Spatial Data Structures and Hierarchies

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Spatial Data Structures

• Store geometric information
• Organize geometric information
• Permit fast access to/of geometric information

• Applications
  – Heightfields
  – Collision detection (core to *many* uses)
  – Simulations (e.g., surgery, games)
  – Rendering (e.g., need to render fast!)
Hierarchical Modeling

• Concept is old but fundamental
  – “Hierarchical geometric models for visible surface algorithms”, James Clark - 1976
Hierarchical Modeling

- Trees and Scene Graphs
Hierarchical Modeling

• Trees and Scene Graphs
Hierarchical Modeling

• Trees and Scene Graphs
Bounding Volumes

• Problem:
  – Suppose you need to intersect rays with a scene...
  – Suppose you have a scene divided into objects...

• Solution: bottom-up
  – Wrap complex objects into simple ones
    • Boxes, spheres, other shapes...
  – Organize into a tree
Bounding Sphere

- Simplest way to bound an object
- Good for small or round objects
Bounding Boxes

- Axis Aligned Vs Orientated

- Orientated
  More Expensive

- Axis Aligned
  Cheaper
Bounding Volume Hierarchy (BVH)

• How to building an axis aligned bounding box (AABB) BVH?
• How to intersect?
• Complexity? Problem cases?
AABB BVH

• Example construction
  – Given M 2D points, use k-means clustering to determine clusters
  – Then group nearby clusters (e.g., use Voronoi diagram or Delaunay triangulation)
  – And iteratively form a tree from the bottom-up
  – In each node, approximate the contained points using an axis-aligned bounding box
    • e.g., box = [min of all contained pts, max of all contained pts]
Bounding Volume Hierarchy (BVH)

• How to building an oriented bounding box (OBB) BVH?
• How to intersect?
• Complexity? Problem cases? Advantages over axis-aligned?
OBB BVH

• Example construction
  – Similar to AABB BVH but “fit” an oriented box to the points within each cluster/node of the tree
  – Methods:
    • Sample possible rotations and sizes in order to pick the best box
    • Compute distance of points to a line and optimize the line equation parameters until finding the line that best approximates all points
    • Then compute a box width – consider the benefit/cost of the box size
      – e.g., totally containing all points might make the box very large; could also choose to mostly contain the points – however, what does this mean with regards to operations using the BVH?
An Application of BVH: Collision Detection

- Turn complex objects into bounded volumes for collision testing
- Fast, but not reliable
- Great initial test, but should be followed by another more precise test
An Application of BVH: Collision Detection

• Intersect these!
Bounding Volume Hierarchy

• Enclose objects into BVs
• Check BV first
Bounding Volume Hierarchy

- Enclose objects into BVs
- Check BV first
- Decompose into two
Bounding Volume Hierarchy

- Enclose objects into BVs
- Check BV first
- Decompose into two
- Proceed hierarchically
• Enclose objects into BVs
• Check BV first
• Decompose into two
• Proceed hierarchically
• BVH is pre-computed for each object
Bounding Volume Hierarchy in 3D
Collision Detection

Two objects described by their precomputed BVHs
Collision Detection

Search tree

pruning

A

A

A
Collision Detection

Search tree

A

BB
BC
CB
CC

B
D
E

C
F
G

A

A
Collision Detection

Search tree

pruning
If the pieces contained in G and D overlap → collision
AABB
- Not invariant
- Efficient to test
- Not tight
- Invariant
- Less efficient to test
- Tight
### Comparison

<table>
<thead>
<tr>
<th></th>
<th>Sphere</th>
<th>AABB</th>
<th>OBB</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tightness</strong></td>
<td>-</td>
<td>--</td>
<td>+</td>
</tr>
<tr>
<td><strong>Testing</strong></td>
<td>+</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td><strong>Invariance</strong></td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>

No type of BV is optimal for all situations.
Space Subdivision

- Binary tree / Quadtree / Octree
- k-D tree
- Binary Space Partitioning (BSP) Tree
Binary Tree

- A directed edge refers to the link from the parent to the child (the arrows in the picture of the tree).
- The root node of a tree is the node with no parents; there is at most one root node in a rooted tree.
- A leaf is a node that has no children.
- The depth of a node is the length of the path from the root to the node. The root node is at depth zero.
- The height of a tree is the depth of its furthest leaf. A tree with only a root node has a height of zero.
- Siblings are nodes that share the same parent node.

Size = 9
Height = 3
Root node = 2
Binary Tree

• Operations
  – Search
  – Insert
  – Delete
Quadtree

- Similar to binary-tree, but have 4 children per node.
- Each node corresponds to one of four rectangular regions of the current quad.
Quadtree

• Similar to binary-tree, but have 4 children per node
• Each node corresponds to one of four rectangular regions of the current quad
• Point quadtree
  – Partitions depend on the data
  – The quad is divided using the previous point within it
Quadtree

• Point quadtree
  – Partitions depend on the data
  – The quad is divided using the previous point within it

• Advantage
  – Data dependent subdivision reduces (unnecessary) number of quads

• Disadvantage
  – Quads do not tightly approximate region surrounding the point
Quadtree

- **Matrix (MX) quadtree**
  - Location of partition lines independent of the data
  - The occupied nodes are all subdivided until a tight fitting box
• **MX quadtree**
  – Location of partition lines independent of the data
  – The occupied nodes are all subdivided until a tight fitting box

• **Advantage**
  – Quads leaf nodes always tightly approximate region surrounding the point

• **Disadvantage**
  – Potentially lots of levels from root to a point
Quadtree

- Point Region (PR) quadtree
  - Location of partition lines independent of the data
  - The nodes are all subdivided until $p$ or less points per node (e.g., $p=1$)
Quadtrees

- **PR quadtree**
  - Location of partition lines independent of the data
  - The nodes are all subdivided until \( p \) or less points per node (e.g., \( p=1 \))

- **Advantage**
  - Partition lines are known and paths from root to point is only as long as needs to be

- **Disadvantage**
  - Quads do not tightly approximate region surrounding the point
Quadtree

• Comparison

Point QT  MX QT  PR QT
Demo

- [http://donar.umiacs.umd.edu/quadtree/](http://donar.umiacs.umd.edu/quadtree/)
Octree

- Analogous to Quadtree but extended to 3D
- Each node is divided into eight subboxes
Octree

- Analogous to Quadtree but extended to 3D
- Each node is divided into eight subboxes
- Similar, there are
  - Point octrees
  - MX octrees
  - PR octrees
K-D tree

• Partition each dimension in a cyclical fashion
  – Thus, can be applied to 2D, 3D, or higher dimensions

• Each node stores a next partitioned “half-space” of data points (or of the data space)
**k-D tree**

- The first split (red) cuts the root cell (white) into two
- Each of which is then split (green) into two subcells
- Each of those four is split (blue) into two subcells
- The final eight called leaf cells
- The yellow spheres represent the tree vertices
Demo

- http://donar.umiacs.umd.edu/quadtree/
Binary Space Partitioning (BSP)

• Similar to k-D tree but splitting lines/planes are not necessarily axis-aligned
• Can adapt better to data
• Was algorithm used for visibility sorting...
Binary Space Partitioning (BSP)

• Suitable for any number of dimensions

Separating planes are shown in black and objects in blue.

BSP trees
Demo

• More stuff at
  – http://donar.umiacs.umd.edu/quadtree

• See