Image/View Morphing and Warping

CS334

Daniel G. Aliaga
Department of Computer Science
Purdue University

Motivation – Rendering from Images

- Given
  - left image
  - right image
- Create intermediate images
  - simulates camera movement

Related Work

- Panoramas (e.g., QuicktimeVR, etc)
  – user can look in any direction at few given locations but camera translations are ***not*** allowed…

Topics

- Image morphing (2D)
- View morphing (2D+)
- Image warping (3D)

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Image Morphing

Image Morphing

- Identify correspondences between input/output image
- Produce a sequence of images that allow a smooth transition from the input image to the output image

Image Morphing

1. Correspondences

Image Morphing

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Image Morphing

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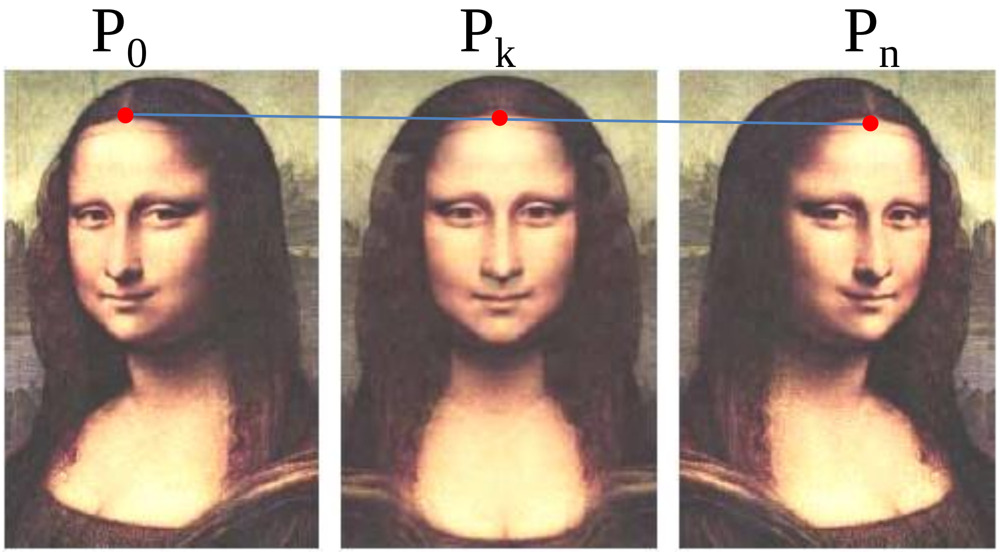
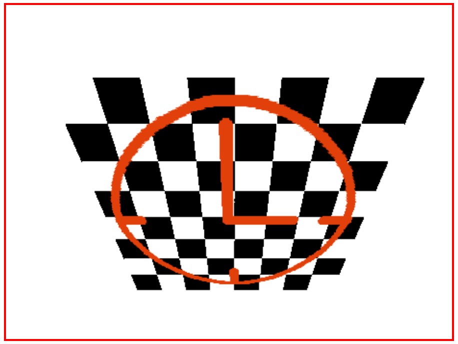
Image Morphing



1. Correspondences
2. Linear interpolation

$$P_k = (1 - \frac{k}{n}) P_0 + \frac{k}{n} P_n$$

Image Morphing



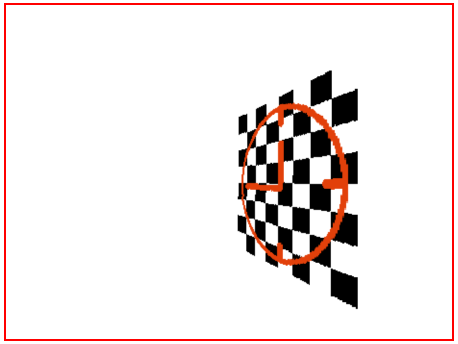
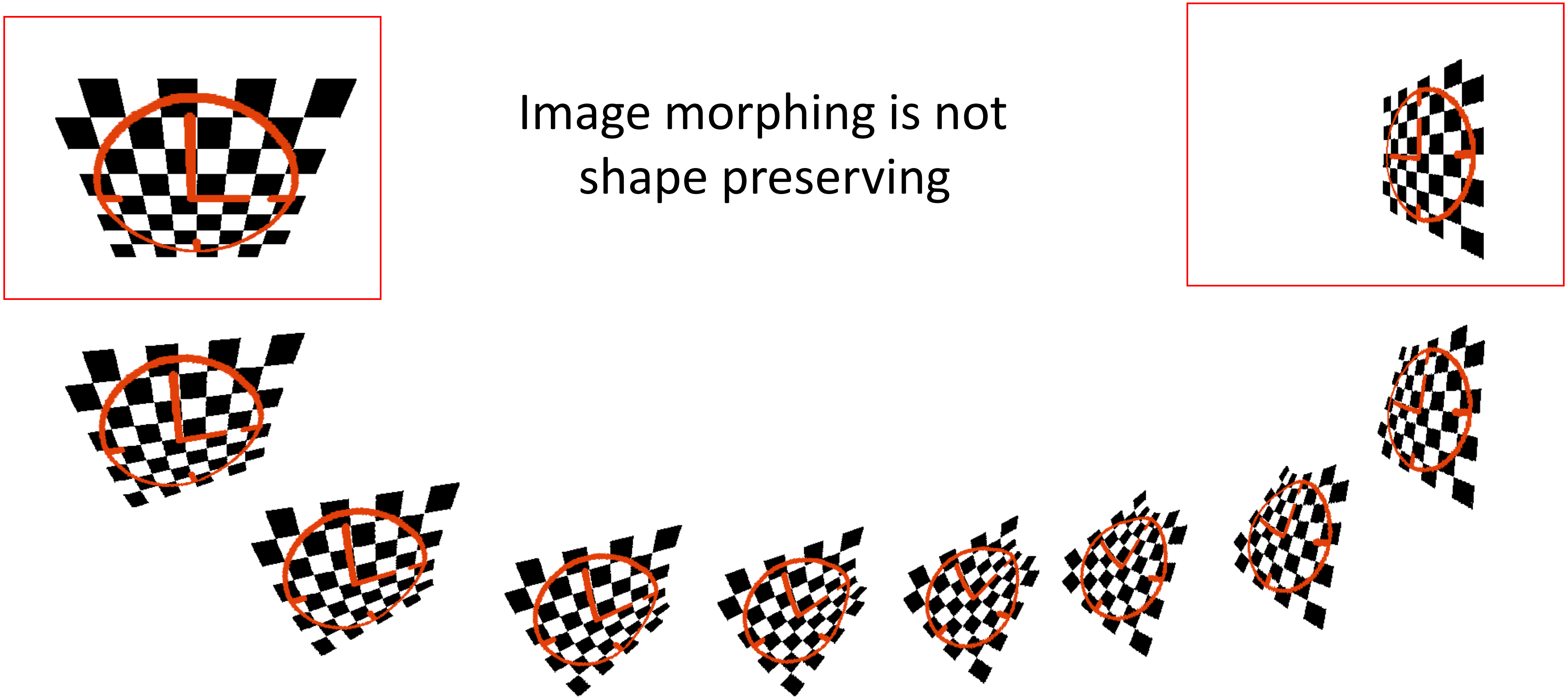


Image Morphing

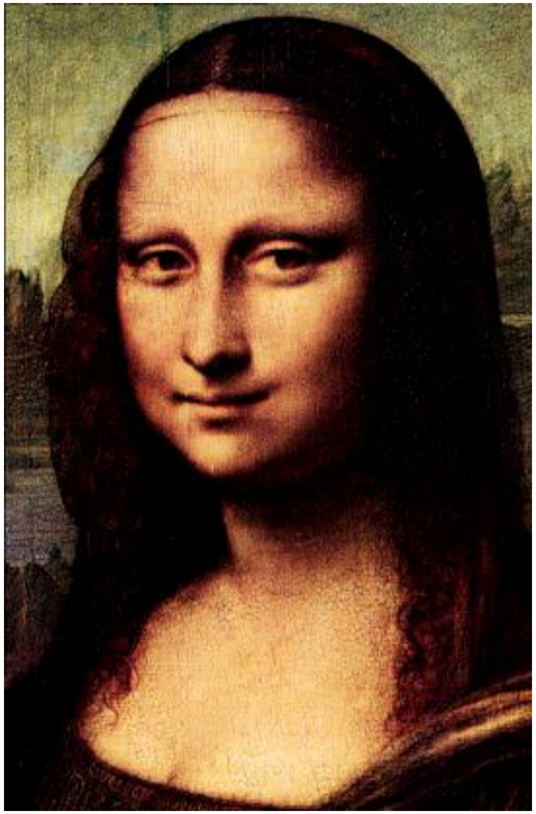
Image morphing is not shape preserving

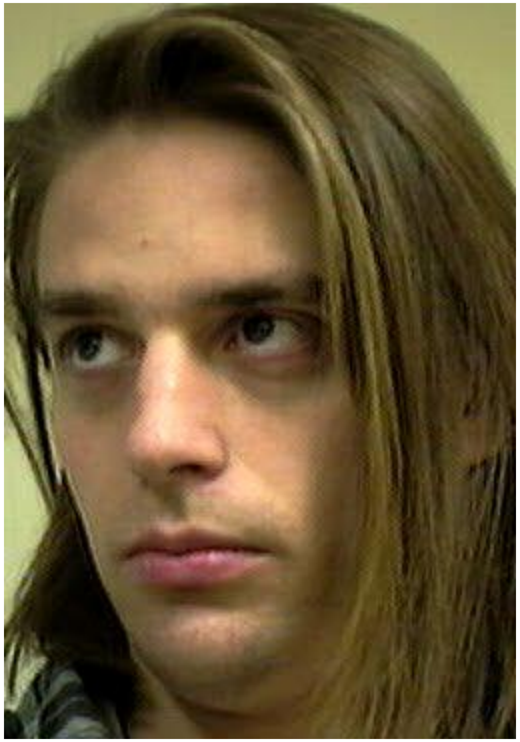


Topics

- Image morphing (2D)
- View morphing (2D+)
- Image warping (3D)

View Morphing

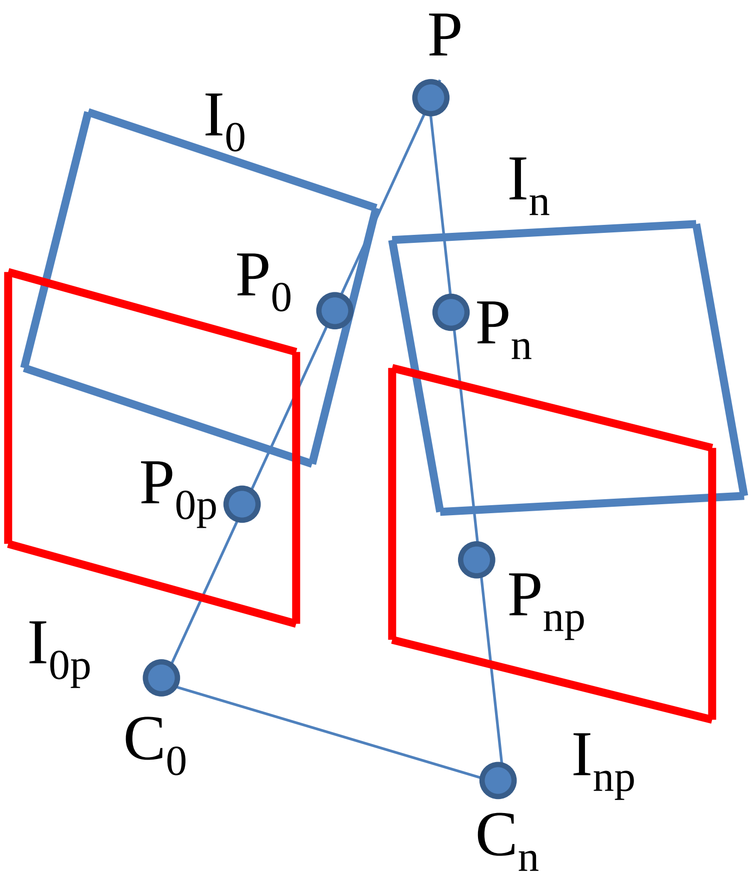




View Morphing

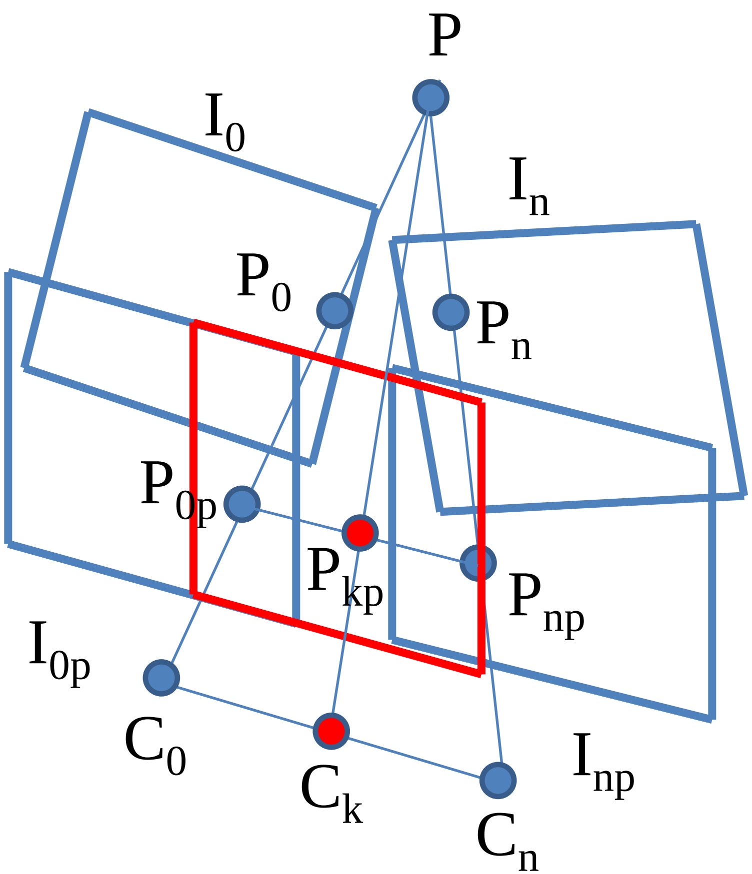
- Shape preserving morph
- Three step algorithm
  - Prewarp first and last images to parallel views
  - Image morph between prewarped images
  - Postwarp to interpolated view

Step 1: prewarp to parallel views



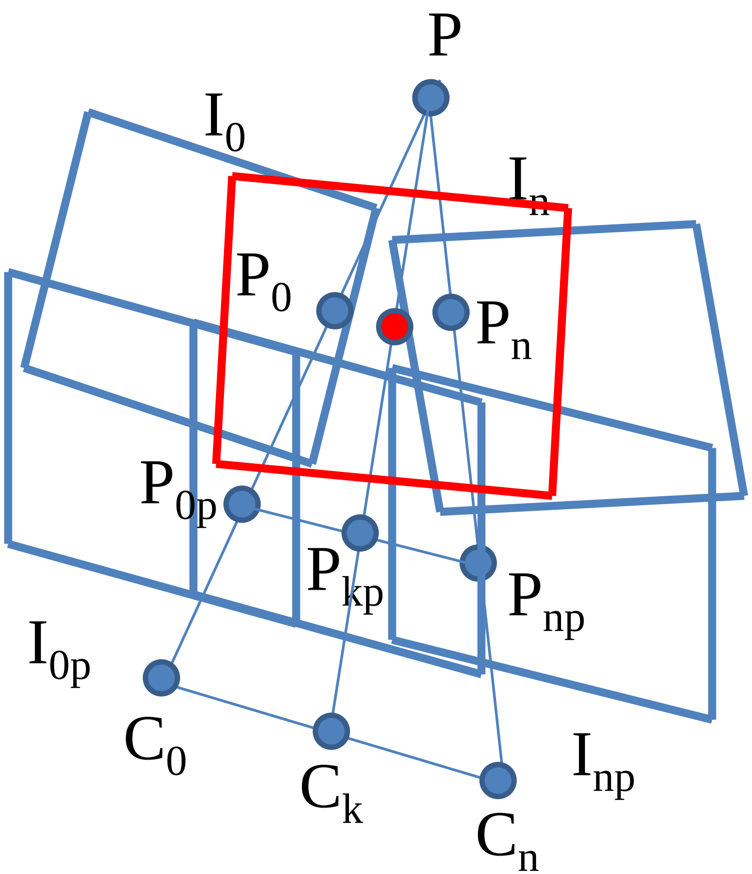
- Parallel views
  - same image plane
  - image plane parallel to segment connecting the two centers of projection
- Prewarp
  - compute parallel views I_{0p}, I_{np}
  - rotate I_0 and I_n to parallel views
  - prewarp correspondence is (P_0, P_n) -> (P_{op}, P_{np})

Step 2: morph parallel images



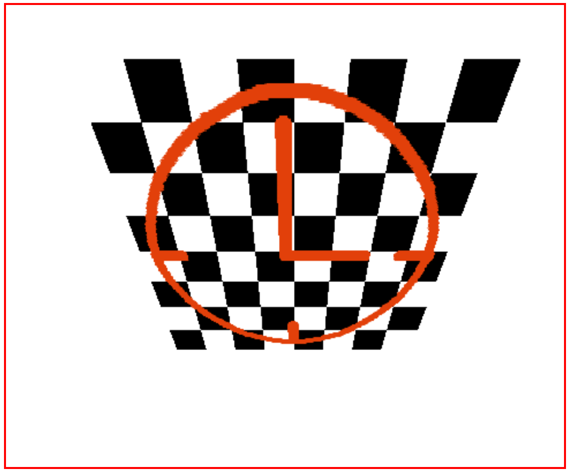
- Shape preserving
- Use prewarped correspondences
- Interpolate C_k from C_0 C_n

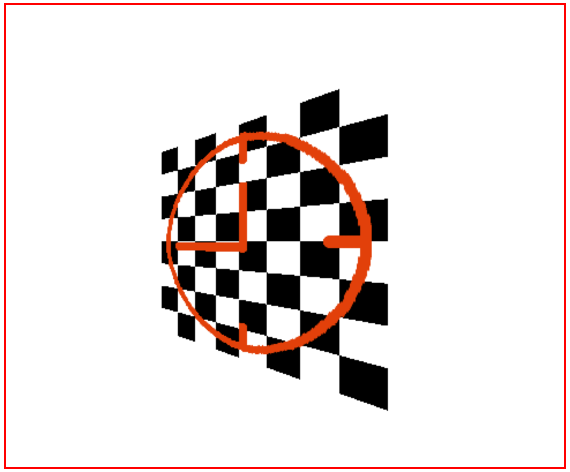
Step 3: postwarp image



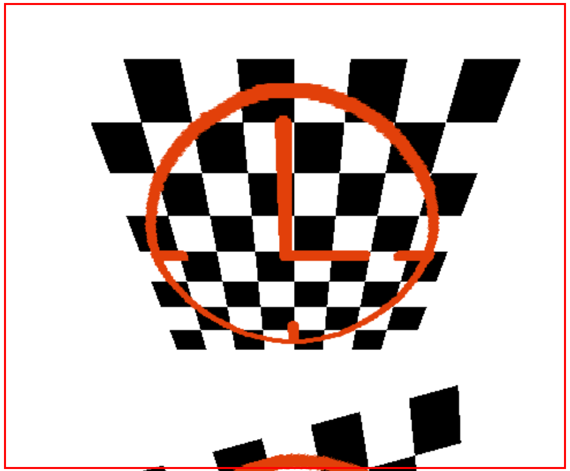
- Postwarp morphed image
  - create intermediate view
    - C_k is known
    - interpolate view direction and tilt
  - rotate morphed image to intermediate view

View morphing

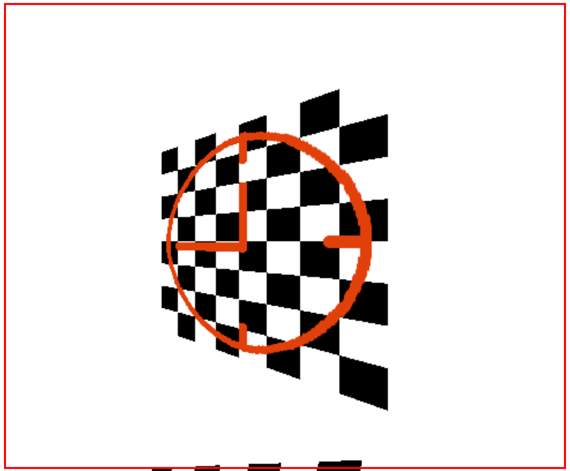


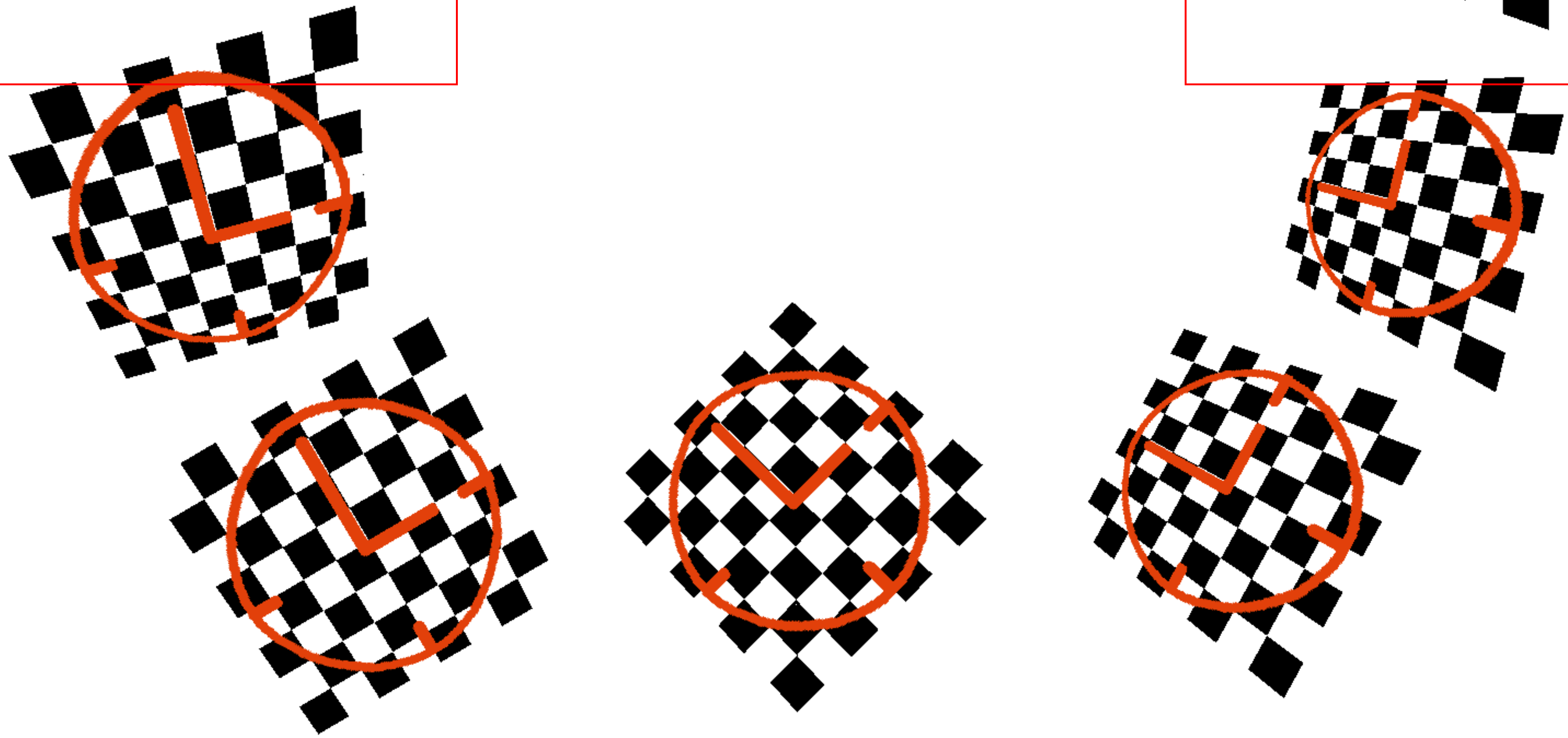


View morphing



- View morphing is shape preserving





View Morphing Examples

- Using computer vision/stereo reconstruction techniques

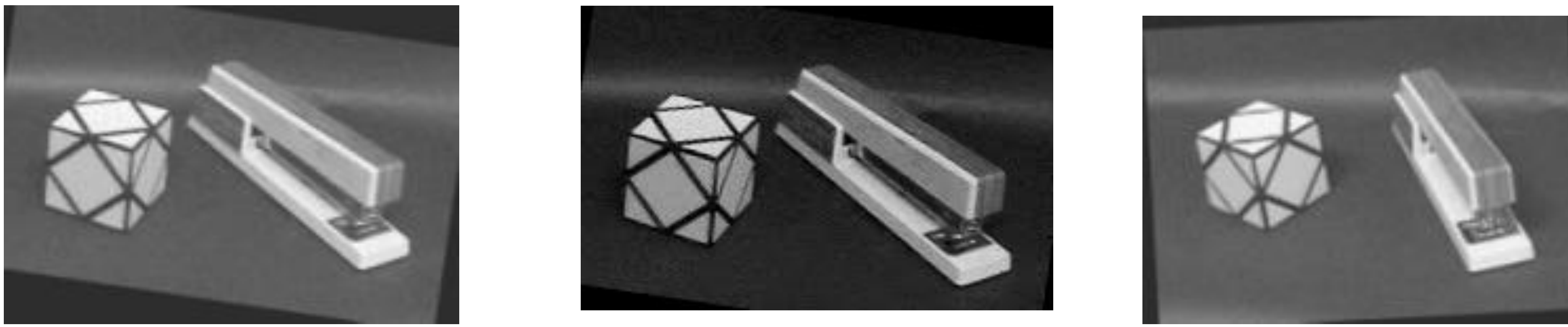
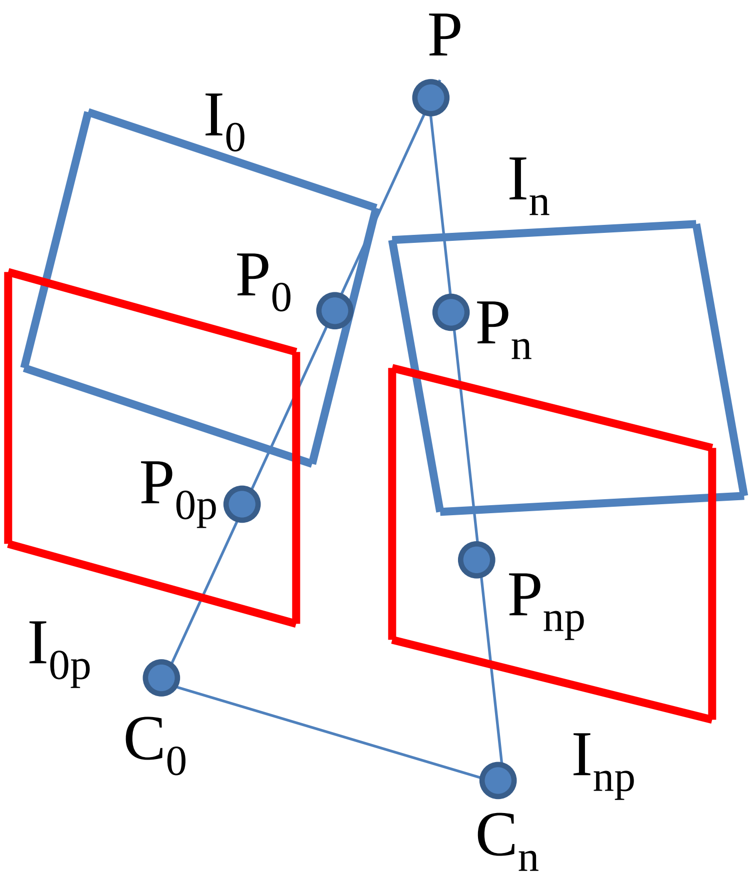


Image Transformations



- Intuitively, how do you compute the matrix M by which to transform P_0 to P_{0p} ?

Image Transformations

- A geometric relationship between input (u,v) and output pixels (x,y)

  – Forward mapping:

    $(x,y) = (X(u,v), Y(u,v))$

  – Inverse mapping:

    $(u,v) = (U(x,y), V(x,y))$

Image Transformations

- General matrix form is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and operates in the “homogeneous coordinate system”.

Affine Transformations

- Matrix form is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and accommodates translations, rotations, scale, and shear.

- How many unknowns? How to create matrix?

Affine Transformations

- Transformation can be inferred from correspondences; e.g.,

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} \Leftrightarrow \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

- Given ≥3 correspondences can solve for T

Perspective/Projective Transformations

- Matrix form is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and it accommodates foreshortening of distant line and convergence of lines to a vanishing point;

also, straight lines are maintained but not their mutual angular relationships, and

only parallel lines parallel to the projection plane remain parallel

Perspective/Projective Transformations

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- How many unknowns?
- How many correspondences are needed?

Direct Linear Transform

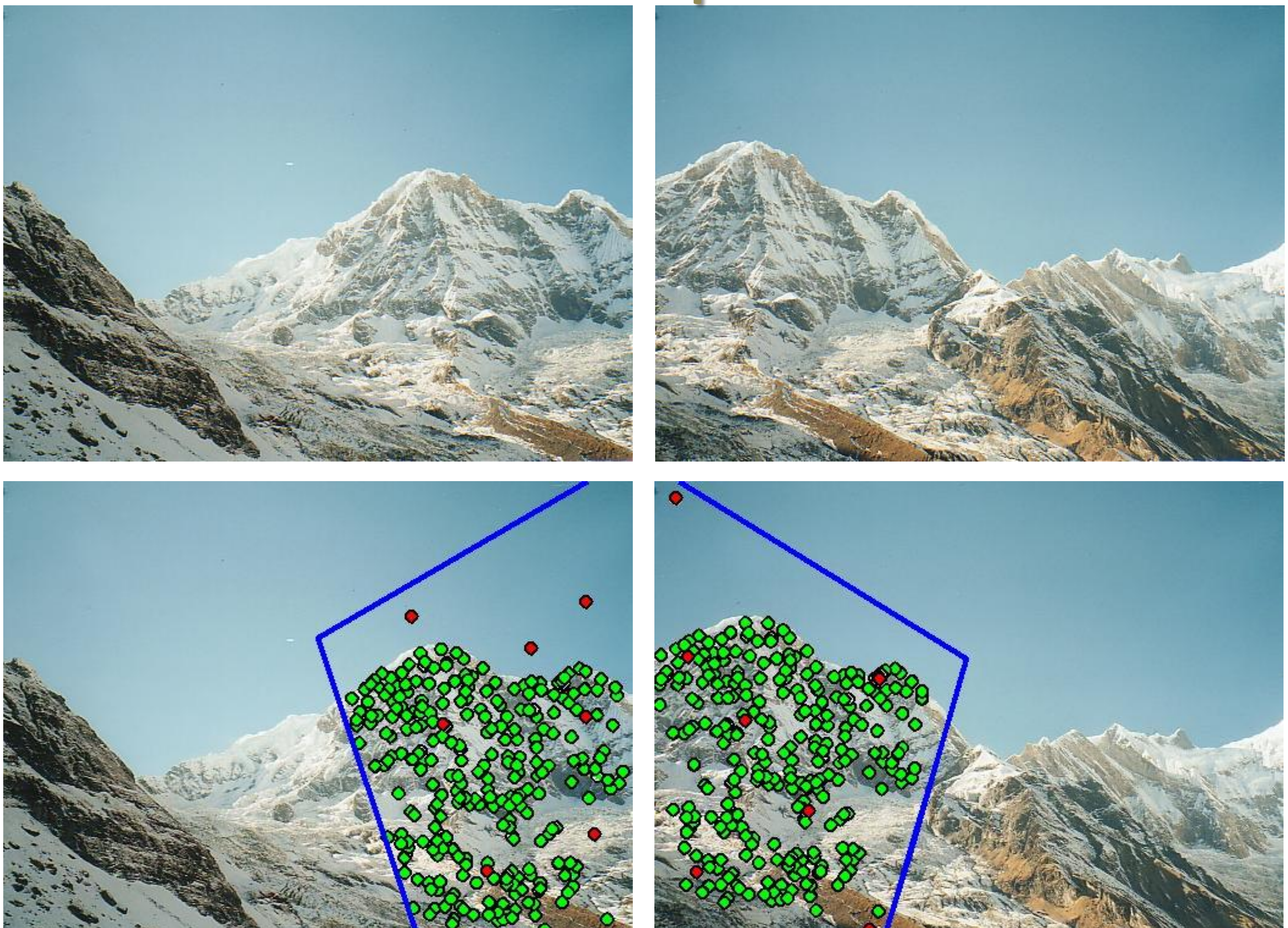
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- Set $w = 1$ and $z = 1$, then have

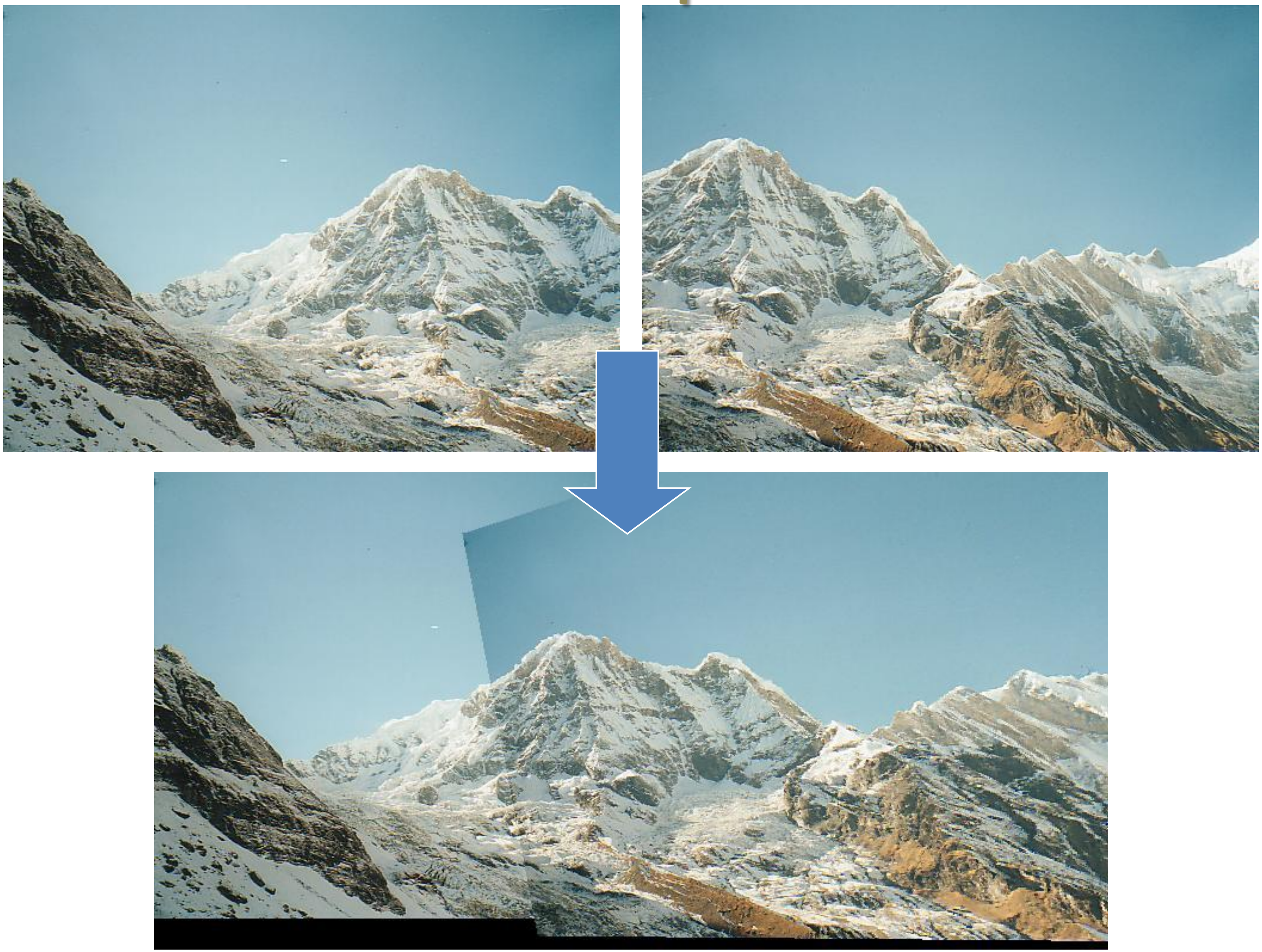
$$\alpha \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Divide line 1 and 2 by 3
- Rearrange terms to form…

Example



Example



“Image Stitching”

- A colloquial term for the same thing…

See blackboard…

$$\alpha(a_{11}u + a_{12}v + a_{13}) = x$$
$$\alpha(a_{21}u + a_{22}v + a_{23}) = y$$
$$\alpha(a_{31}u + a_{32}v + a_{33}) = 1$$

Divide 1st and 2nd line by 3rd line:

$$(a_{11}u + a_{12}v + a_{13}) = x(a_{31}u + a_{32}v + a_{33})$$
$$(a_{21}u + a_{22}v + a_{23}) = y(a_{31}u + a_{32}v + a_{33})$$

Rearrange terms:

$$a_{11}u + a_{12}v + a_{13} - a_{31}xu - a_{32}yv - a_{33}x = 0$$
$$a_{21}u + a_{22}v + a_{23} - a_{31}xu - a_{32}yv - a_{33}y = 0$$

See blackboard…

$$a_{11}u + a_{12}v + a_{13} - a_{31}xu - a_{32}yv - a_{33}x = 0$$
$$a_{21}u + a_{22}v + a_{23} - a_{31}xu - a_{32}yv - a_{33}y = 0$$

Assume $a_{33} = 1$,

$$a_{11}u + a_{12}v + a_{13} - a_{31}xu - a_{32}yv = x$$
$$a_{21}u + a_{22}v + a_{23} - a_{31}xu - a_{32}yv = y$$

Setup for 4+ points, yields 8 equations for 8 unknowns…

Perspective/Projective Transformations

- Solve Direct Linear Transform (DLT):

$$\begin{pmatrix} u_0 & v_0 & 1 & 0 & 0 & 0 & -u_0x_0 & -v_0x_0 \\ u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1x_1 & -v_1x_1 \\ u_2 & v_2 & 1 & 0 & 0 & 0 & -u_2x_2 & -v_2x_2 \\ u_3 & v_3 & 1 & 0 & 0 & 0 & -u_3x_3 & -v_3x_3 \\ 0 & 0 & 0 & u_0 & v_0 & 1 & -u_0y_0 & -v_0y_0 \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1y_1 & -v_1y_1 \\ 0 & 0 & 0 & u_2 & v_2 & 1 & -u_2y_2 & -v_2y_2 \\ 0 & 0 & 0 & u_3 & v_3 & 1 & -u_3y_3 & -v_3y_3 \end{pmatrix} A = b$$

where A is the vector of unknown coefficients a_{ij}

Topics

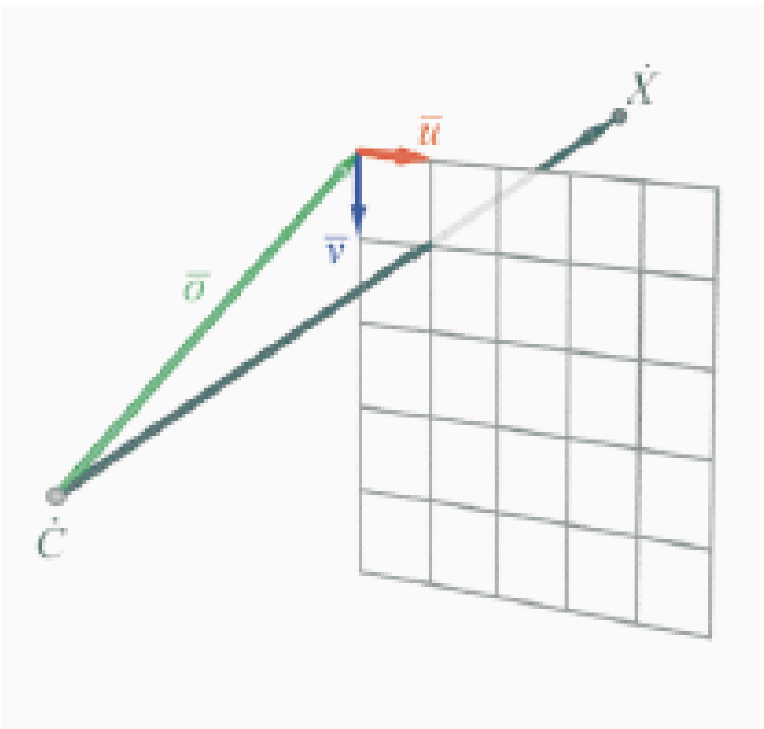
- Image morphing (2D)
- View morphing (2D+)
- Image warping (3D)

3D Image Warping

- Goal: “warp” the pixels of the image so that they appear in the correct place for a new viewpoint
- Advantage:
  - Don’t need a geometric model of the object/environment
  - Can be done in time proportional to screen size and (mostly) independent of object/environment complexity
- Disadvantage:
  - Limited resolution
  - Excessive warping reveals several visual artifacts

    (see examples)

3D Image Warping Equations

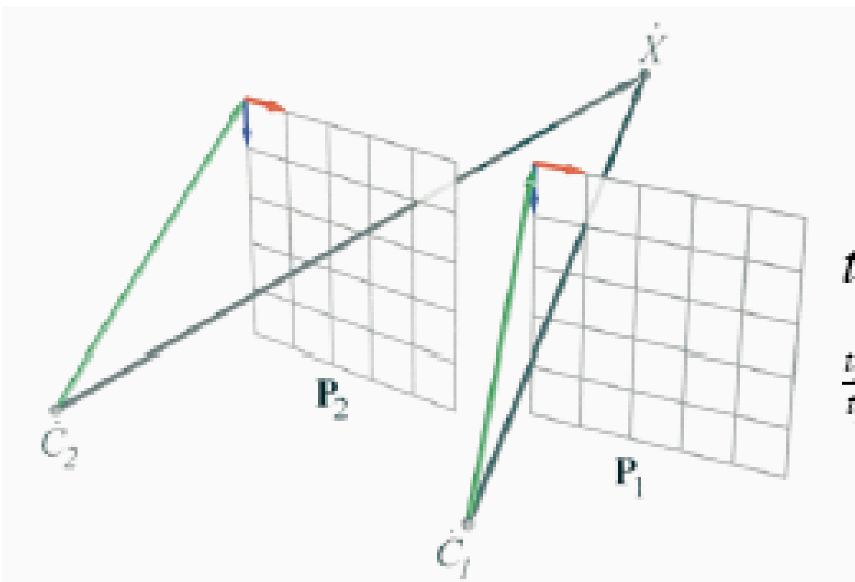


$$P = \begin{bmatrix} u_x & v_x & o_x \\ u_y & v_y & o_y \\ u_z & v_z & o_z \end{bmatrix}$$

$$\dot{X} = \dot{C} + t\,P\,\vec{x}$$

Some pictures courtesy of SIGGRAPH ’99 course notes
(Leonard McMillan)

3D Image Warping Equations



$$\dot{C}_2 + t_2 P_2 \vec{x}_2 = \dot{C}_1 + t_1 P_1 \vec{x}_1$$

$$t_2 P_2 \vec{x}_2 = \dot{C}_1 - \dot{C}_2 + t_1 P_1 \vec{x}_1$$

$$t_2 \vec{x}_2 = P_2^{-1}\left(\dot{C}_1 - \dot{C}_2\right) + t_1 P_2^{-1} P_1 \vec{x}_1$$

$$\frac{t_2}{t_1} \vec{x}_2 = \frac{1}{t_1} P_2^{-1}\left(\dot{C}_1 - \dot{C}_2\right) + P_2^{-1} P_1 \vec{x}_1$$

$$\vec{x}_2 \doteq \underbrace{\frac{1}{t_1}}_{\delta} \underbrace{P_2^{-1}\left(\dot{C}_1 - \dot{C}_2\right)}_{e_{21}} + \underbrace{P_2^{-1} P_1}_{H_{21}} \vec{x}_1$$

3D Image Warping Equations

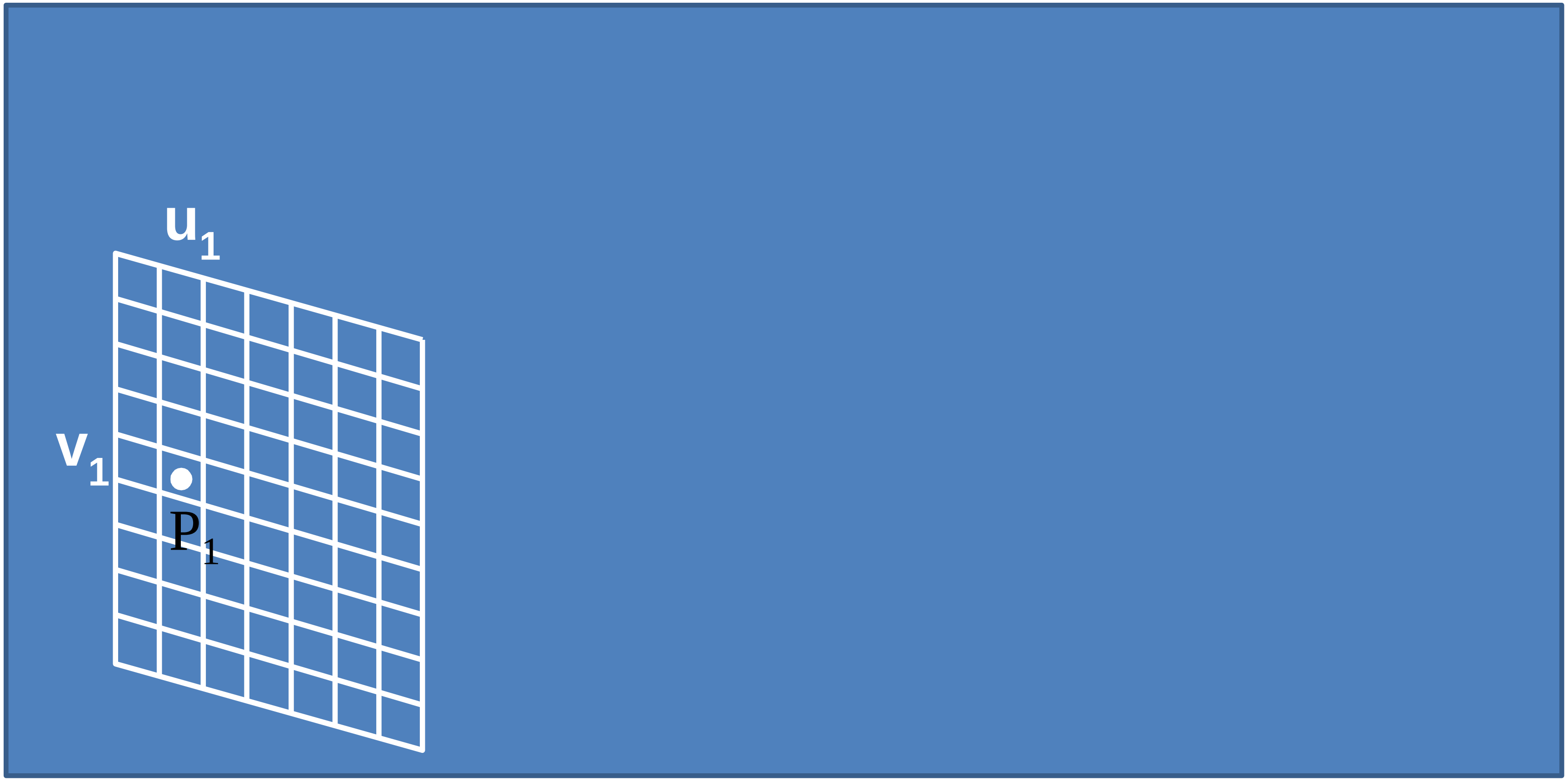
McMillan & Bishop Warping Equation:

$$x_2 = \delta\underbrace{(x_1)\, P_2^{-1} (c_1 - c_2)}_{\textit{Move pixels based on distance to eye}} + \underbrace{P_2^{-1} P_1\, x_1}_{\textit{\sim Texture mapping}}$$

- Per-pixel distance values are used to warp pixels to their correct location for the current eye position

3D Image Warping Equations

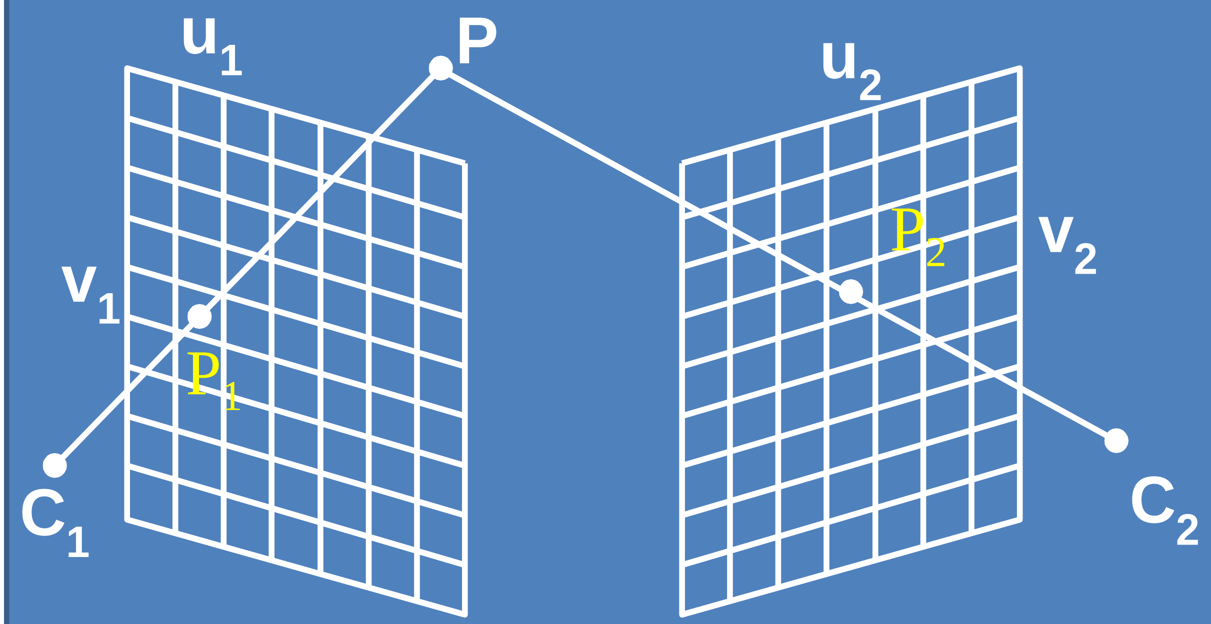
- Images enhanced with per-pixel depth [McMillan95]



3D Image Warping Equations

$$\dot{P} = \dot{C}_1 + (\bar{c}_1 + u_1\bar{a}_1 + v_1\bar{b}_1)w_1$$

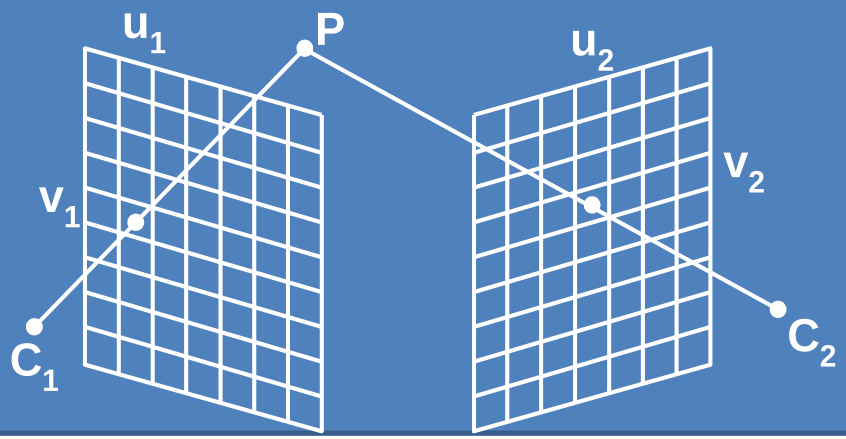
$$\dot{P} = \dot{C}_2 + (\bar{c}_2 + u_2\bar{a}_2 + v_2\bar{b}_2)w_2$$



3D Image Warping Equations

$$u_2 = \frac{w_{11} + w_{12} \cdot u_1 + w_{13} \cdot v_1 + w_{14} \cdot \delta(u_1, v_1)}{w_{31} + w_{32} \cdot u_1 + w_{33} \cdot v_1 + w_{34} \cdot \delta(u_1, v_1)}$$

$$v_2 = \frac{w_{21} + w_{22} \cdot u_1 + w_{23} \cdot v_1 + w_{24} \cdot \delta(u_1, v_1)}{w_{31} + w_{32} \cdot u_1 + w_{33} \cdot v_1 + w_{34} \cdot \delta(u_1, v_1)}$$



3D Image Warping Example

3D Image Warping Example

- DeltaSphere
  - Lars Nyland *et al.*

3D Image Warping Example

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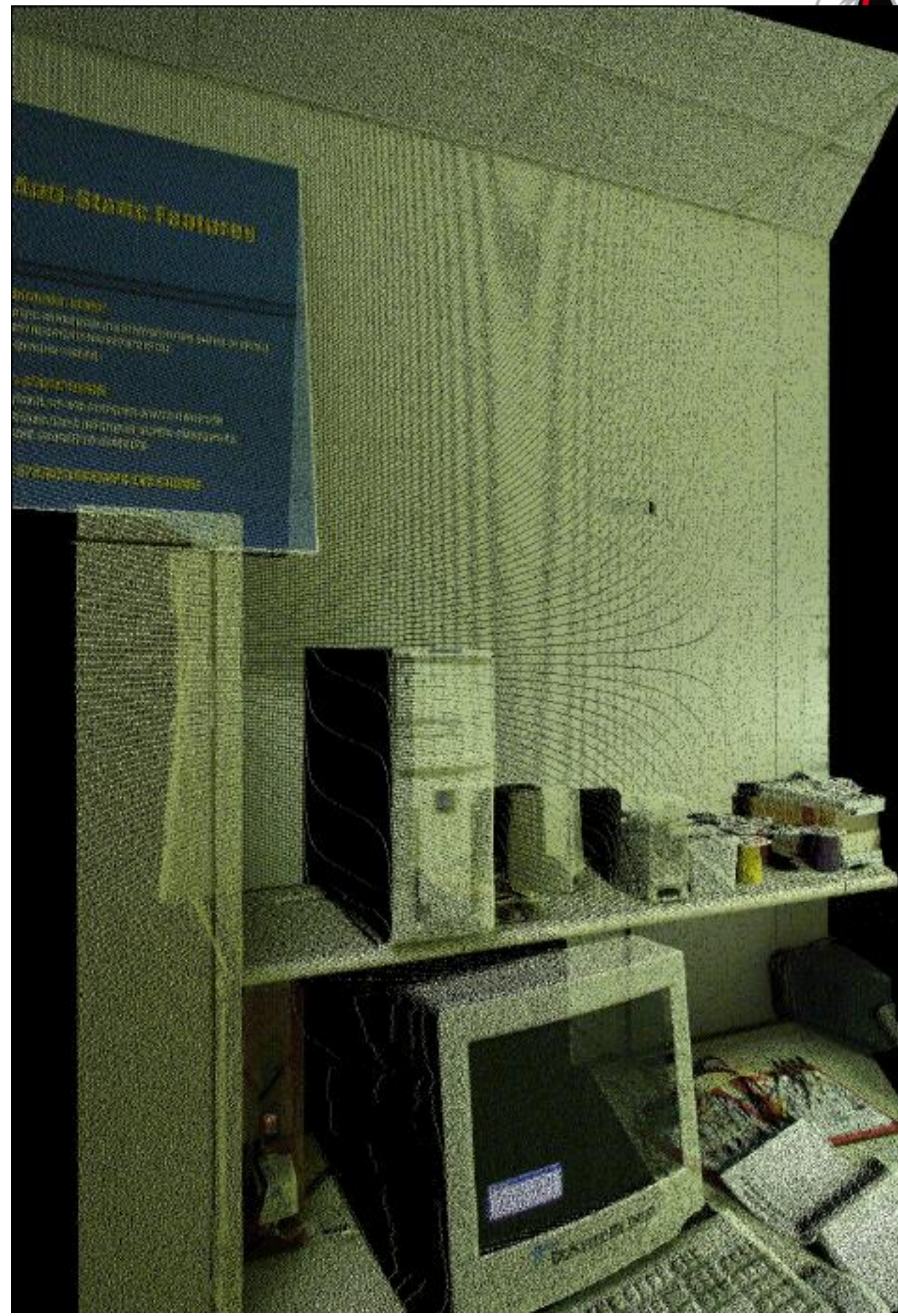
3D Image Warping Example



3D Image Warping Example

Disocclusions

- Disocclusions (or exposure events) occur when unsampled surfaces become visible…

What can we do?

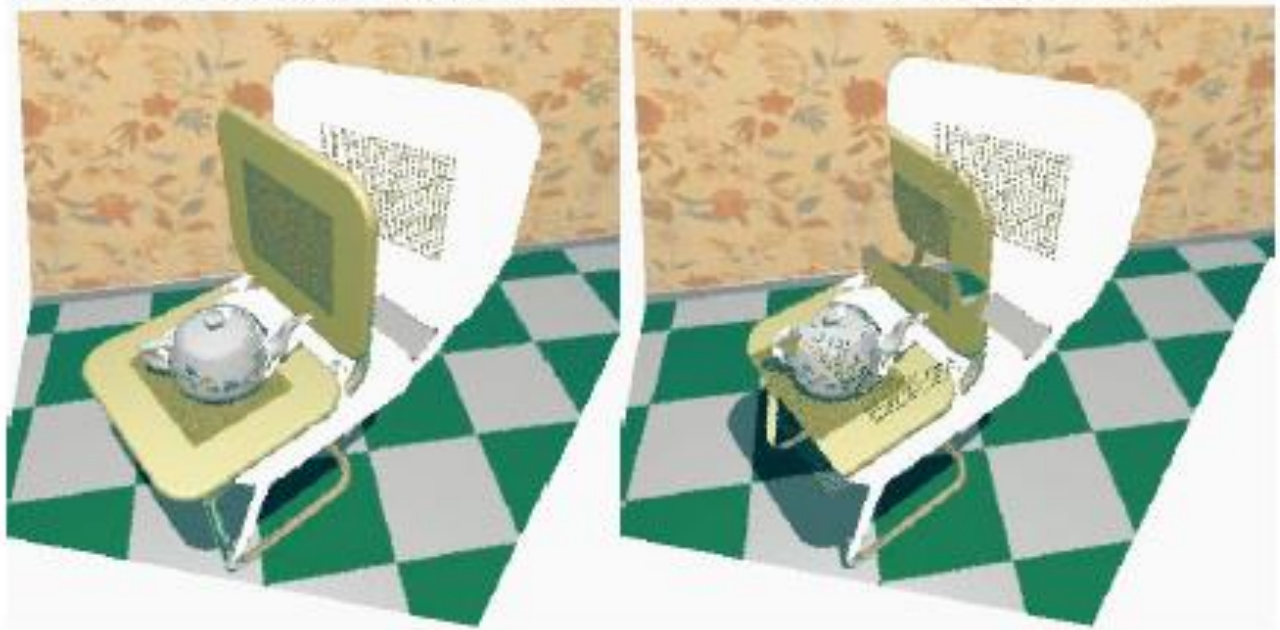
Disocclusions

- Bilinear patches: fill in the areas

What else?

Rendering Order

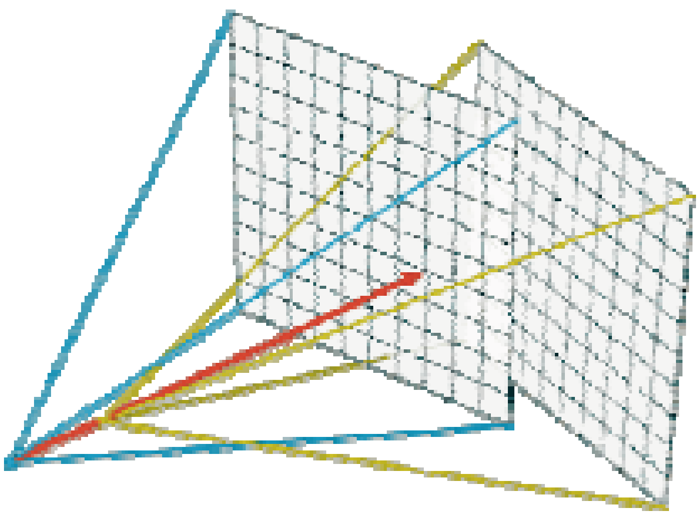
✓ The warping equation determines where points go…

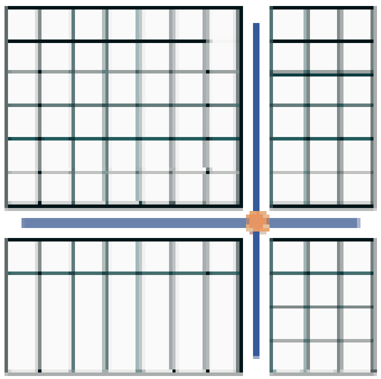


… but that is not sufficient

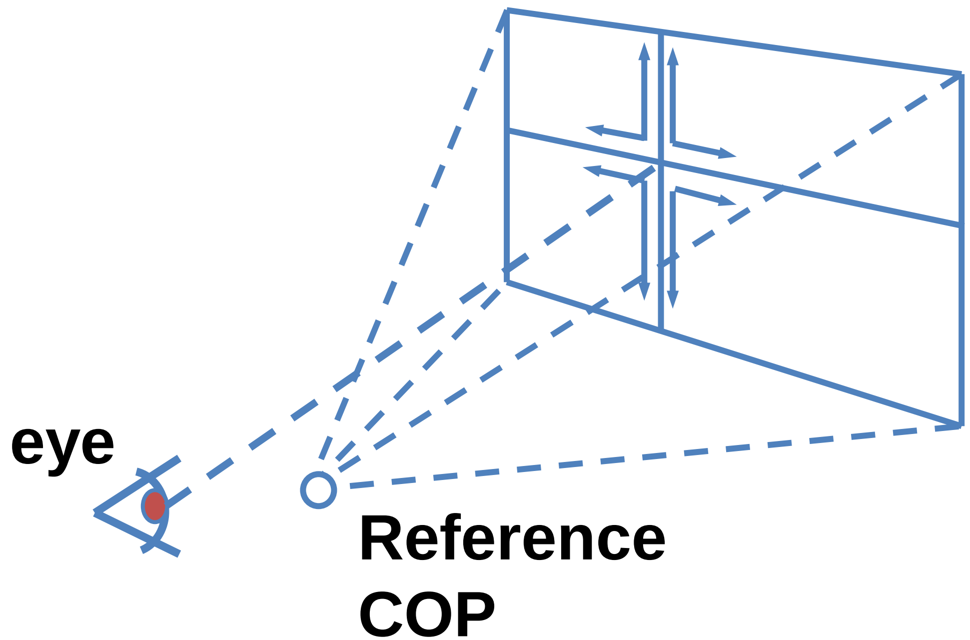
Occlusion Compatible Rendering Order

- Epipolar geometry
- Project the new viewpoint onto the original image and divide the image into 1, 2 or 4 "sheets"





Occlusion Compatible Rendering Order



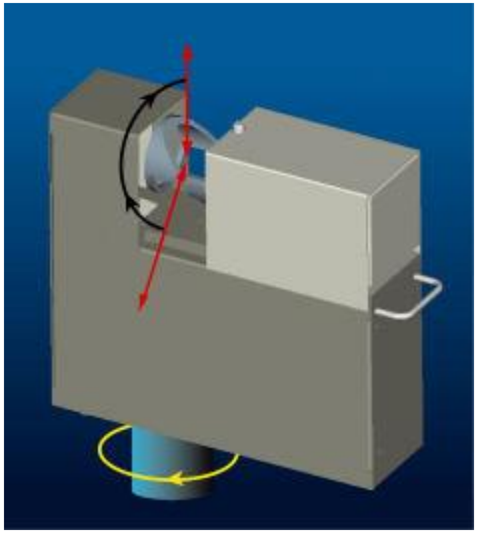
- A raster scan of each sheet produces a back-to-front ordering of warped pixels

Splatting

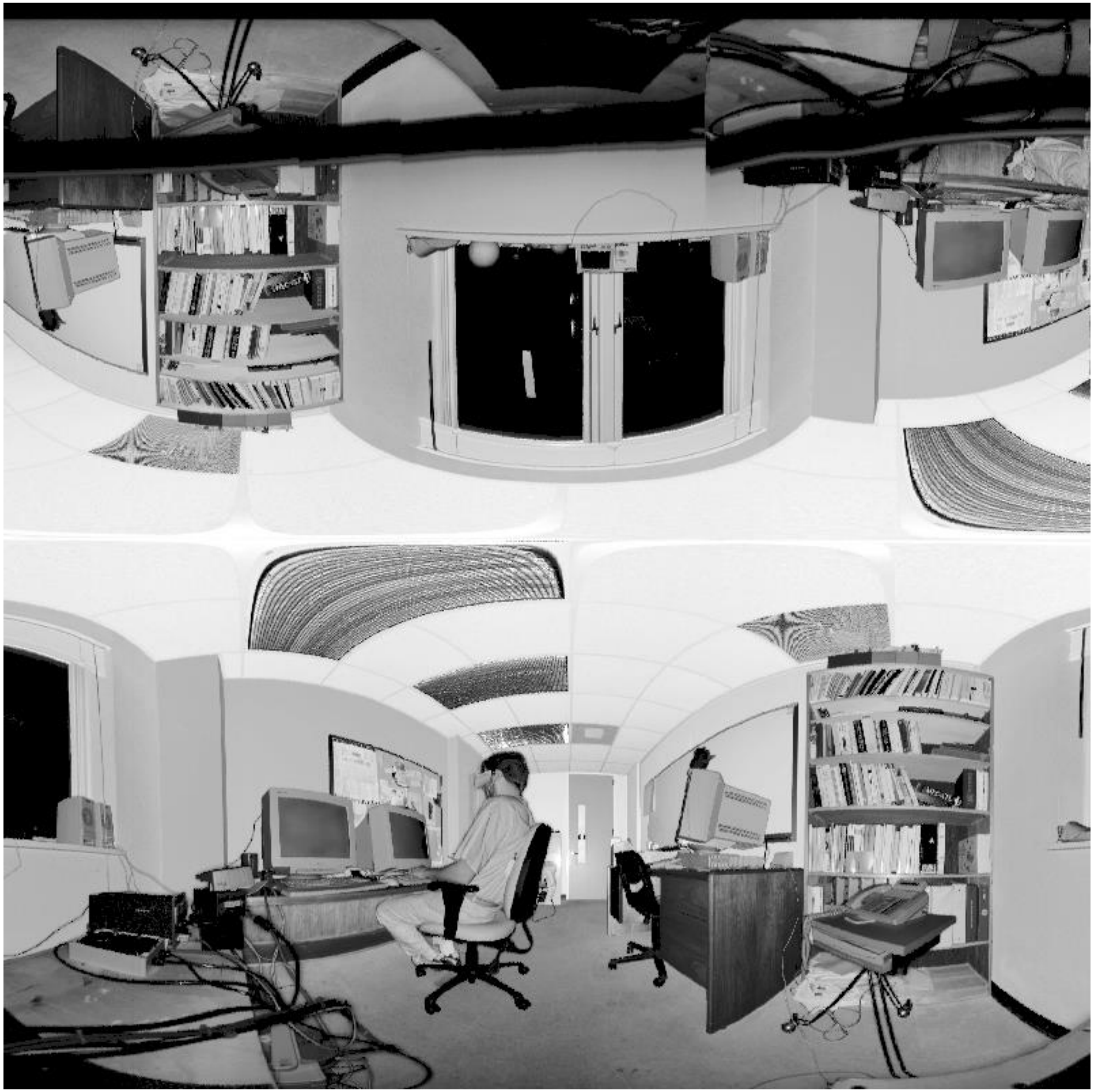
- One pixel in the source image does not necessarily project to one pixel in the destination image
  - e.g., if you are walking towards something, the sample might get larger…
- A solution: estimate shape and size of footprint of warped samples
  - expensive to do accurately
  - square/rectangular approximations can be done quickly (3x3 or 5x5 splats)
  - occlusion-compatible rendering will take care of oversized splats
  - *BUT large splats can make the image seem blocky/low-res*

More Examples Using the DeltaSphere

- Lars Nyland *et al.*

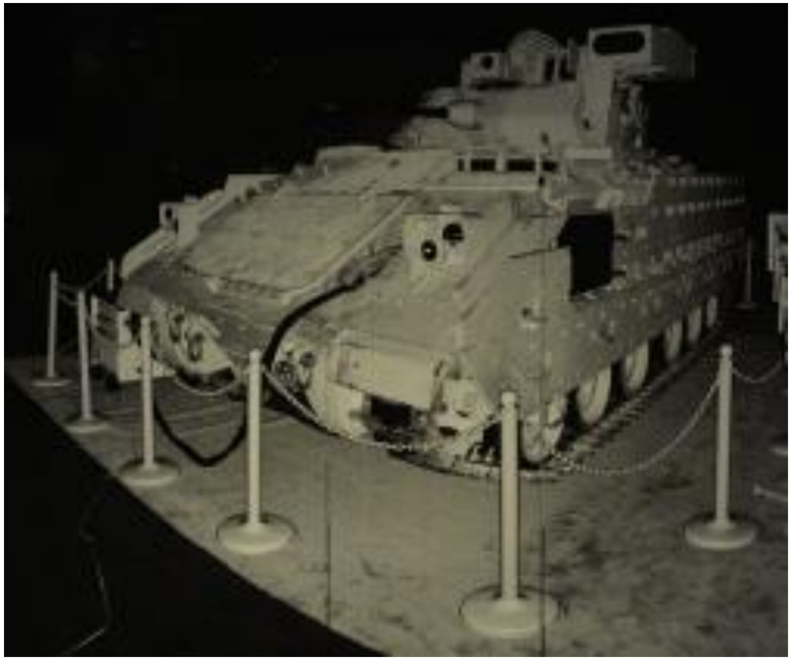


courtesy 3rd Tech Inc.



- 300º x 300º panorama
- this is the reflected light

- 300º x 300º panorama
- this is the range light

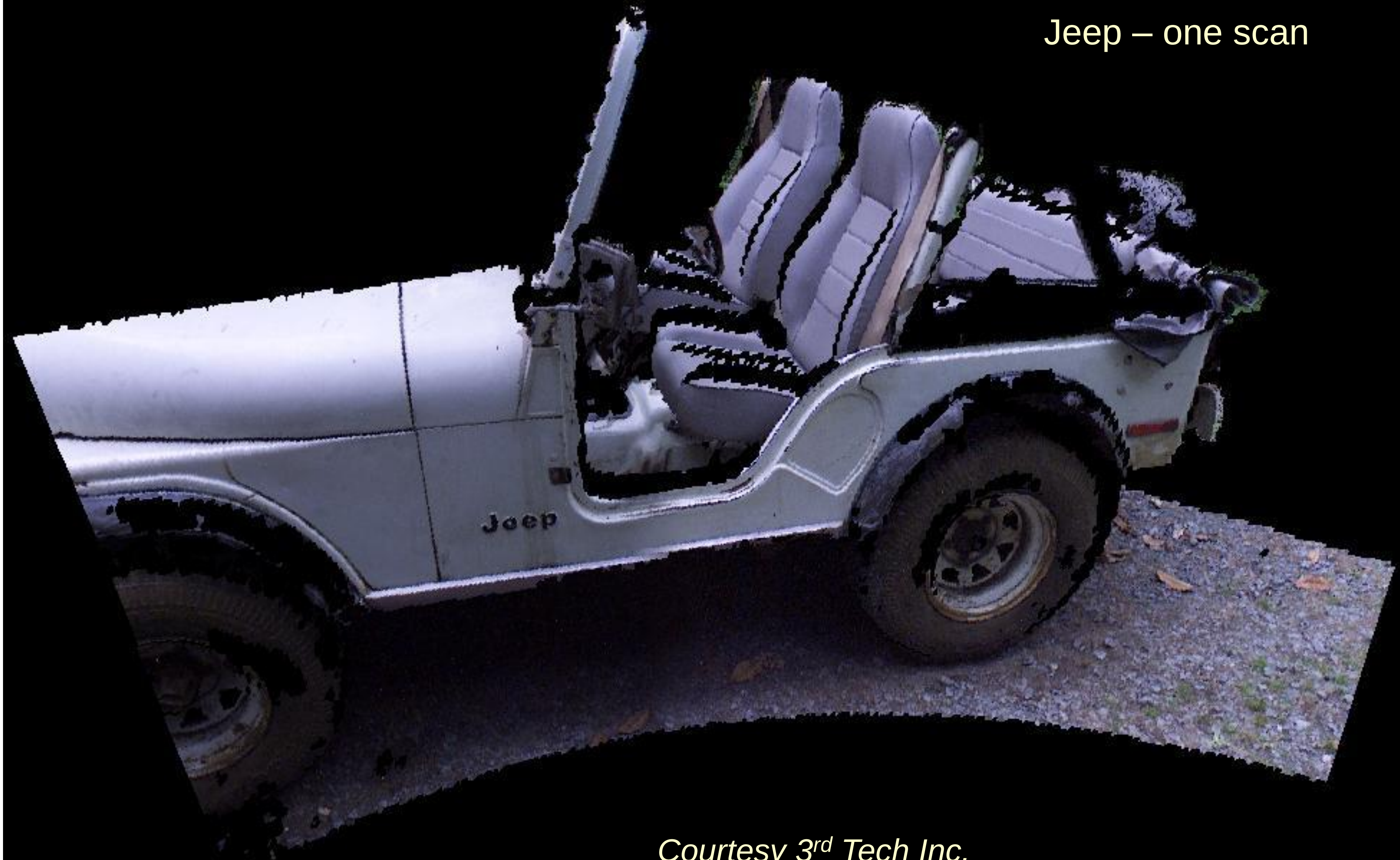


spherical range panoramas

planar re-projection

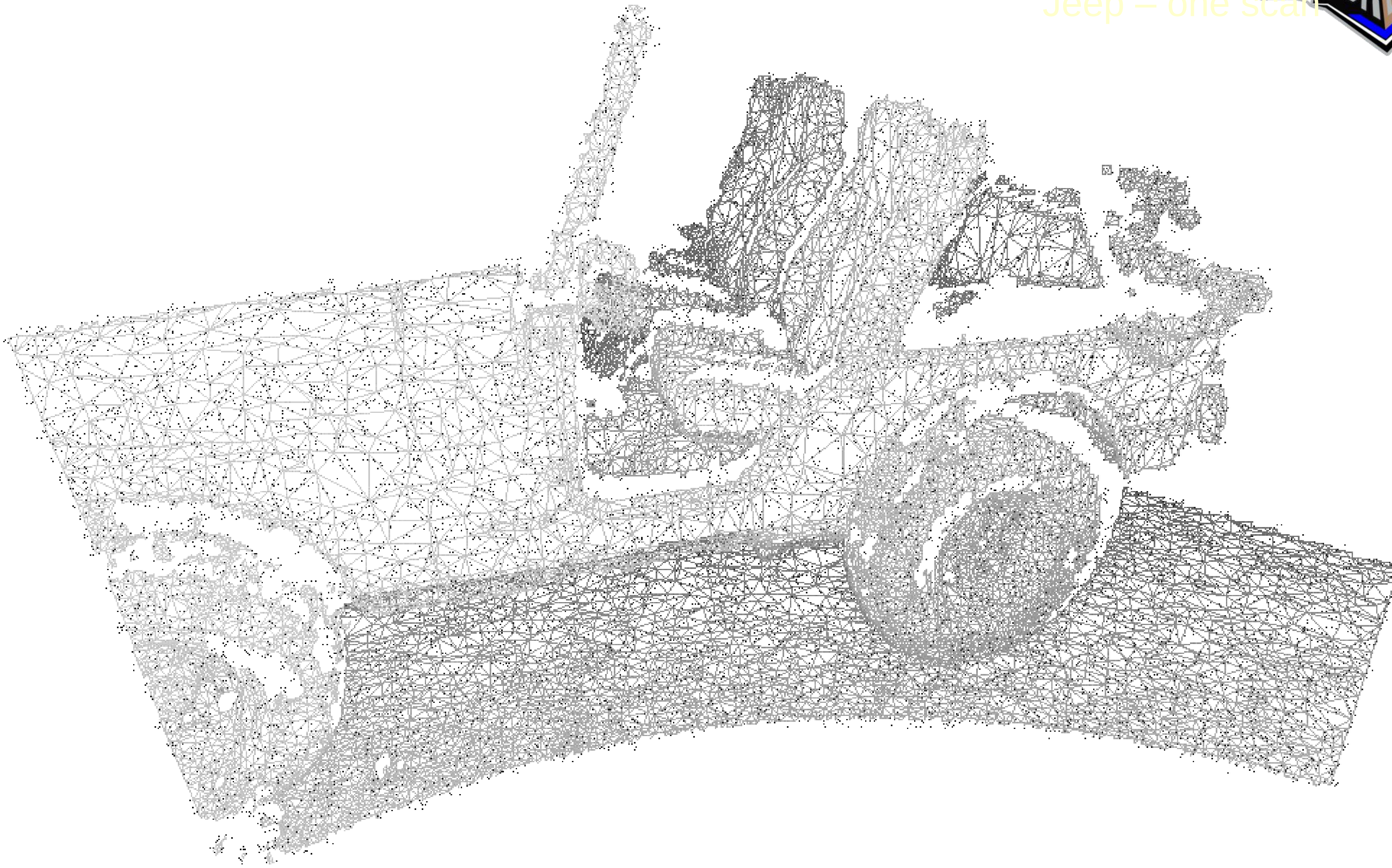
Courtesy 3rd Tech Inc.

Jeep – one scan



Courtesy 3rd Tech Inc.

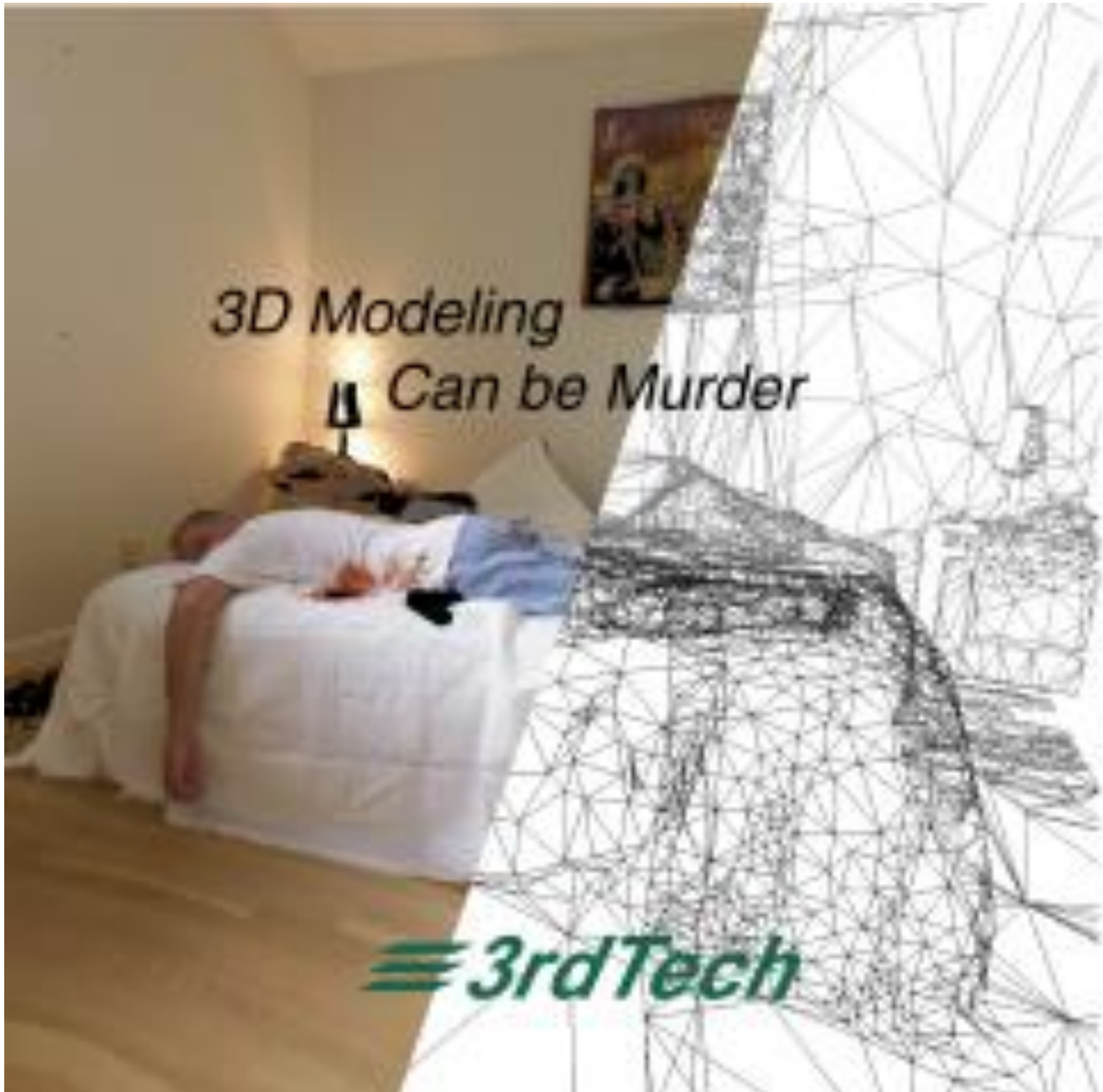
Jeep – one scan



Courtesy 3rd Tech Inc.

Complete Jeep model

Courtesy 3rd Tech Inc.


3D Modeling
Can be Murder
3rdTech