Graphics Pipeline: Transformation, Shading/Lighting, Projection, Texturing, and more!

Spring 2022

Daniel G. Aliaga
Department of Computer Science
Purdue University

Computer Graphics Pipeline

Geometry

Modeling Transformation	Transform into 3D *world* coordinate system
Lighting	Simulate illumination and reflectance
Viewing Transformation	Transform into 3D *camera* coordinate system
Clipping	Clip primitives outside camera's view
Projection	Transform into 2D camera coordinate system
Scan Conversion	Draw pixels (incl. texturing, hidden surface…)

Image

Computer Graphics Pipeline

Geometry

Modeling Transformation	Transform into 3D *world* coordinate system
Lighting	Simulate illumination and reflectance
Viewing Transformation	Transform into 3D *camera* coordinate system
Clipping	Clip primitives outside camera's view
Projection	Transform into 2D camera coordinate system
Scan Conversion	Draw pixels (incl. texturing, hidden surface…)

Image

Modeling Transformations

- Most popular transformations in graphics
  - Translation
  - Rotation
  - Scale
  - Projection
- In order to use a single matrix for all, we use homogeneous coordinates…

Modeling Transformations

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Identity

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Translation

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Mirror over X axis

Modeling Transformations

Rotate around Z axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 & 0 \\ \sin\Theta & \cos\Theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Rotate around Y axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} \cos\Theta & 0 & -\sin\Theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\Theta & 0 & \cos\Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

And many more…

Rotate around X axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Theta & -\sin\Theta & 0 \\ 0 & \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Computer Graphics Pipeline

Geometry

↓

Stage	Description
Modeling Transformation	Transform into 3D *world* coordinate system
Lighting	Simulate illumination and reflectance
Viewing Transformation	Transform into 3D *camera* coordinate system
Clipping	Clip primitives outside camera's view
Projection	Transform into 2D camera coordinate system
Scan Conversion	Draw pixels (incl. texturing, hidden surface…)

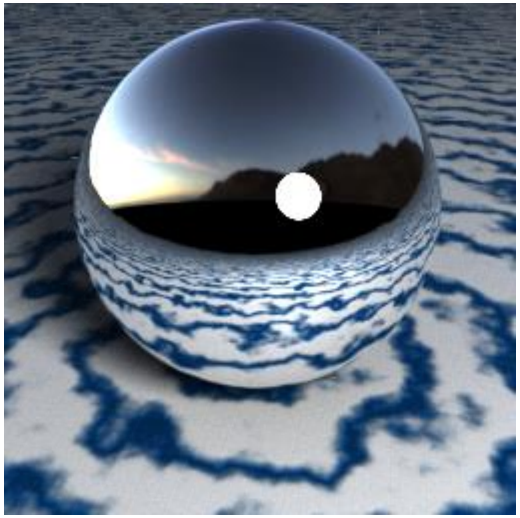
↓

Image

Diffuse

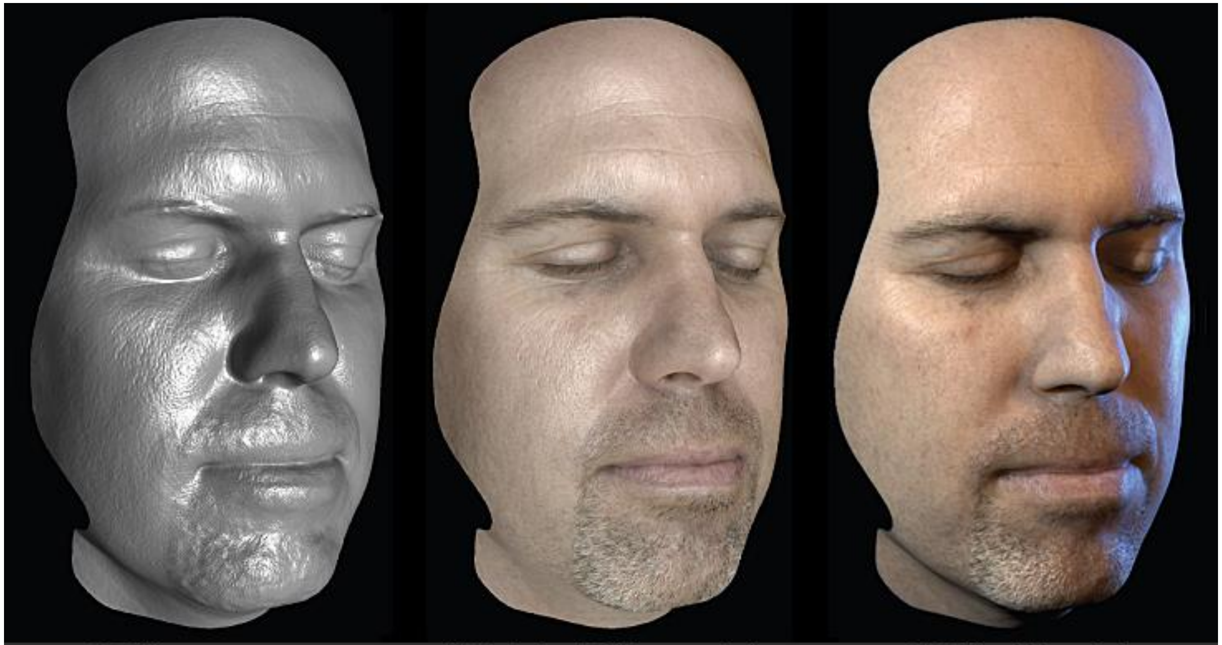
(mostly)

Specular++



Environment Mapping

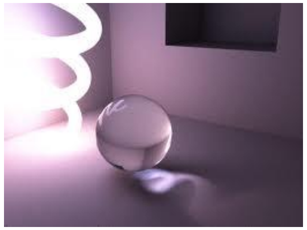
Subsurface Scatterring

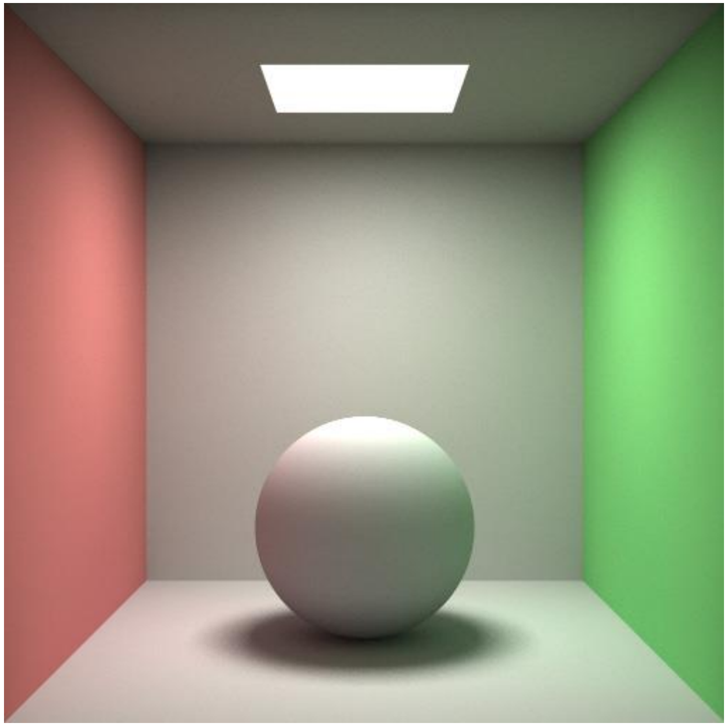


(a) High-res geometry (b) Real-time hybrid map rendering (c) Offline SSS rendering

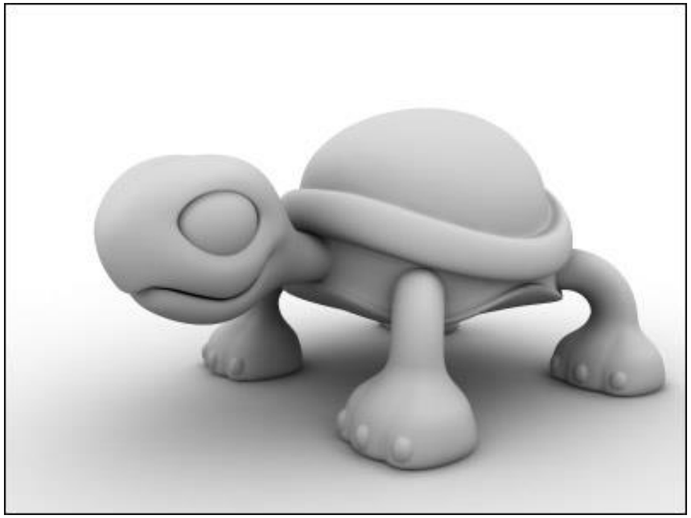
Others

Transparency



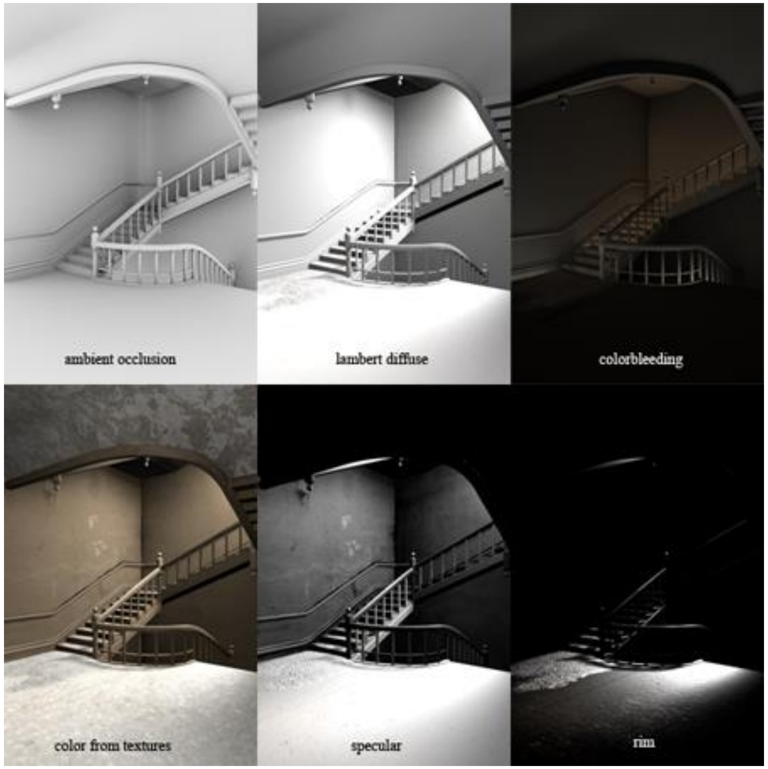


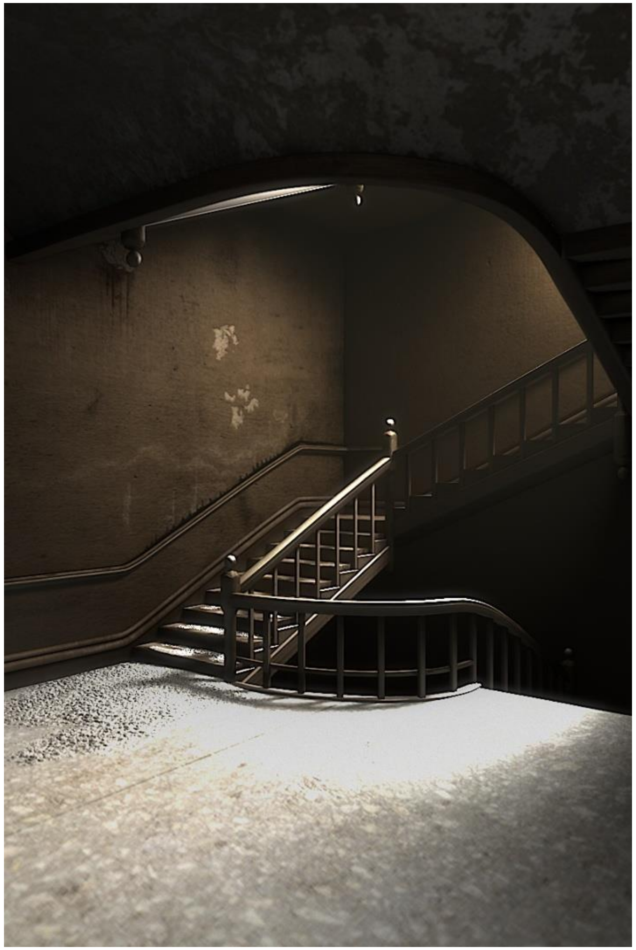
Radiosity



Ambient occlusion

Others


ambient occlusion
lambert diffuse
colorbleeding
color from textures
specular
rim



Lighting and Shading

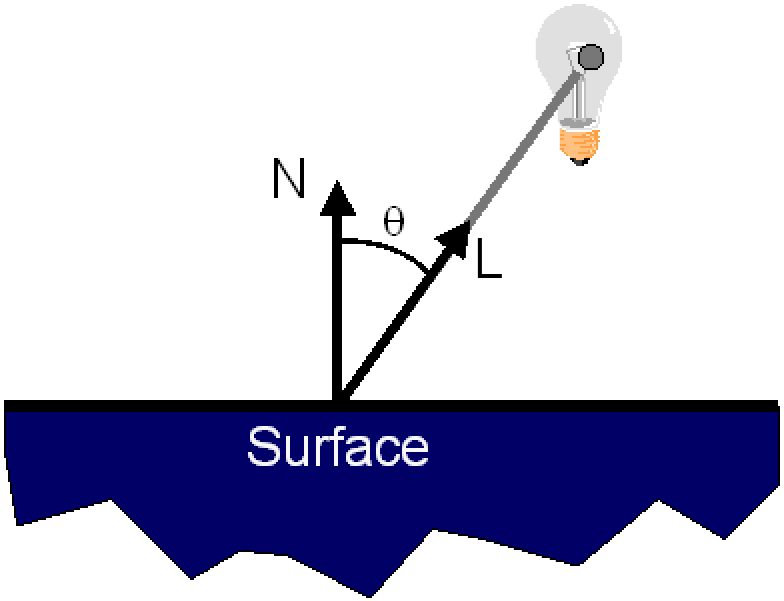
- Light sources
  - Point light
    - Models an omnidirectional light source (e.g., a bulb)
  - Directional light
    - Models an omnidirectional light source at infinity
  - Spot light
    - Models a point light with direction
- Light model
  - Ambient light
  - Diffuse reflection
  - Specular reflection

Lighting and Shading

- Diffuse reflection
  - Lambertian model

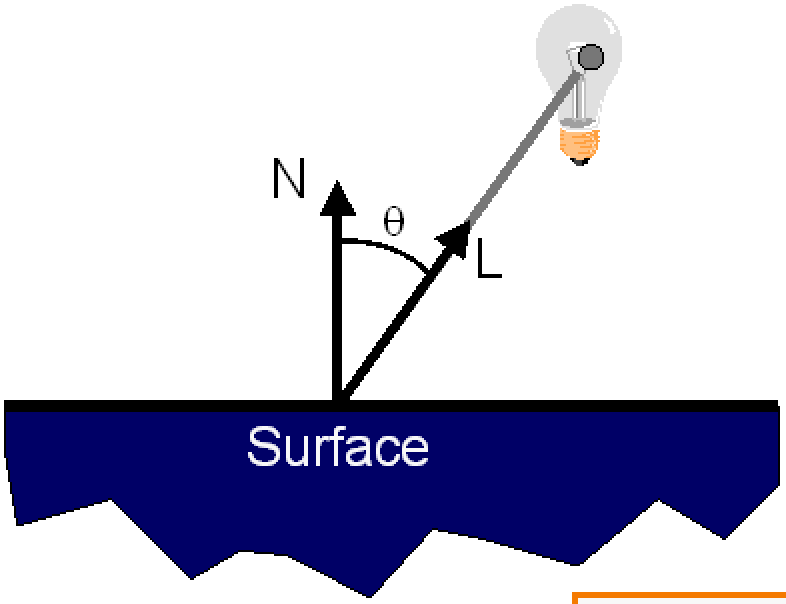
Lighting and Shading

- Diffuse reflection
  - Lambertian model



Lighting and Shading

- Diffuse reflection
  - Lambertian model



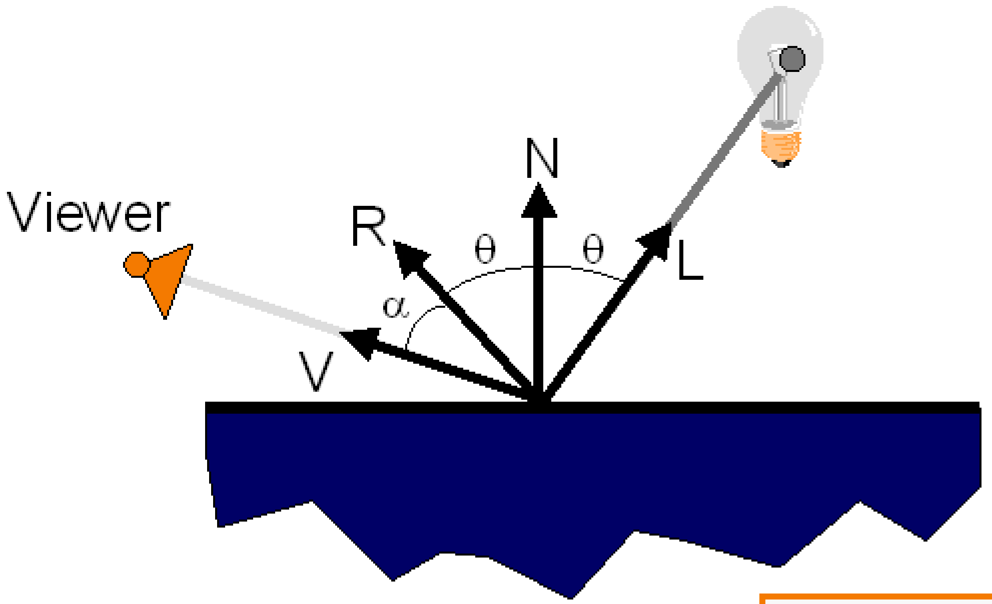
$$I_D = K_D (N \bullet L) I_L$$

Lighting and Shading

- Specular reflection
  - Phong model

Lighting and Shading

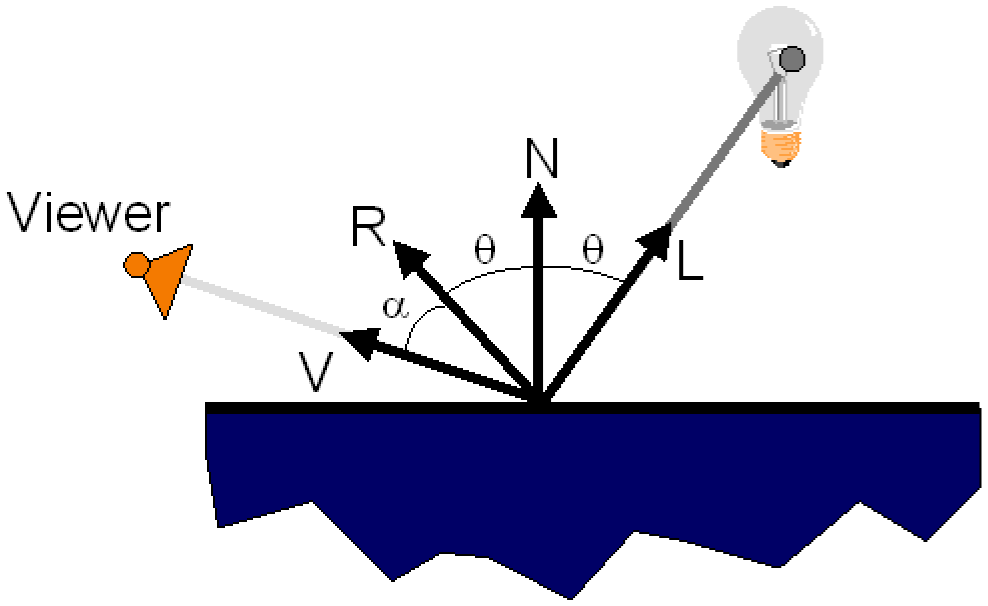
- Specular reflection
  - Phong model



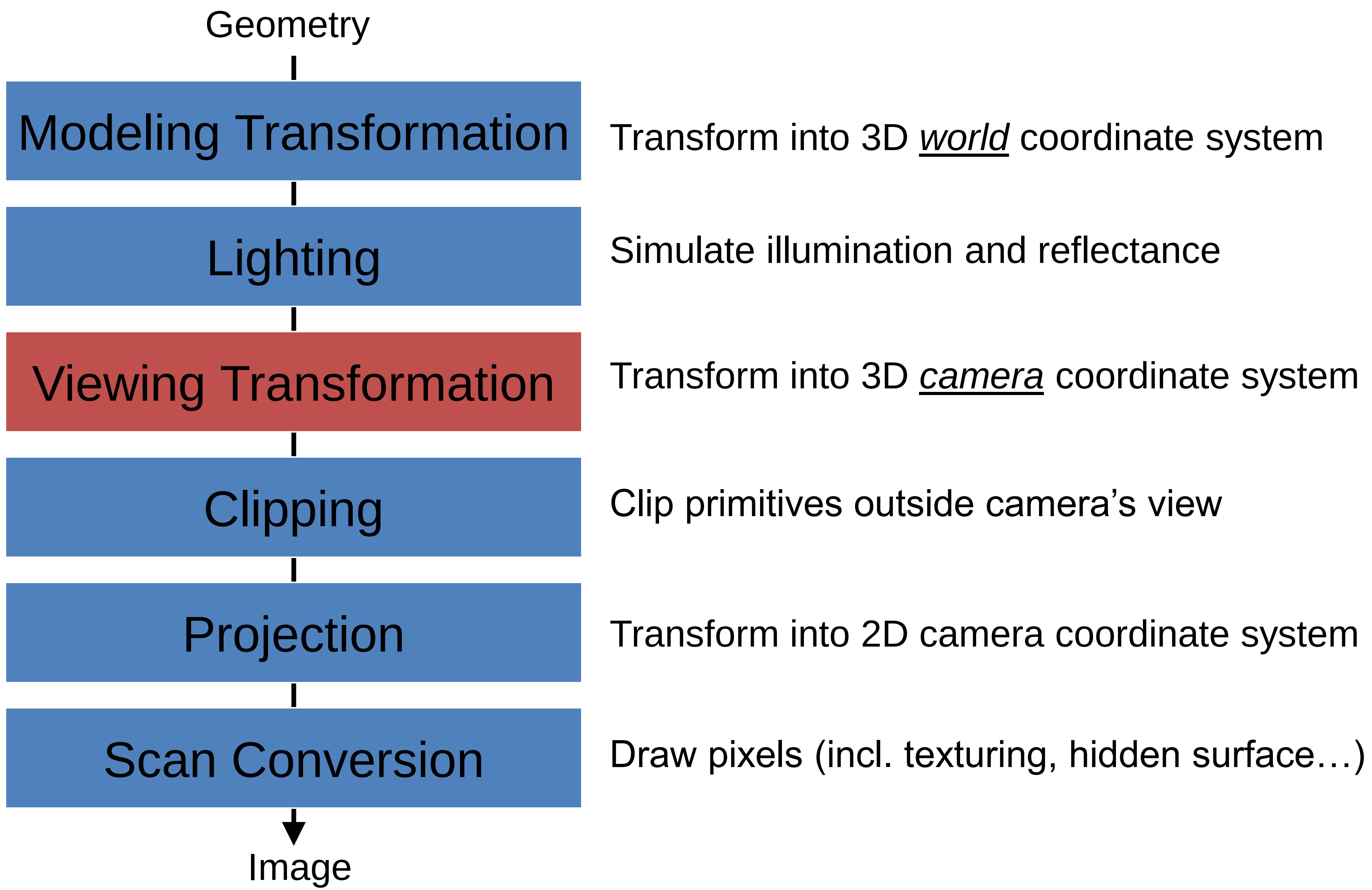
$$I_S = K_S (V \bullet R)^n I_L$$

Lighting and Shading

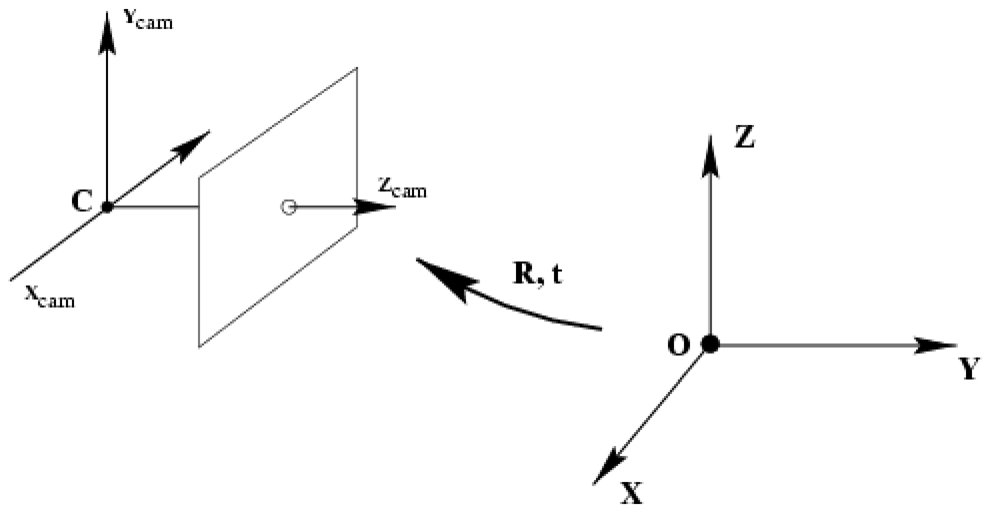
- Specular reflection
  - Phong model



Computer Graphics Pipeline



Viewing Transformation



$$\tilde{x}_c = R(\tilde{X} - C)$$

$$\tilde{x}_c = R\tilde{X} - RC$$

$$-t$$

$$\tilde{x}_c = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

World-to-camera matrix M

$$R = R_x R_y R_z$$

3x3 rotation matrices

$$t = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix}^T$$

translation vector

Computer Graphics Pipeline

Geometry

Stage	Description
Modeling Transformation	Transform into 3D *world* coordinate system
Lighting	Simulate illumination and reflectance
Viewing Transformation	Transform into 3D *camera* coordinate system
Clipping	Clip primitives outside camera's view
Projection	Transform into 2D camera coordinate system
Scan Conversion	Draw pixels (incl. texturing, hidden surface…)

Image

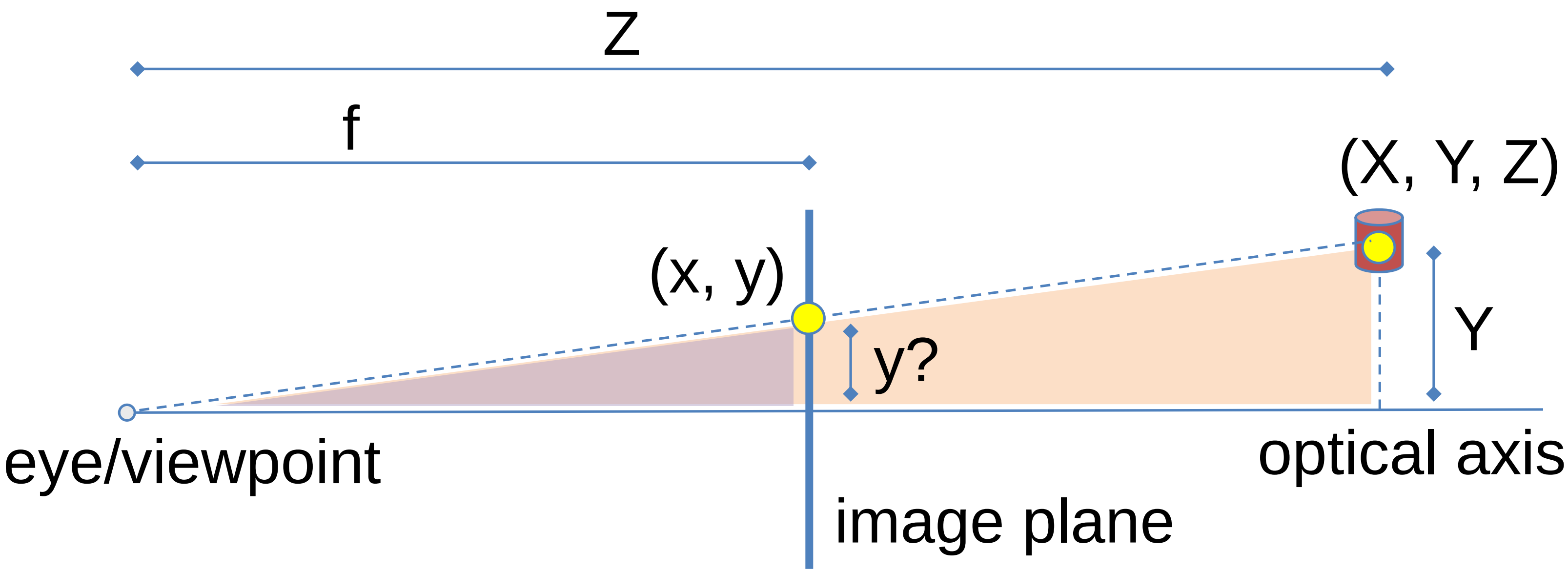
Computer Graphics Pipeline

Geometry

Modeling Transformation	Transform into 3D *world* coordinate system
Lighting	Simulate illumination and reflectance
Viewing Transformation	Transform into 3D *camera* coordinate system
Clipping	Clip primitives outside camera's view
Projection	Transform into 2D camera coordinate system
Scan Conversion	Draw pixels (incl. texturing, hidden surface…)

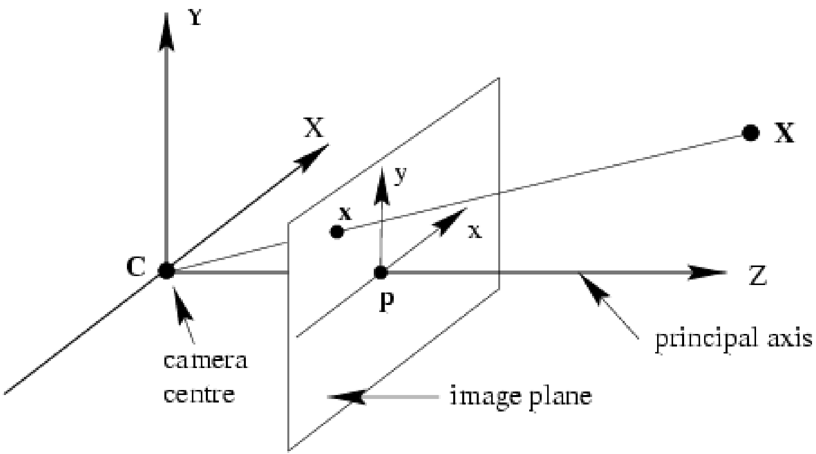
Image

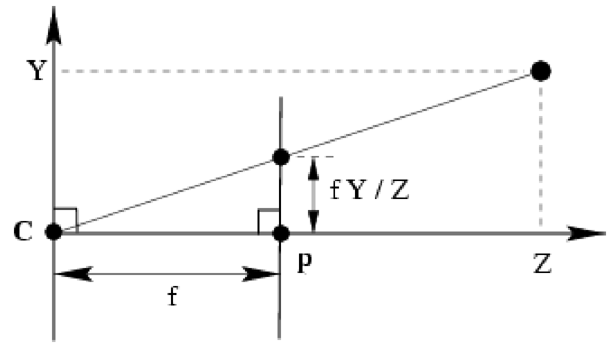
Perspective projection



$$\frac{y}{f} = \frac{Y}{Z} \quad \Rightarrow \quad y = f\frac{Y}{Z} \quad \& \quad x = f\frac{X}{Z}$$

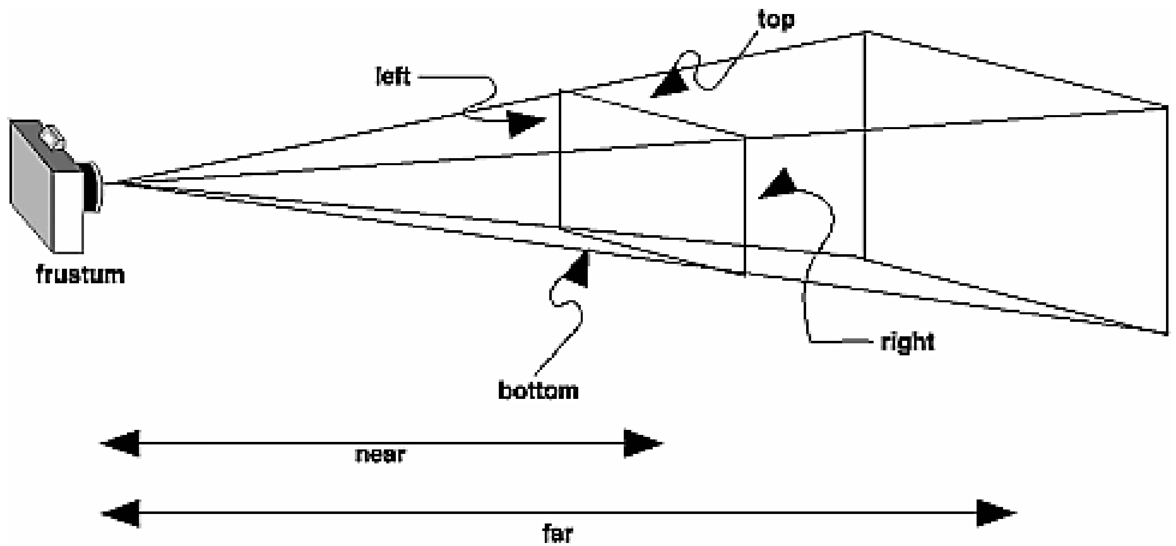
Perspective Projection





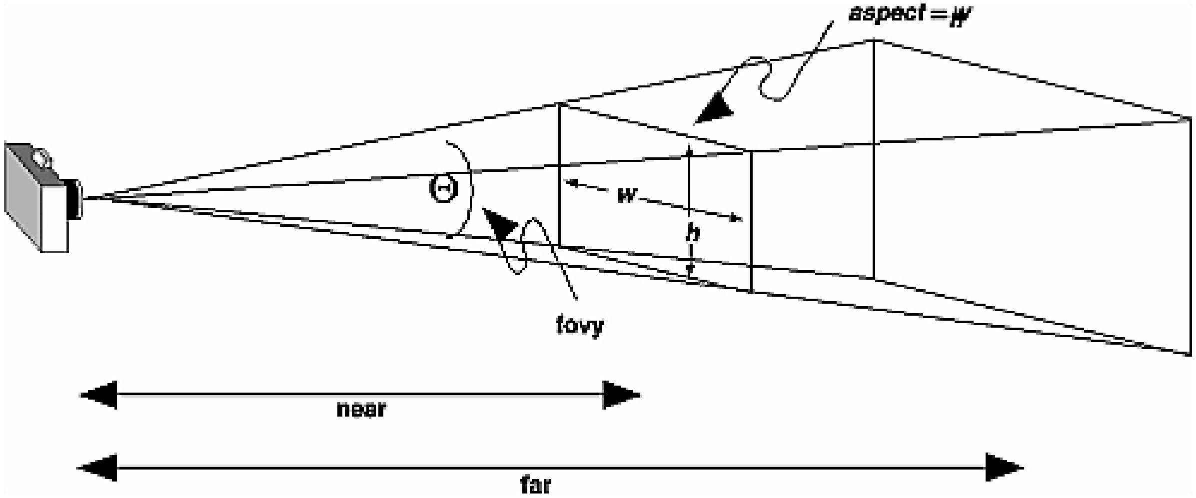
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} fX/Z \\ fY/Z \end{pmatrix} \Leftarrow \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Projection Transformations



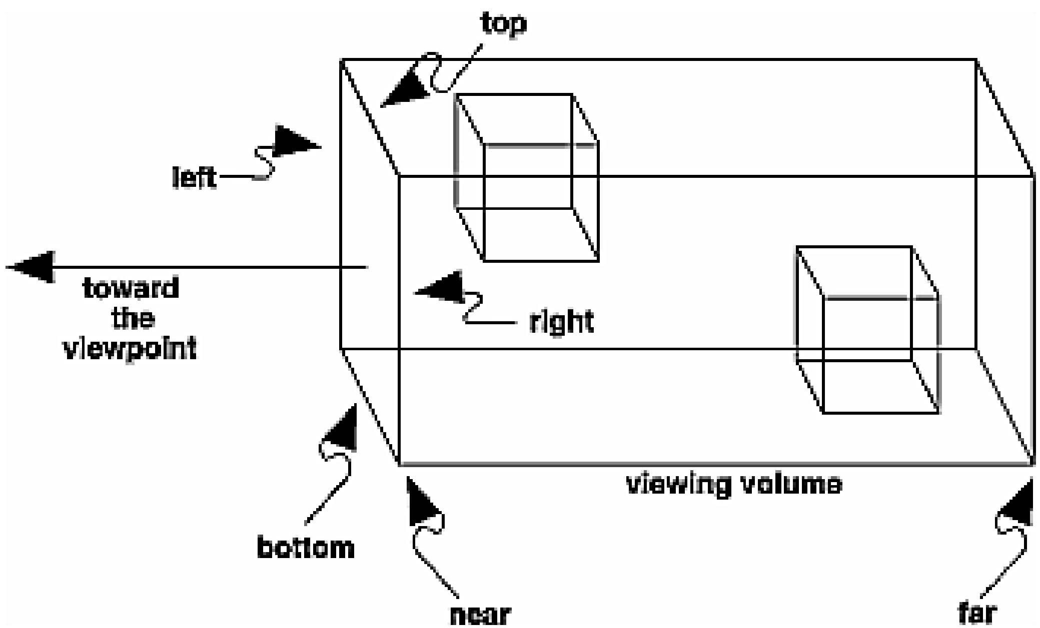

```
void glFrustum(GLdouble left, GLdouble right, GLdouble
  bottom, GLdouble top, GLdouble near, GLdouble far);
```

Projection Transformations




```
void gluPerspective(GLdouble fovy, GLdouble aspect, GLdouble
   near, GLdouble far);
```

Projection Transformations




```
void glOrtho(GLdouble left, GLdouble right, GLdouble
  bottom,
  GLdouble top, GLdouble near, GLdouble far);

void gluOrtho2D(GLdouble left, GLdouble right,
  GLdouble bottom, GLdouble top);
```

Computer Graphics Pipeline

Geometry

↓

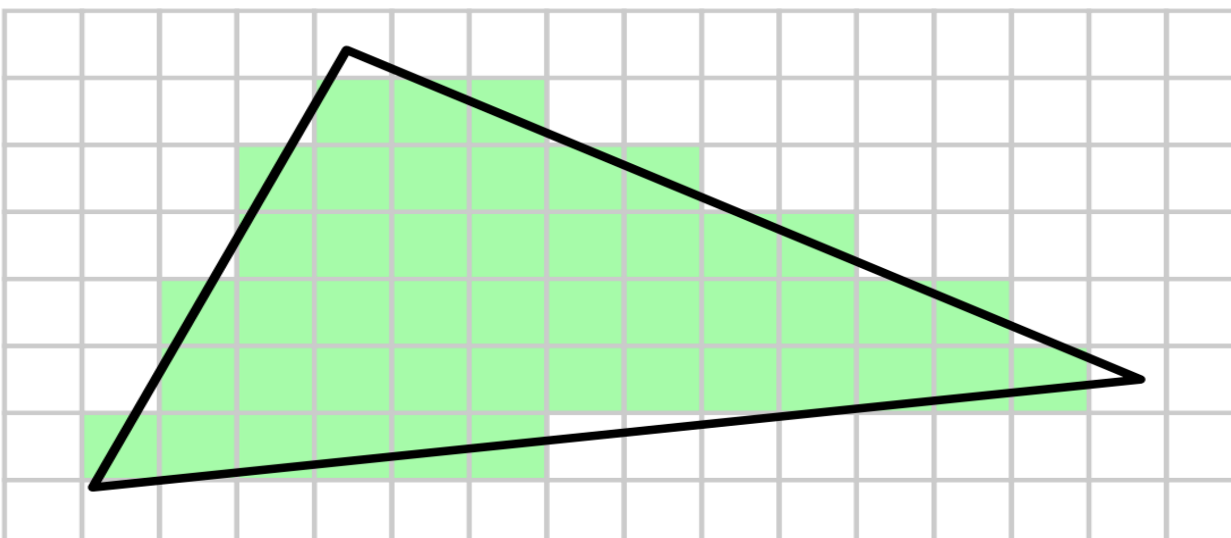
Stage	Description
Modeling Transformation	Transform into 3D *world* coordinate system
Lighting	Simulate illumination and reflectance
Viewing Transformation	Transform into 3D *camera* coordinate system
Clipping	Clip primitives outside camera's view
Projection	Transform into 2D camera coordinate system
Scan Conversion	Draw pixels (incl. texturing, hidden surface…)

↓

Image

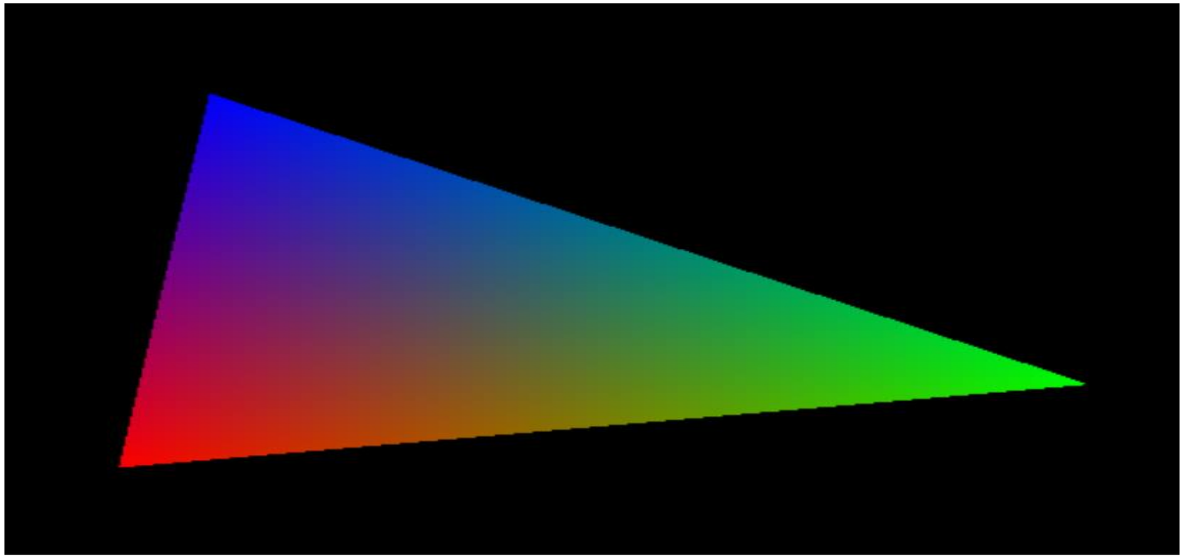
Scan Conversion/Rasterization

- Determine which fragments get generated
- Interpolate parameters (colors, textures, normals, etc.)



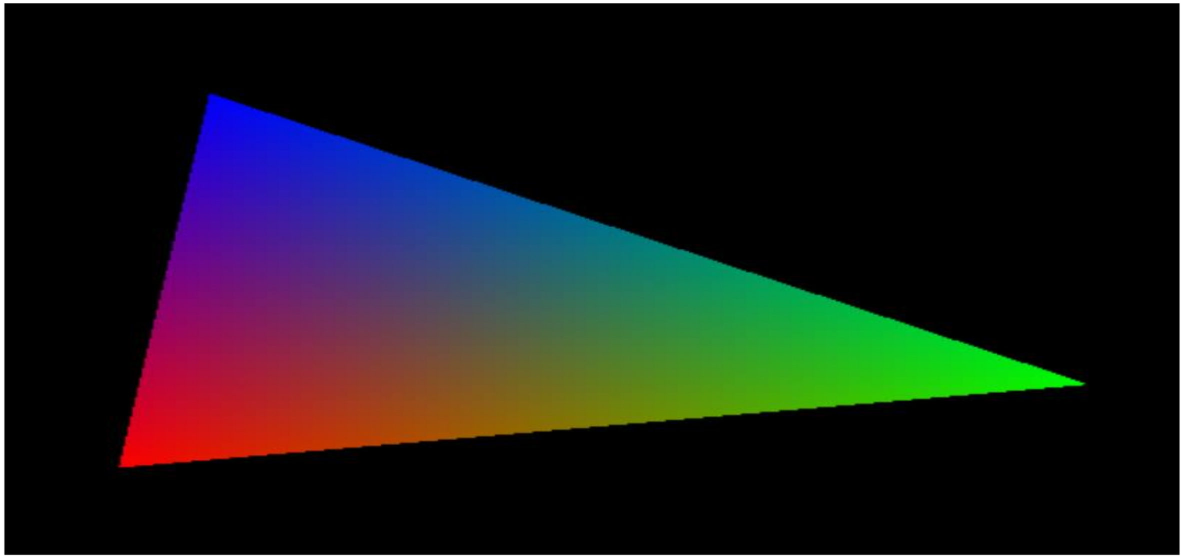
Scan Conversion/Rasterization

- Determine which fragments get generated
- Interpolate parameters (colors, textures, normals, etc.)



Scan Conversion/Rasterization

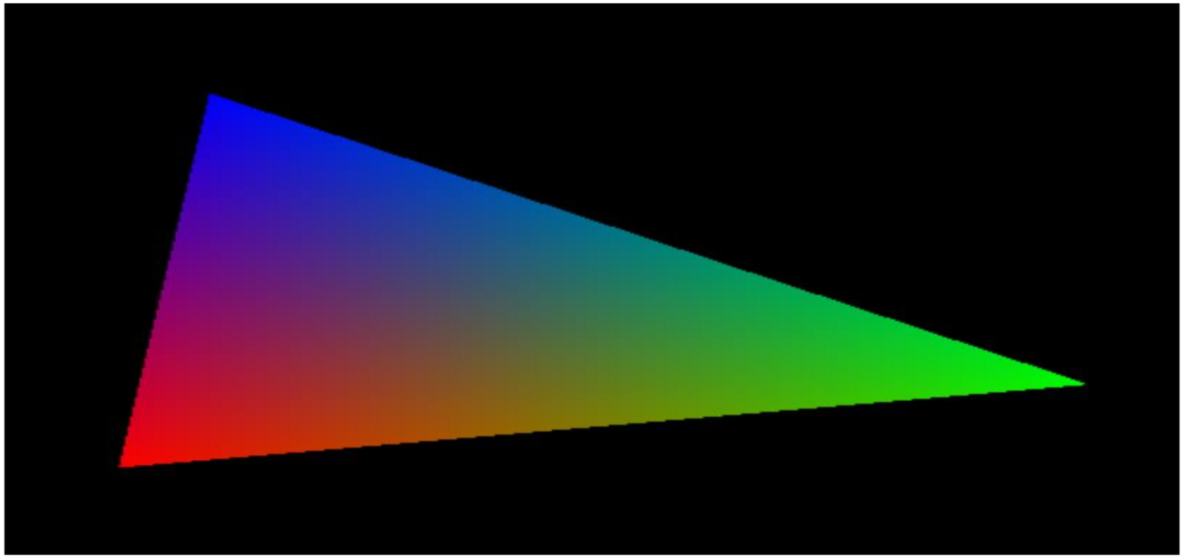
- Determine which fragments get generated
- Interpolate parameters (colors, textures, normals, etc.)



- How?

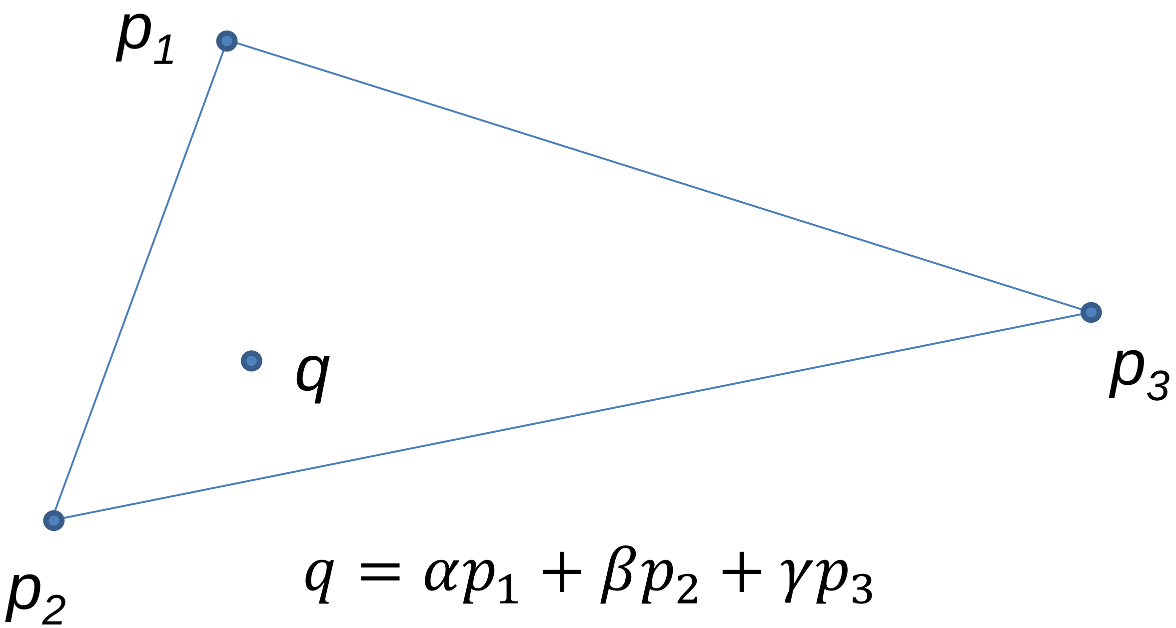
Scan Conversion/Rasterization

- Determine which fragments get generated
- Interpolate parameters (colors, textures, normals, etc.)



- Barycentric coords amongst many other ways…

Barycentric coordinates

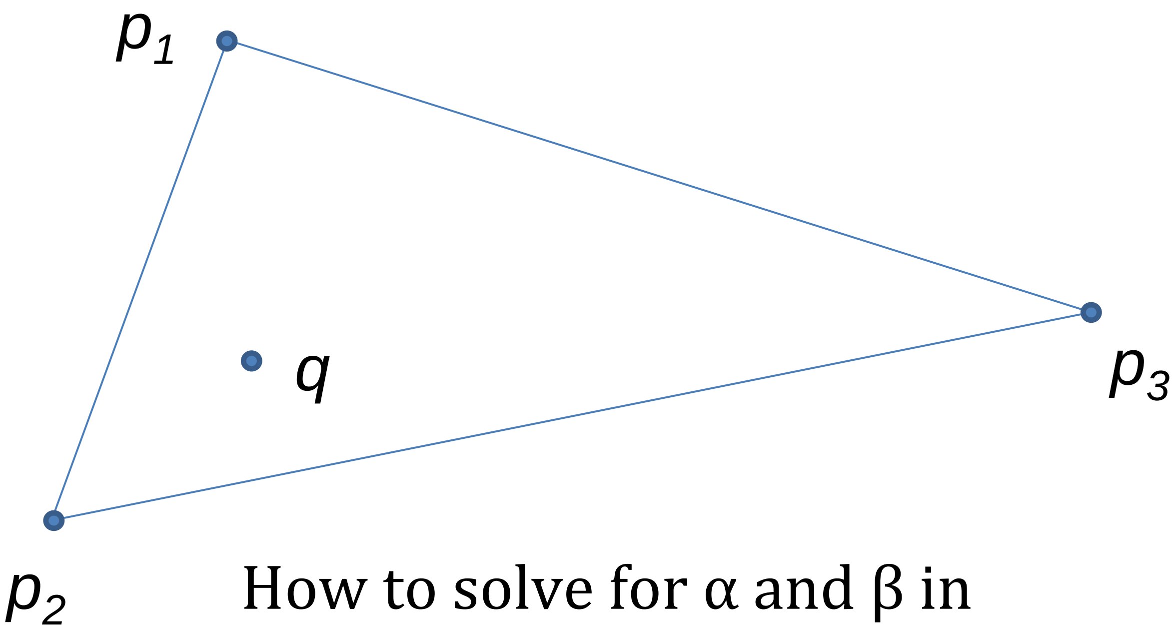


$$q = \alpha p_1 + \beta p_2 + \gamma p_3$$

If $[\alpha + \beta + \gamma = 1 \;\; and \;\; \{\alpha, \beta, \gamma\} \geq 0]$,
then q inside triangle (p_1, p_2, p_3)

Can also write:
$$q = \alpha p_1 + \beta p_2 + (1 - \alpha - \beta) p_3$$

Barycentric coordinates



How to solve for α and β in

$$q = \alpha p_1 + \beta p_2 + (1 - \alpha - \beta) p_3?$$

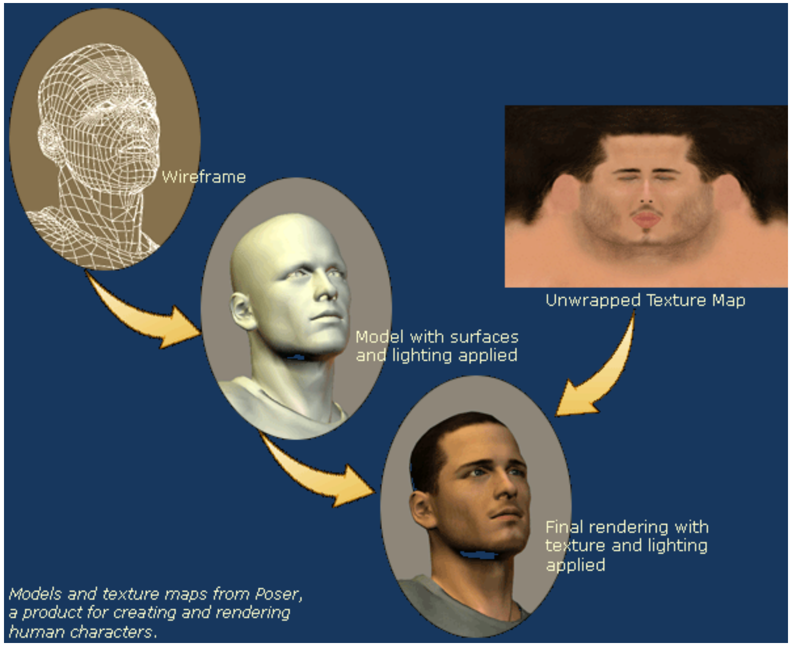
Two equations, two unknowns:
use 2x2 matrix inversion…

Additional concept: Texture mapping

- Model surface-detail with images
  - wrap object with photograph(s)
  - graphics object itself is a simpler model but "looks" more complex

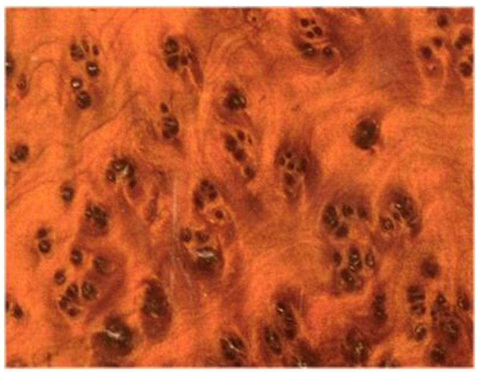
Texture mapping

- Model surface-detail with images
  - wrap object with photograph(s)
  - graphics object itself is a simpler model but "looks" more complex



Texture mapping

bark



veneer

bricks

- Generic image to represent material
  - e.g., tile pattern

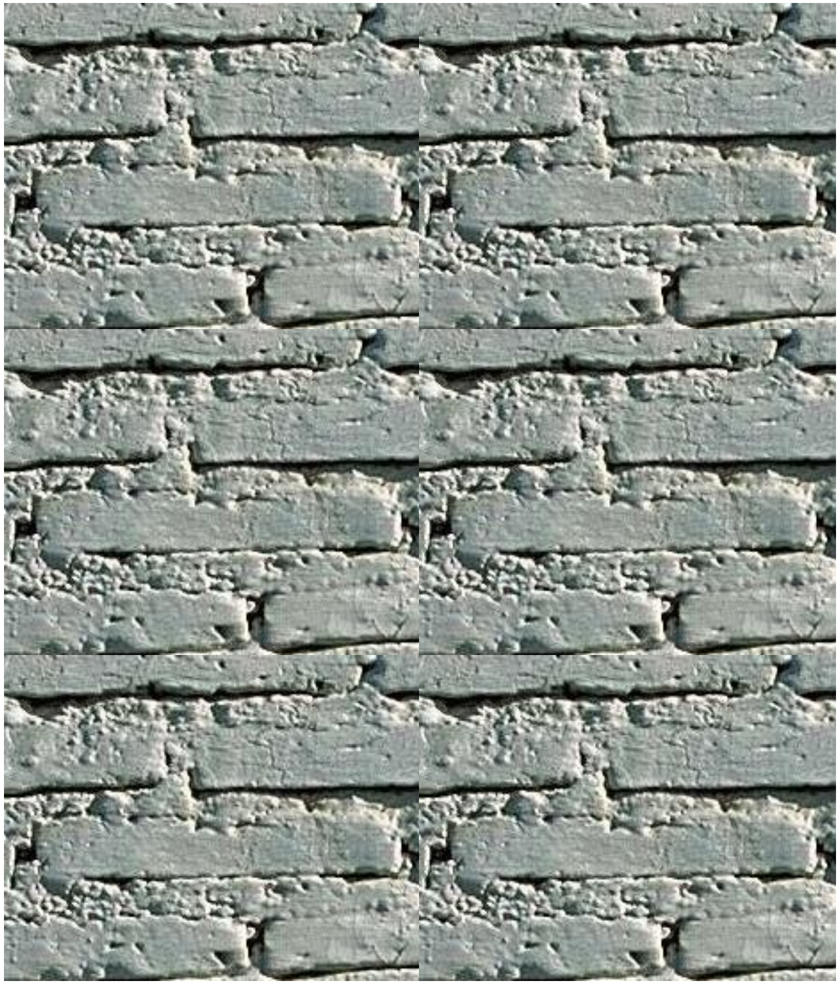
Tiling

- Repeat pattern

Tiling

- Repeat pattern

Tiling



- Repeat pattern
- How can we improve?

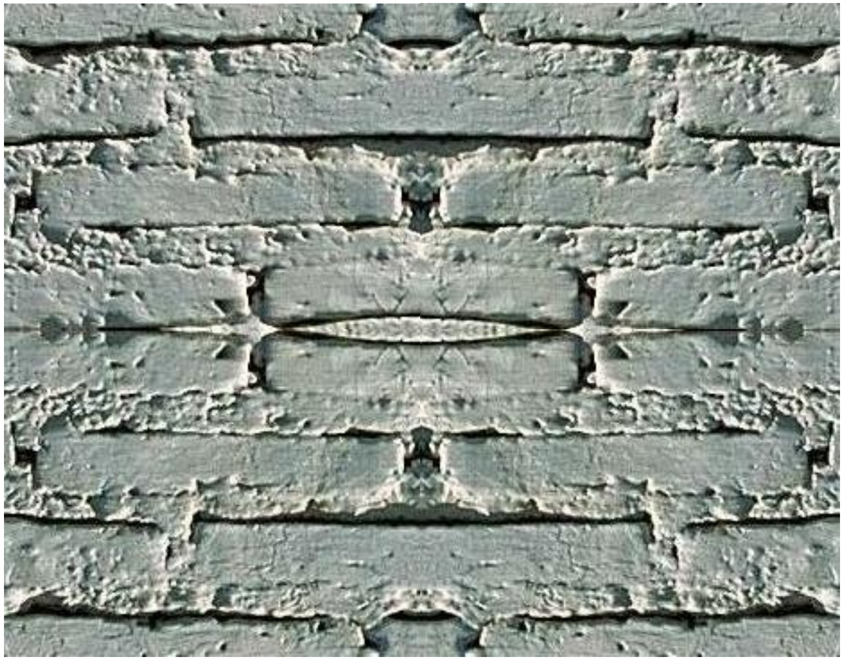
Tiling

- Repeat pattern
  - reduce seems by mirroring

Tiling

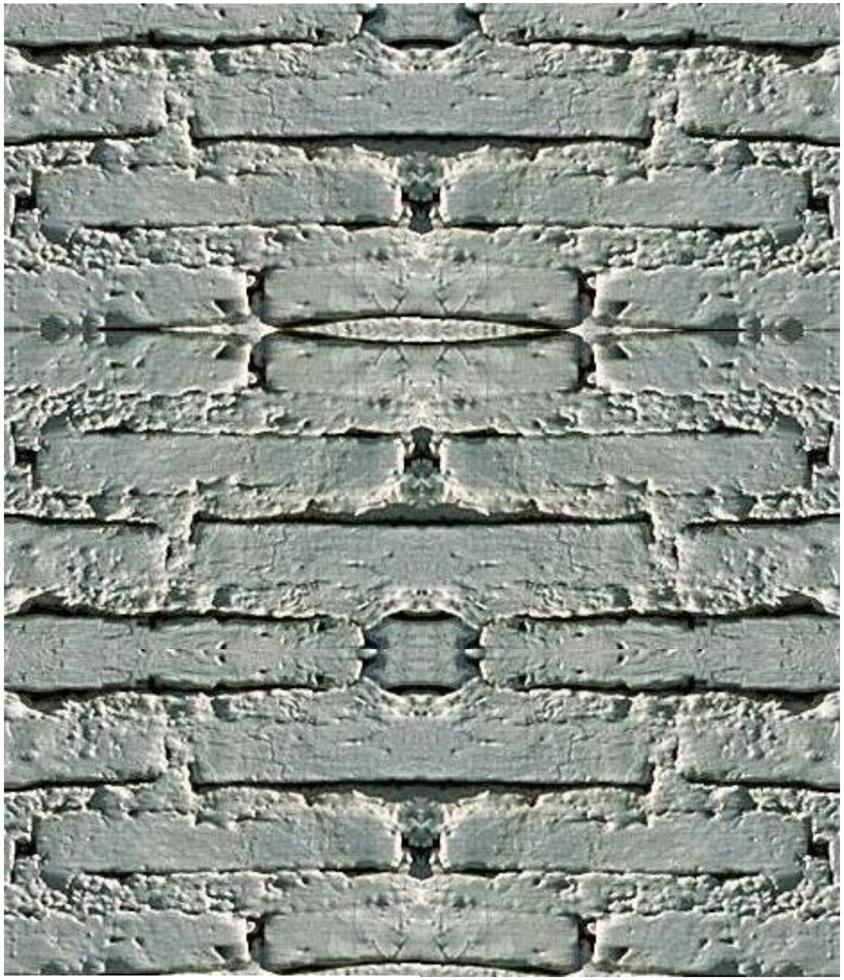
- Repeat pattern
  - reduce seems by mirroring

Tiling



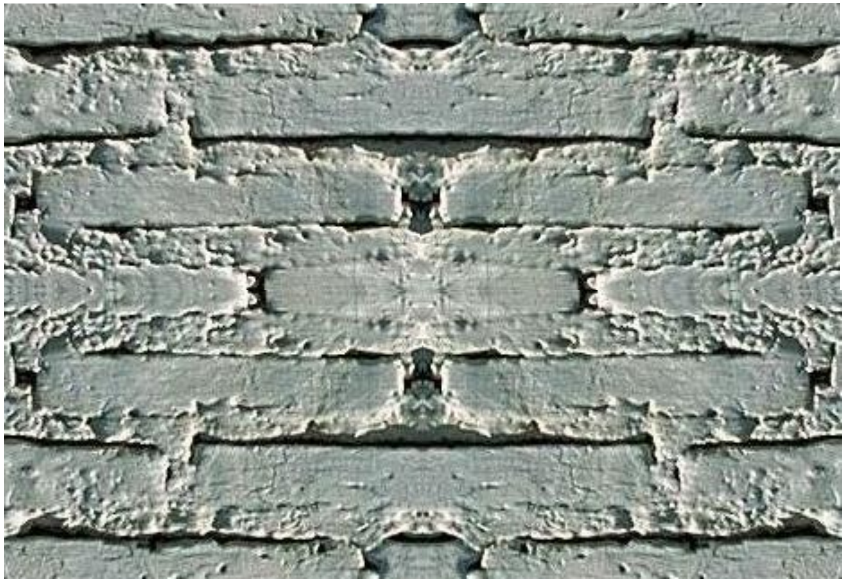
- Repeat pattern
  - reduce seems by mirroring

Tiling



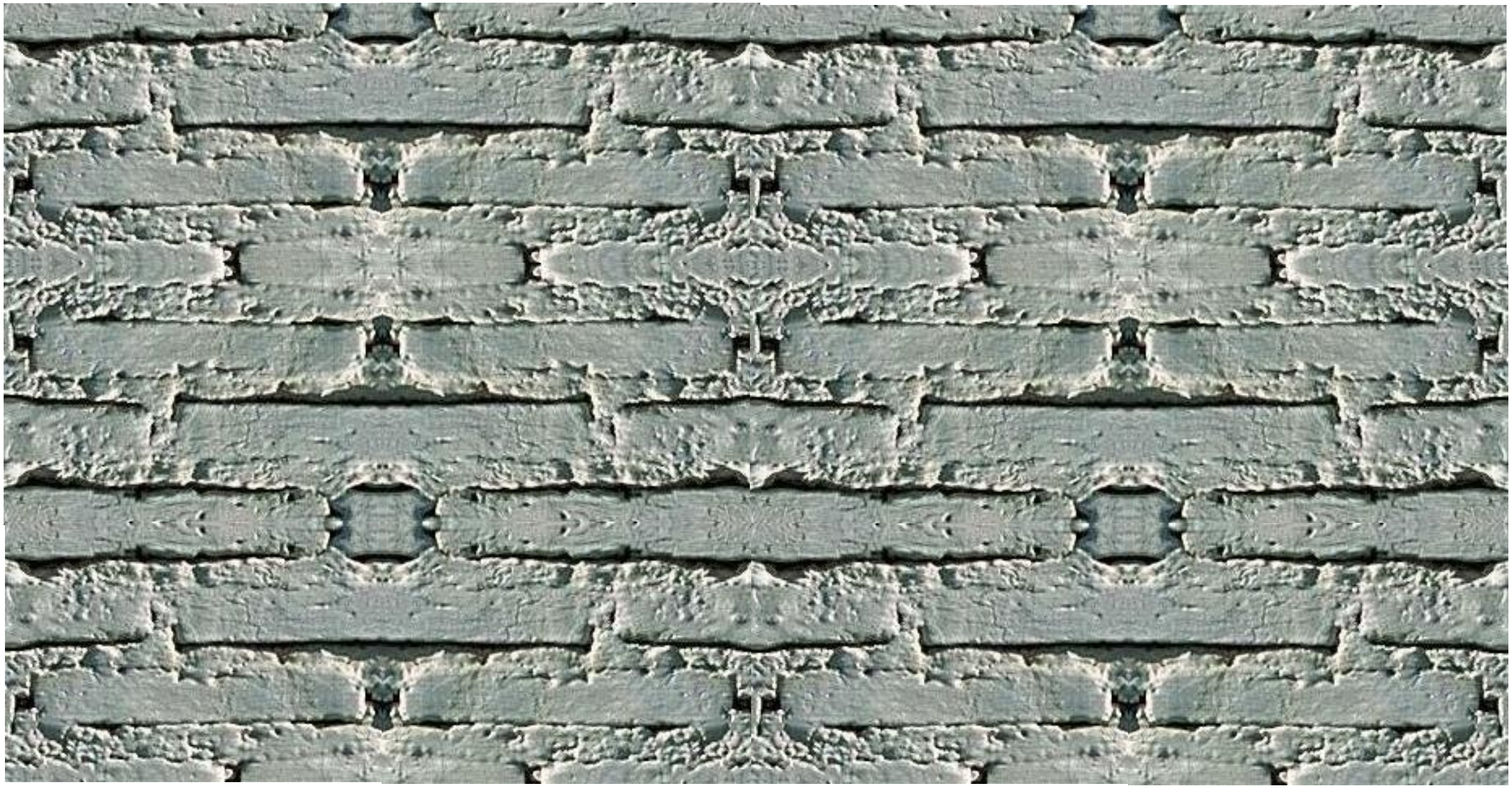
- Repeat pattern
  - reduce seems by mirroring
  - How we can further improve?

Tiling

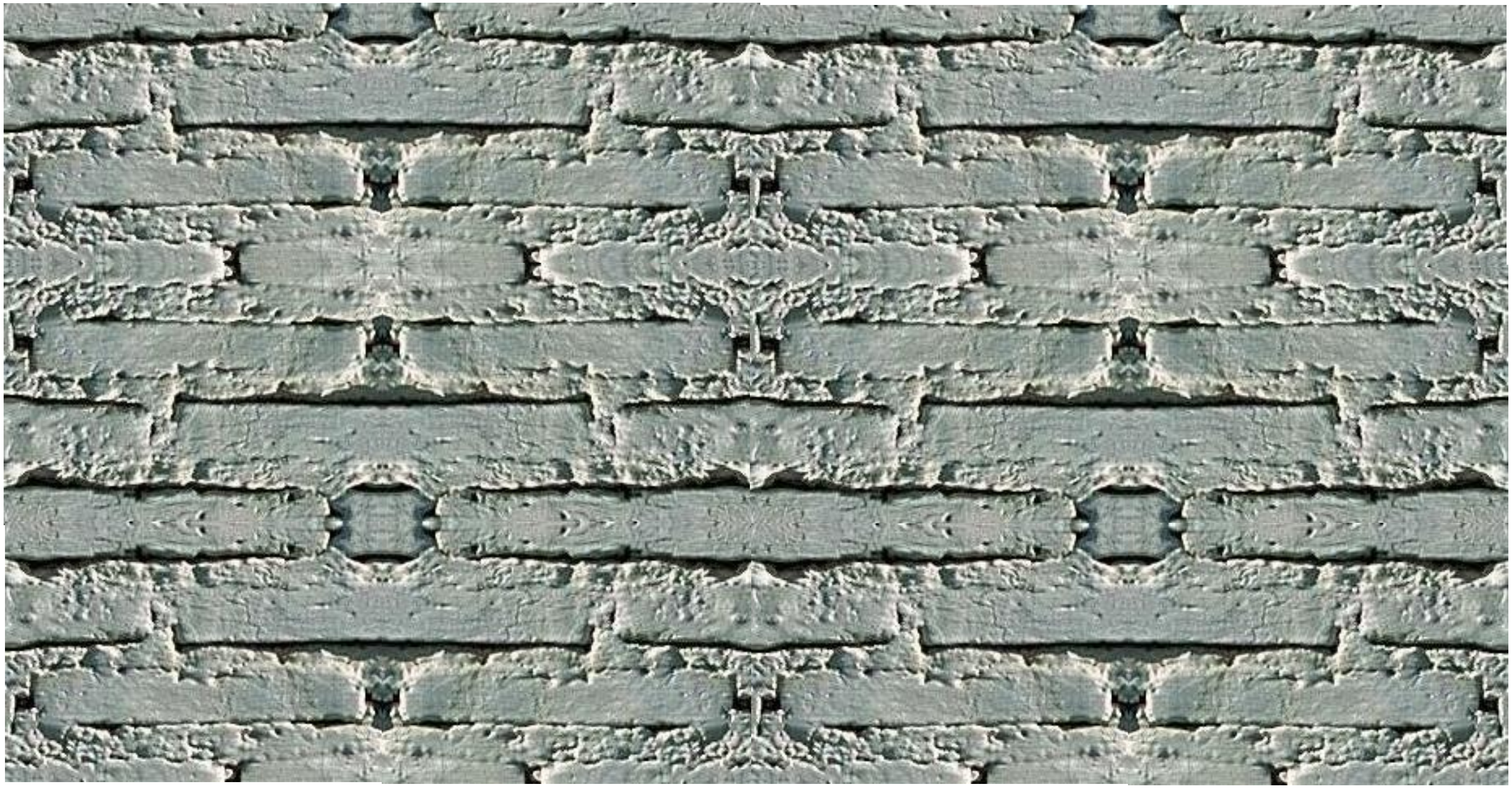


- Repeat pattern
  - reduce seems by mirroring
  - reduce seems by choosing tile that covers one period of repeated texture

Tiling



Texture mapping limitations do exist…



Bricks are similar not identical

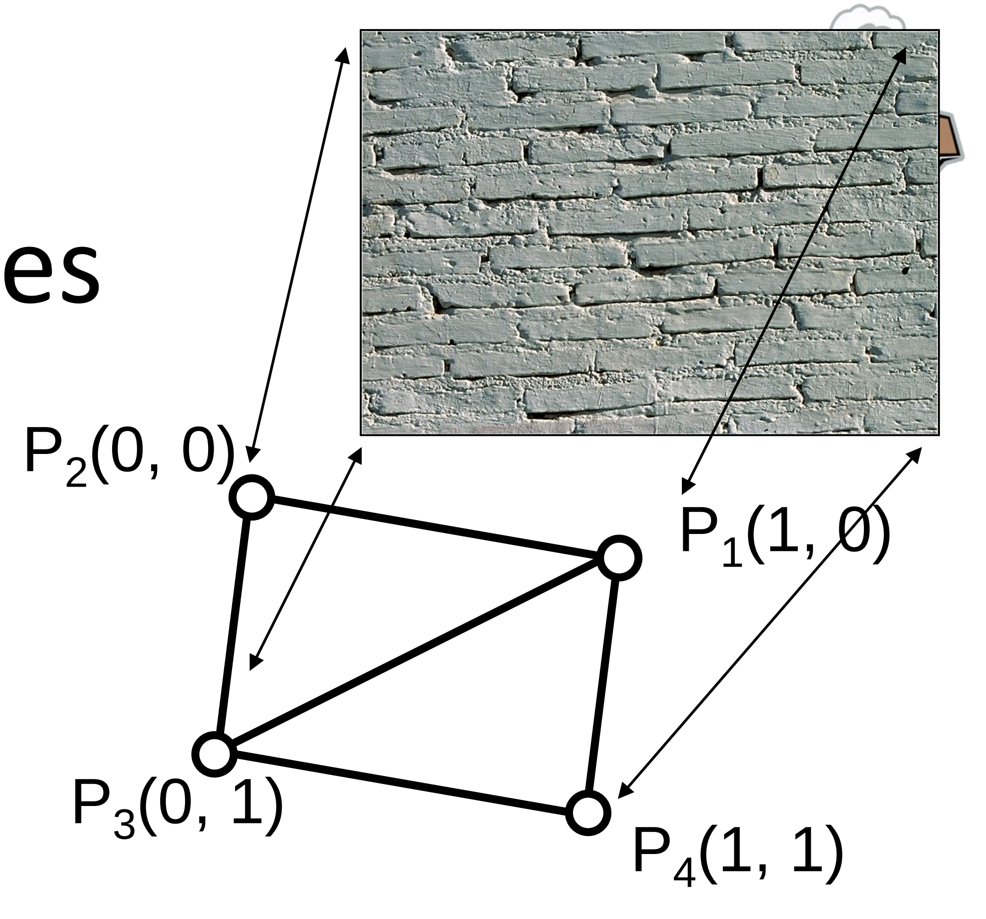
Solution?

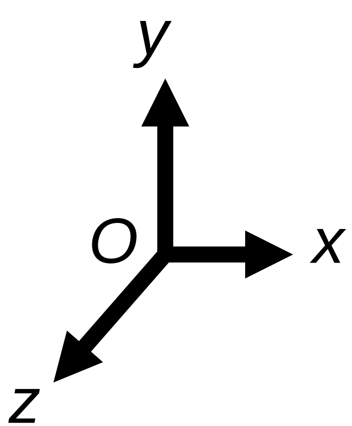
Solution: Texture synthesis…

Texture coordinates

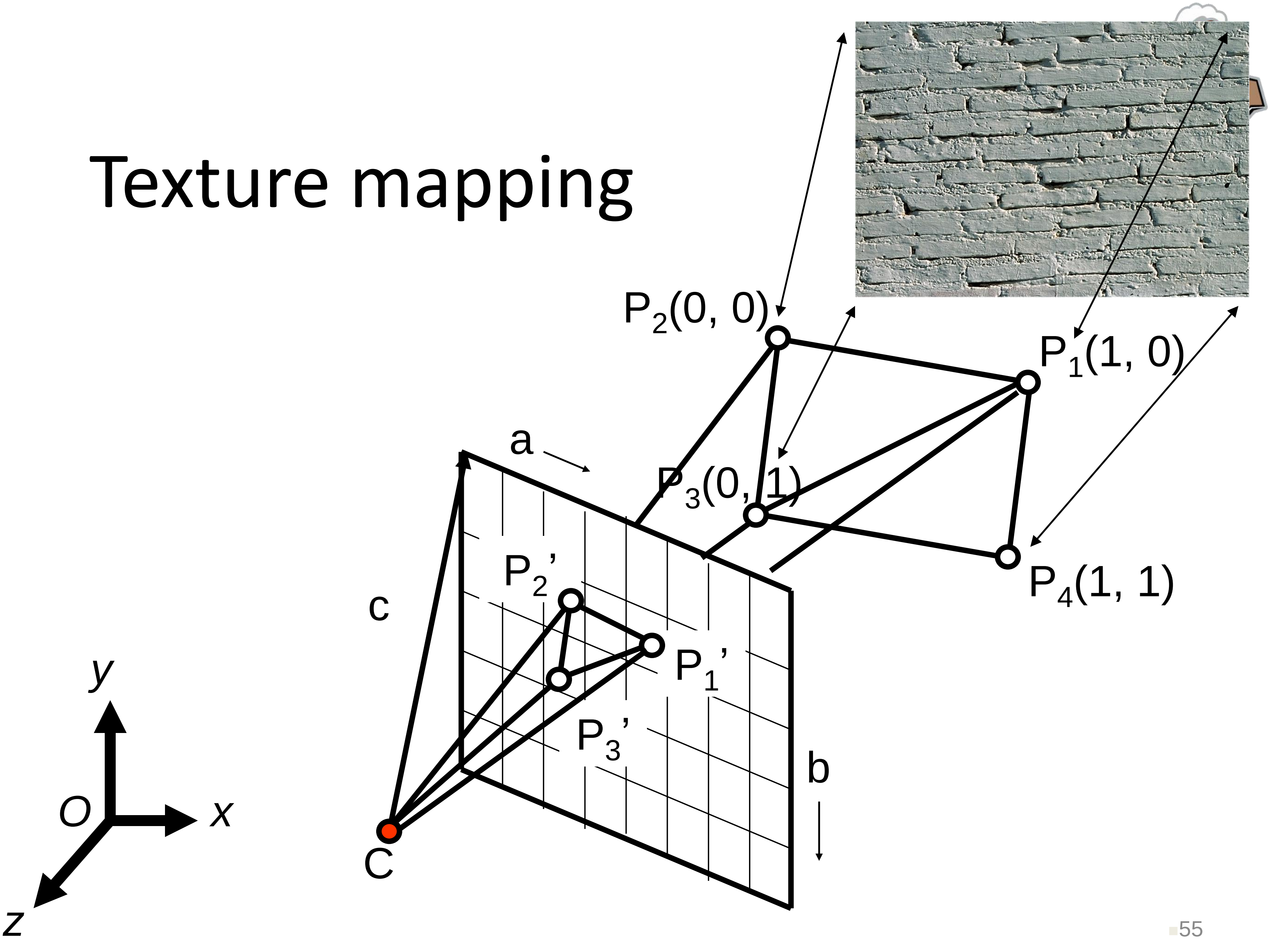
- Mechanism for attaching the texture map to the surface modeled
  - a pair of floats (s, t) for each triangle vertex
  - corners of the image are (0, 0), (0, 1), (1, 1), and (1, 0)
  - tiling indicated with tex. coords. > 1
  - *texels* – color samples in texture maps

Texture coordinates

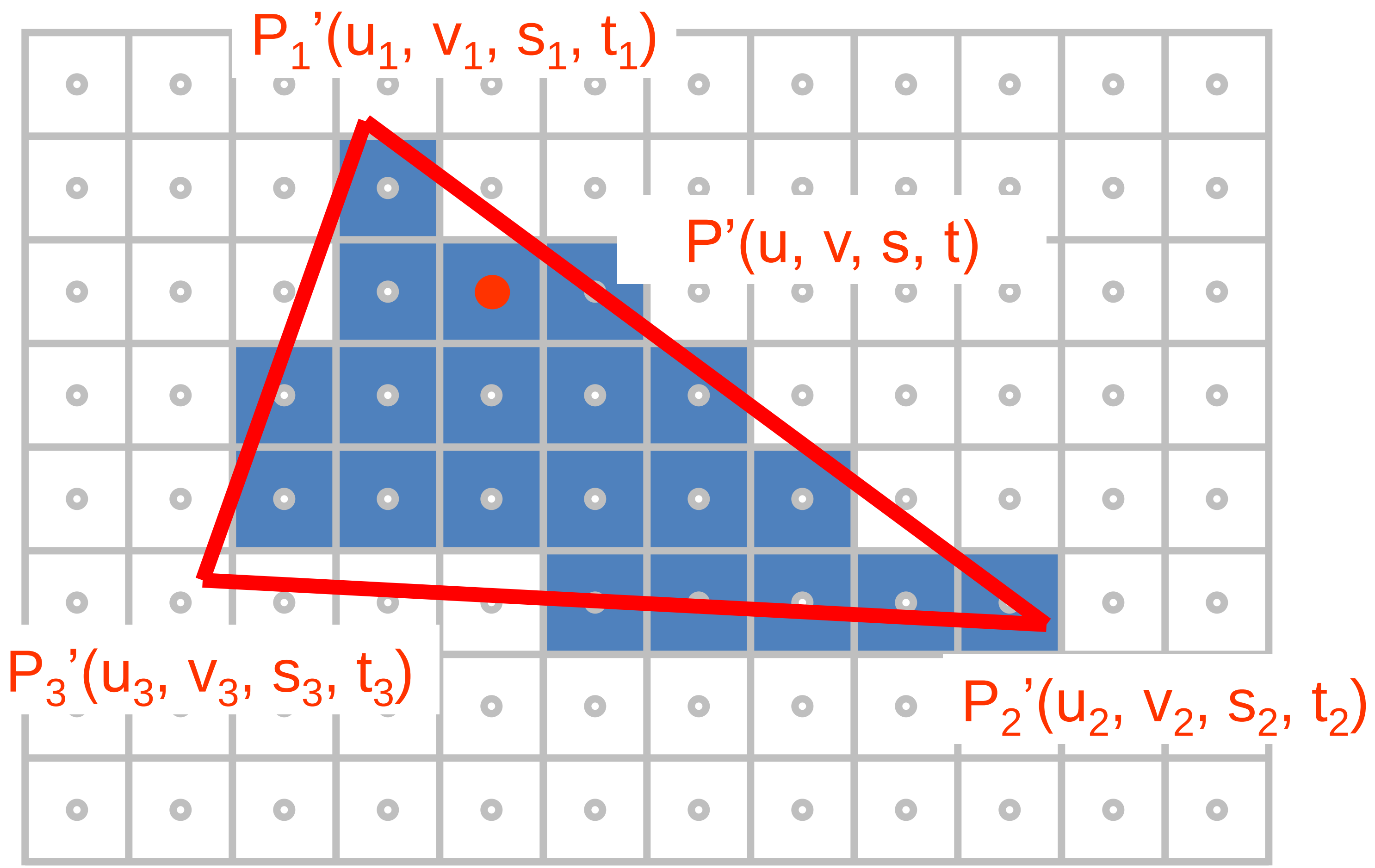




Texture mapping

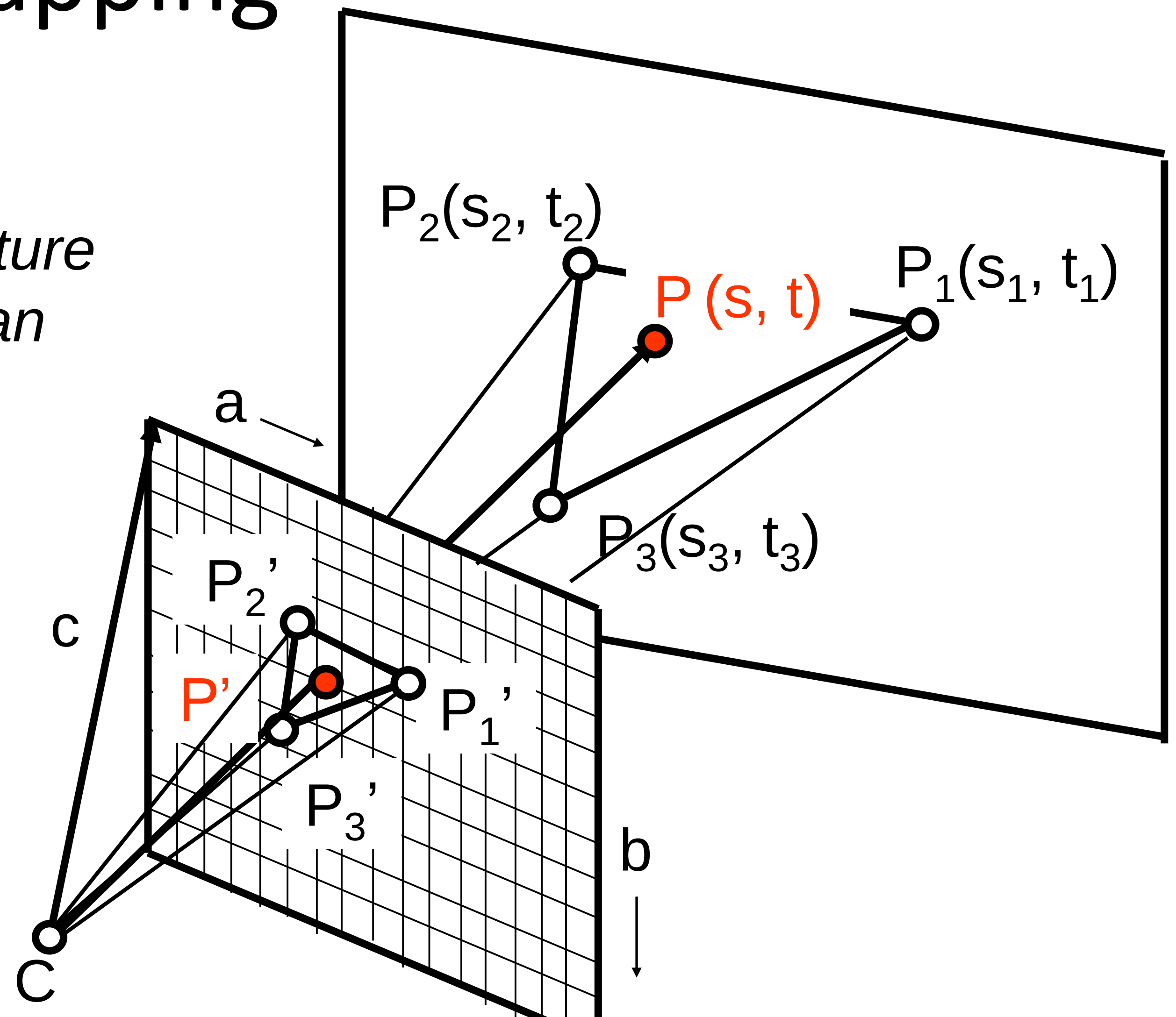


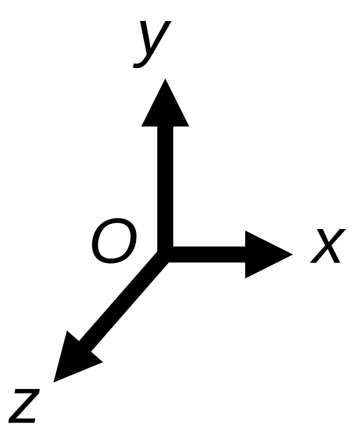
Texels: texture elements



Texture mapping

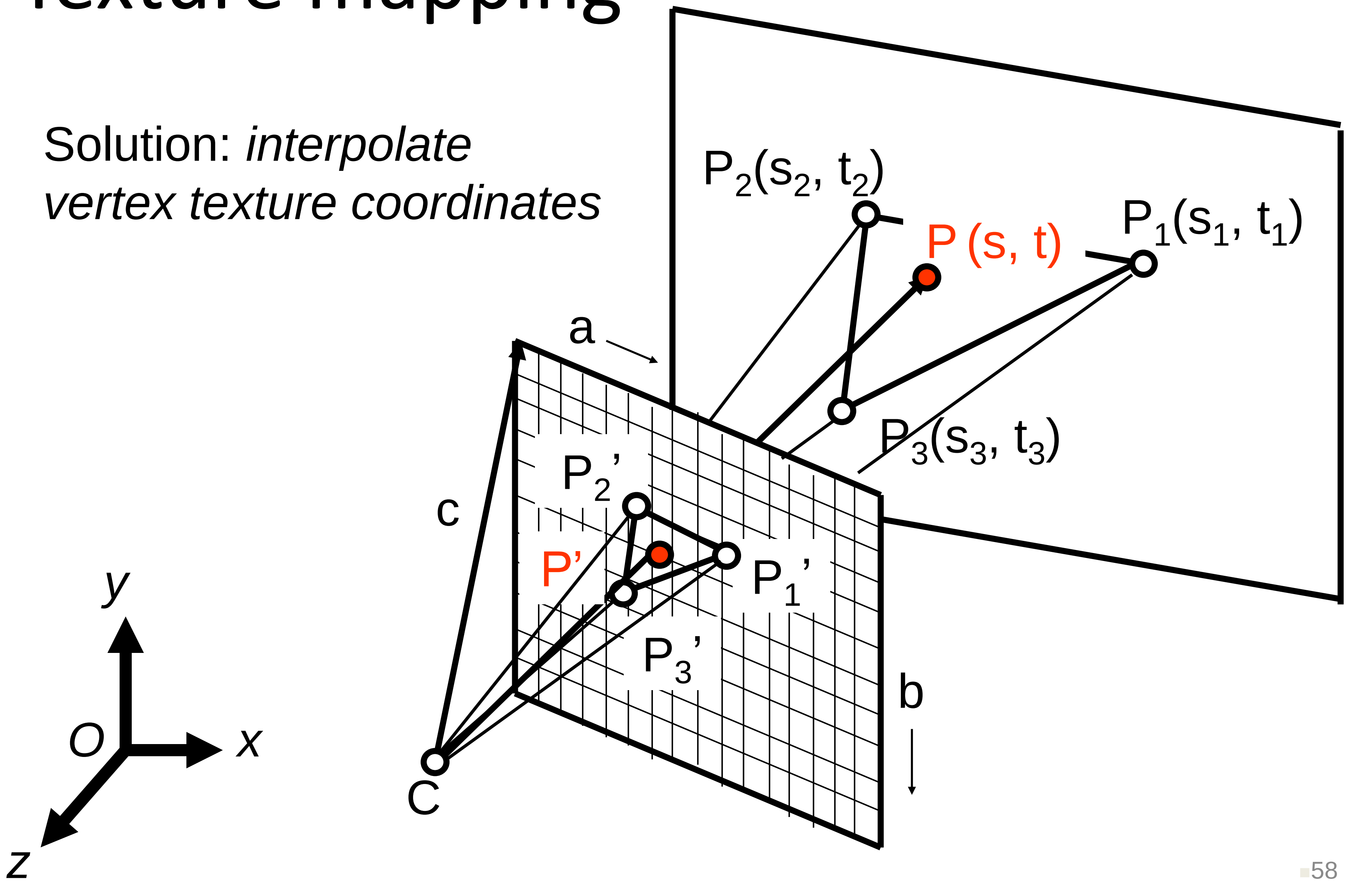
Problem*: how to compute the texture coordinates for an interior pixel?*





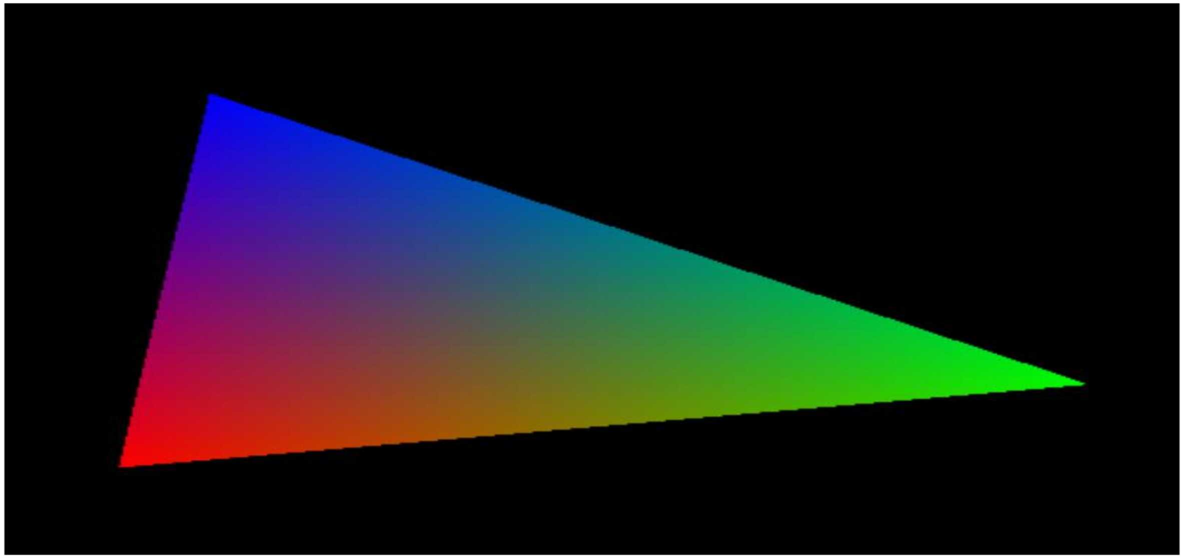
Texture mapping

Solution: *interpolate vertex texture coordinates*



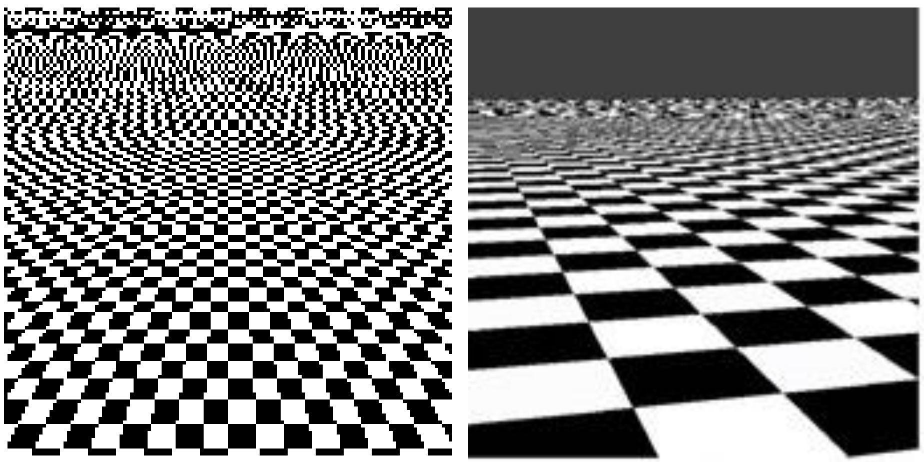
Parameter Interpolation

- Texture coordinates, colors, normals, etc.

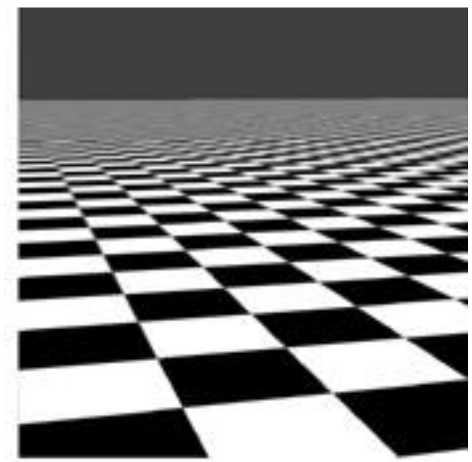


- How?
  - Again, use barycentric coordinates…

Level of detail problem



aliasing



anti-aliased

If curious, you can read more on this subject!