotate(-90)  
Translate(a)
```

Rotate(-90)



Ant position = a

Rotate(d): rotate CW by d degrees

Translate(t): translate by vector t

View/Model Transformations



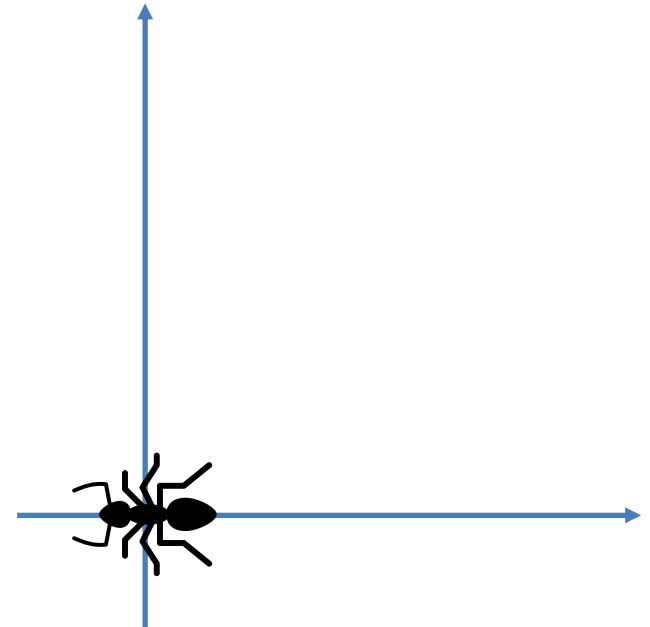
- The order of operations matters!
- How to rotate CCW 90° ?
- What if I rotate first?

Previous solution:

```
Translate(-a)  
Rotate(-90)  
Translate(a)
```

```
Rotate(-90)  
Translate(-a)
```

[assuming a was updated to
new position]



Ant position = a

Rotate(d): rotate CW by d degrees

Translate(t): translate by vector t

View/Model Transformations

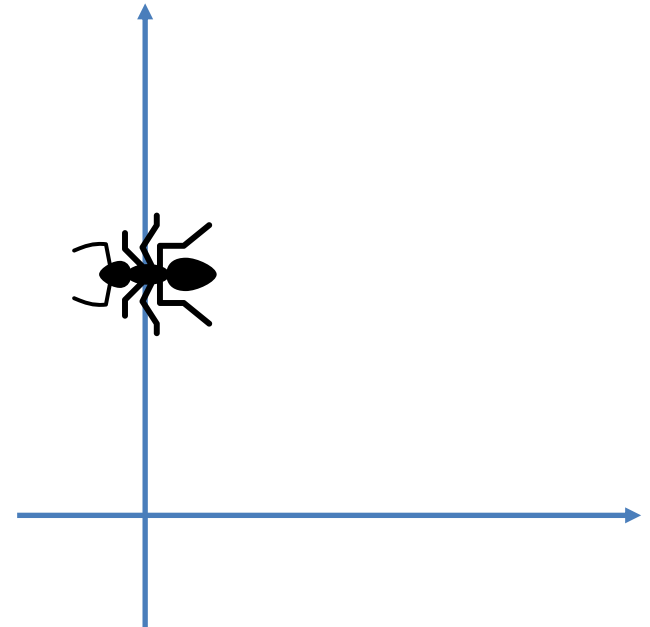


- The order of operations matters!
- How to rotate CCW 90° ?
- What if I rotate first?

Previous solution:

```
Translate (-a)  
Rotate (-90)  
Translate (a)
```

```
Rotate (-90)  
Translate (-a)  
Translate (a)
```



Ant position = a

Rotate(d): rotate CW by d degrees

Translate(t): translate by vector t

View/Model Transformations



- In matrix form:

```
rMat = RotateMat(-90)
inv_tMat = TranslateMat(-a)
tMat = TranslateMat(a)
```

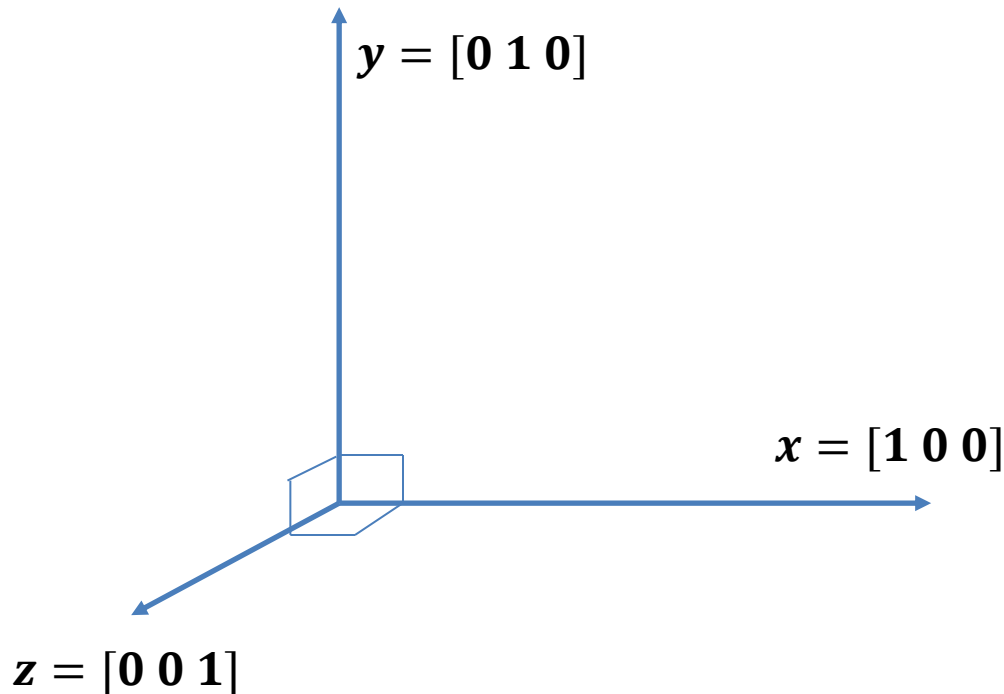
```
p' = tMat * rMat * inv_tMat * p
(rotates points p of the ant "about itself")
```

```
p' = tMat * inv_tMat * rMat * p = rMat * p
(rotates points p of the ant around the origin)
```

Change of Basis Transformation



- Standard basis:



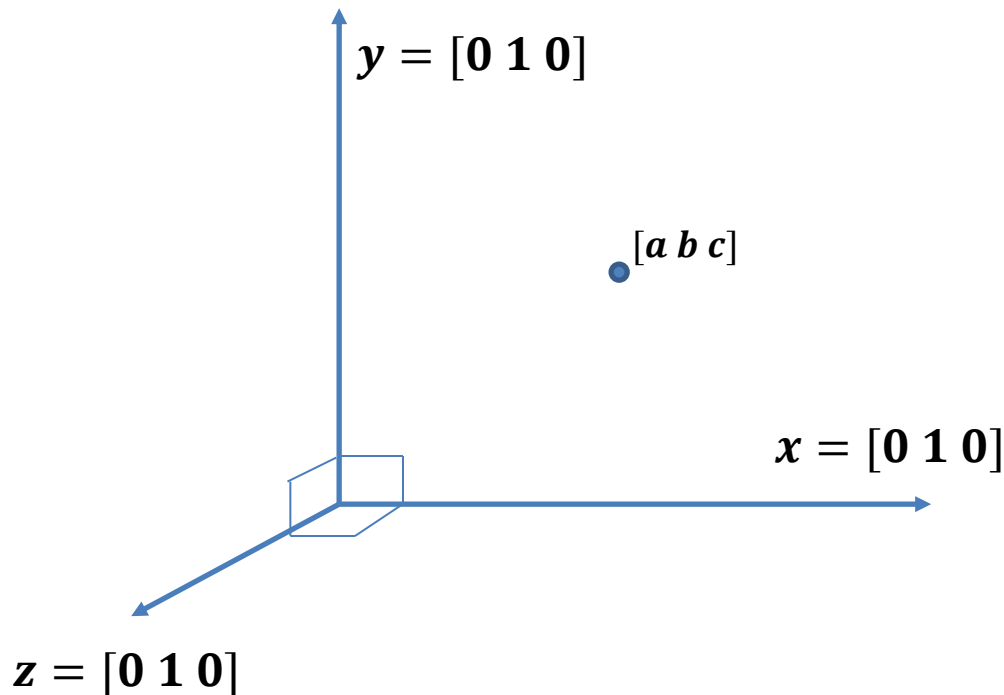
$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

↑ ↑ ↑
 x^T y^T z^T

Change of Basis Transformation



- Standard basis:



$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

↑ ↑ ↑
 x^T y^T z^T

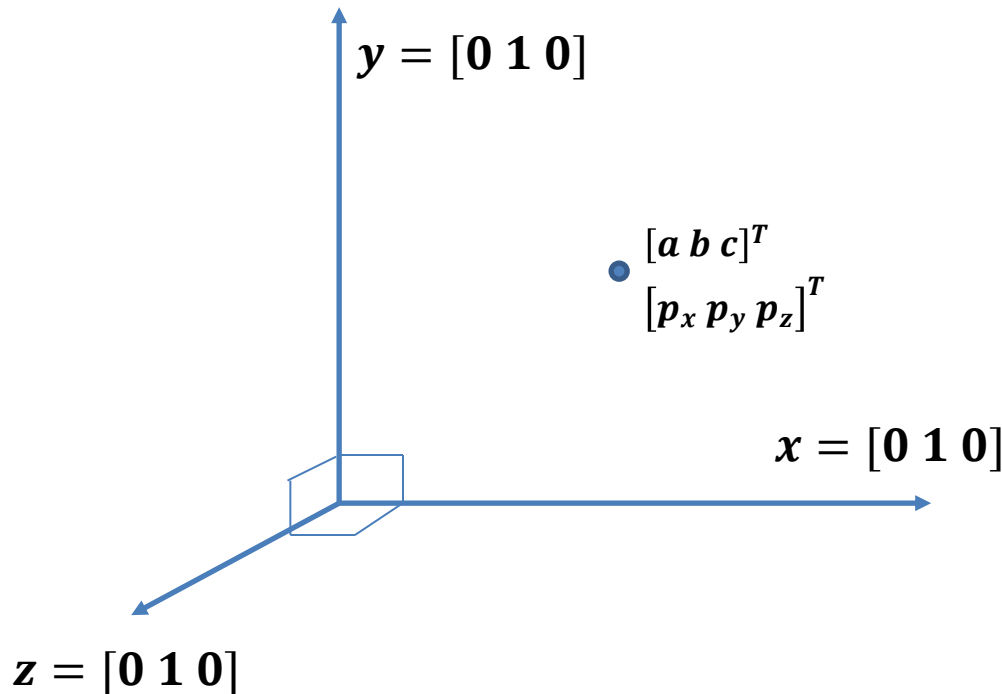
Where is point $[a \ b \ c]$ in basis S ?

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = S \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Change of Basis Transformation



- Standard basis:



$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

↑ ↑ ↑
 x^T y^T z^T

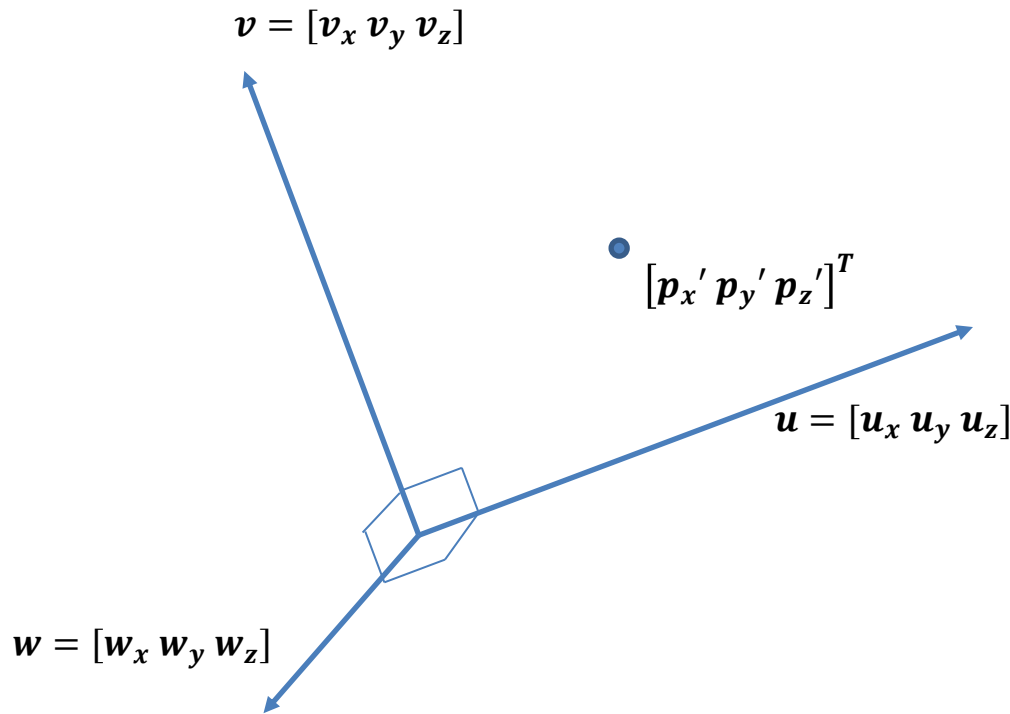
Where is point $[a \ b \ c]$ in basis S ?

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = S \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Change of Basis Transformation



- Basis B:



$$B = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

What is B?

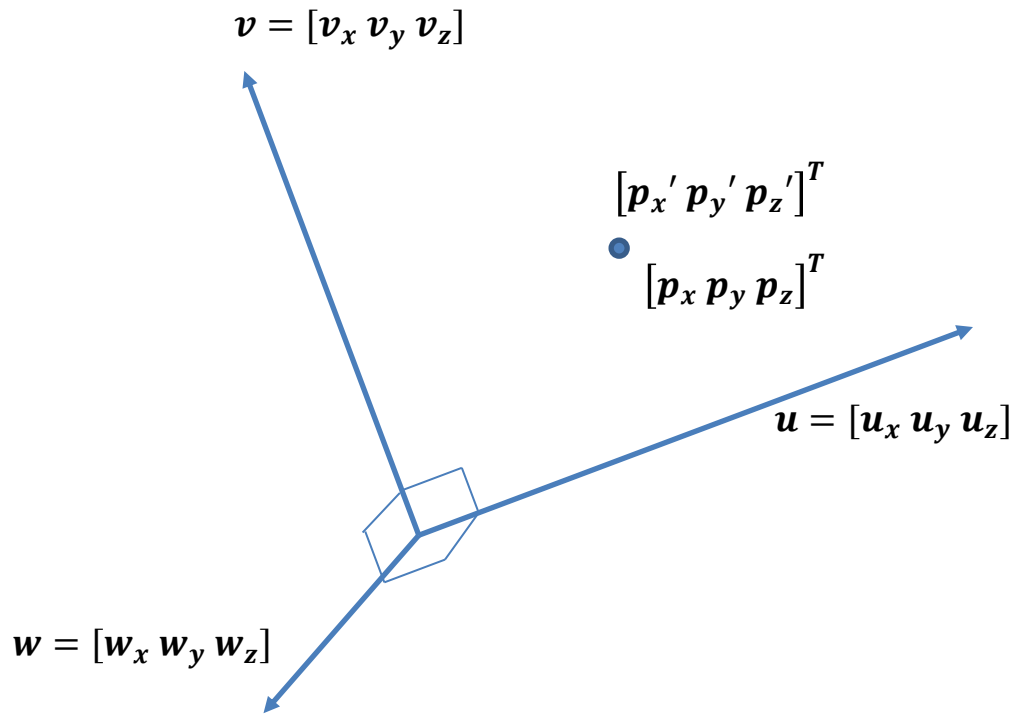
Where is point $[p_x' \ p_y' \ p_z']^T$ from basis B?

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = B \begin{bmatrix} p_x' \\ p_y' \\ p_z' \end{bmatrix}$$

Change of Basis Transformation



- Basis B:



$$B = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix}$$

$\uparrow \quad \uparrow \quad \uparrow$
 $u^T \quad v^T \quad w^T$

Where is point $[p'_x \ p'_y \ p'_z]$ in basis B?

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = B \begin{bmatrix} p'_x \\ p'_y \\ p'_z \end{bmatrix}$$

Change of Basis Transformation



- In matrix form:

```
// change p' from basis b to standard basis
```

```
bMat = makeBasisMat(u,v,w)
```

```
p' = position(1,1,1)
```

```
p = bMat * p'
```

```
// change from standard to basis B
```

```
p' = inverse(bMat) * p
```

Change of Basis Transformation



- What else is this change of basis useful for?
 - Rotating to an arbitrary basis
 - “I was in basis frame (x,y,z) and now I want to rotate to be basis frame (u,v,w) ”

Change of Basis Transformation



- Recall we did “inverse(bMat)”
- What is the inverse of matrix?

$$B^{-1}B = I$$

- A nice property:
 - If B is formed by orthogonal basis vectors, then its inverse is simply:

$$B^{-1} = B^T = \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{bmatrix}$$