



Physically Based Simulations on the GPU (just briefly...)

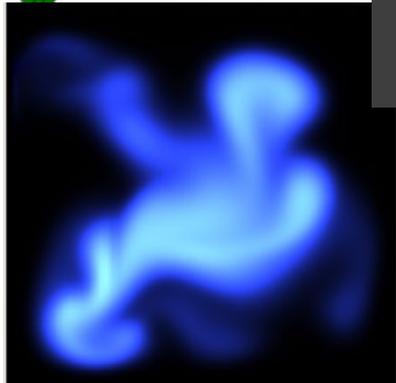
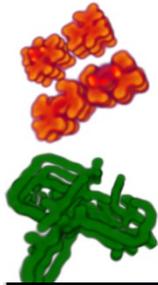
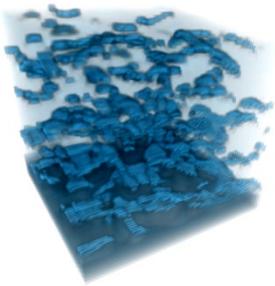
CS334

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Simulating the world

- Floating point arithmetic on GPUs and their speed enable us to simulate a wide variety of phenomena using PDEs





Some Basics

- Operators (on images/lattices)
- Diffusion
- Bouyancy



Operators

- Given an image:
 - Gradient (vector)

$$\nabla f(x, y) = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y}$$

- Laplacian (scalar)

$$\nabla^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$



Discrete Laplacian

- $\nabla^2 f(x, y) =$
 $f(x - 1, y) + f(x + 1, y) +$
 $f(x, y - 1) + f(x, y + 1) -$
 $4f(x, y)$
- Matrix form = ??



Discrete Laplacian

- $\nabla^2 f(x, y) =$
 $f(x - 1, y) + f(x + 1, y) +$
 $f(x, y - 1) + f(x, y + 1) -$
 $4f(x, y)$

- Matrix form =

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$



Heat Equation

$$\frac{\partial f}{\partial t} = \nabla^2 f$$

Diffusion Equation

[Weisstein 1999]



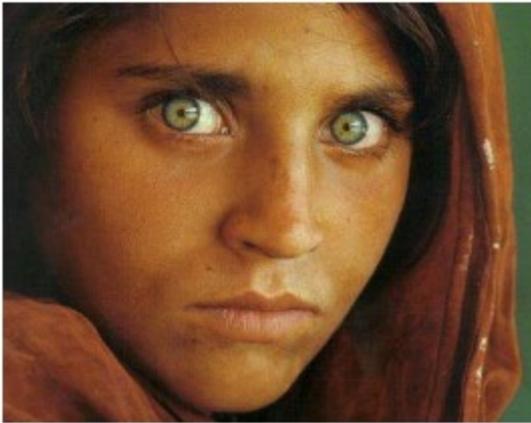
$$f(x, y)' = f(x, y) + \frac{c_d}{4} \nabla^2 f(x, y)$$

where c_d is the coefficient of diffusion...



(Anisotropic) Diffusion

(a) Original Image



(b) Time = 5



(c) Time = 10



(d) Time = 30





Buoyancy

- Used in convection, cloud formations, etc.
- Given a temperature state T :
 - a vertical buoyancy velocity is ‘upwards’ if a node is hotter than its neighbors’ and
 - a vertical buoyancy velocity is ‘downwards’ if a node is cooler than its neighbors



Buoyancy

$$v(x, y)' = v(x, y) + \frac{c_b}{2} (2f(x, y) - f(x + 1, y) - f(x - 1, y))$$

where c_b is the buoyancy strength



Bouyancy (considering neighbors)

- $f(x, y)' =$
 $f(x, y) - \frac{\sigma}{2} f(x, y)$
 $[\rho(f(x, y + 1)) - \rho(f(x, y - 1))]$

where $\rho(f) = \tanh(\alpha(f - f_c))$ (an approx. of density relative to temperature f) and σ is buoyancy strength and α and f_c are constants



Euler Method (for ODE)

- Given:

$$y'(t) = f(t, y(t)) \text{ with } y(t_0) = y_0$$

- Do:

$$y_{n+1} = y_n + hf(t_n, y_n)$$

Classical Runge Kutta Method



- Given:

$$y'(t) = f(t, y(t)) \text{ with } y(t_0) = y_0$$

- Do:

$$y_{n+1} = y_n + h/6(k_1 + 2k_2 + 2k_3 + k_4)$$

$$t_{n+1} = t_n + h$$

where

$$k_1 = f(t_n, y_n),$$

$$k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1),$$

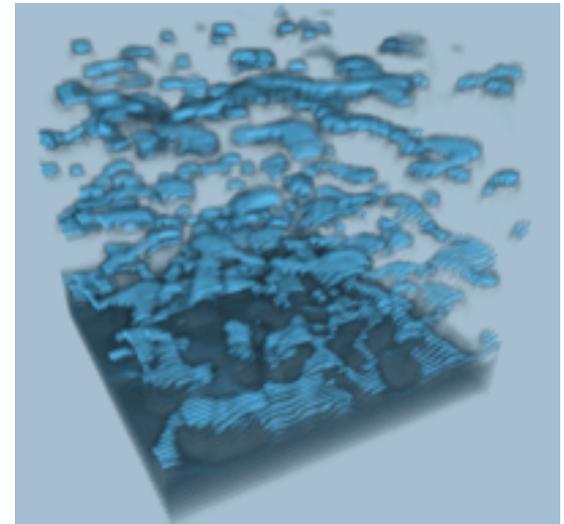
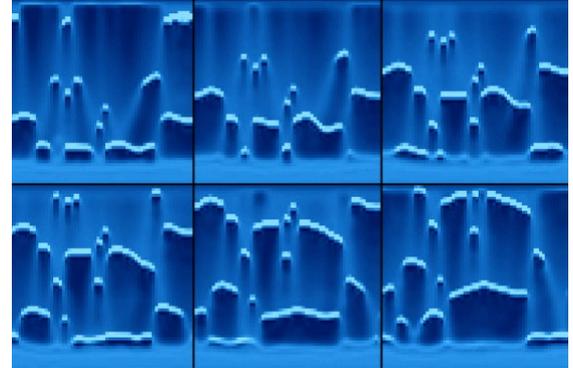
$$k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2),$$

$$k_4 = f(t_n + h, y_n + hk_3).$$



Example: (Water) Boiling

- Based on [Harris et al. 2002]
- State = Temperature
- Three operations:
 - Diffusion, buoyancy, & latent heat
- 3D Simulation
 - Stack of 2D texture slices



Turing: Morphogenesis and Reaction-Diffusion (1952)



“[Alan Turing](#) in 1952 describing the way in which non-uniformity (stripes, spots, spirals, etc.) may arise naturally out of a homogeneous, uniform state. The theory (which can be called a [reaction–diffusion](#) theory of [morphogenesis](#)), has served as a basic model in [theoretical biology](#), and is seen by some as the very beginning of [chaos theory](#).”

$$\frac{\partial U}{\partial t} = D_U \nabla^2 U - k(UV - 16)$$
$$\frac{\partial V}{\partial t} = D_V \nabla^2 V + k(UV - 12 - V)$$

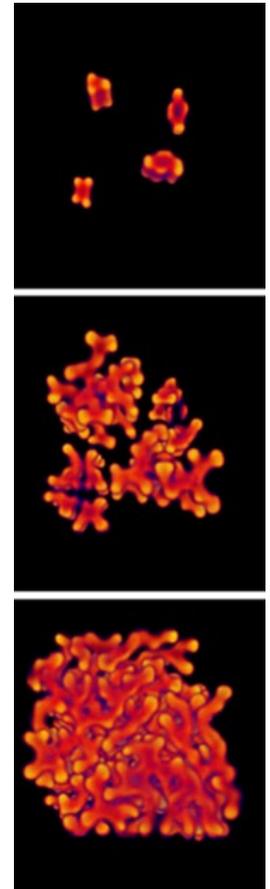
Gray-Scott Reaction-Diffusion



- State = two scalar chemical concentrations
- Simple:
 - just **Diffusion** and **Reaction** ops

$$\begin{aligned}\frac{\partial U}{\partial t} &= D_u \nabla^2 U - UV^2 + F(1 - U), \\ \frac{\partial V}{\partial t} &= D_v \nabla^2 V + UV^2 - (F + k)V\end{aligned}$$

U, V are chemical concentrations,
 F, k, D_u, D_v are constants





Some research...

- http://www.cc.gatech.edu/~turk/reaction_diffusion/reaction_diffusion.html



Navier-Stokes Equations

- Describe flow of an incompressible fluid

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \frac{1}{\rho} \nabla p - \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Advection

Pressure
Gradient

Diffusion
(viscosity)

External Force

$$\nabla \cdot \mathbf{u} = 0$$

← Velocity is divergence-free



Fluid Dynamics

- Solution of Navier-Stokes flow eqs.
 - Stable for arbitrary time steps (=fast!)
 - [Stam 1999], [Fedkiw et al. 2001]
- Can be implemented on latest GPUs
 - Quite a bit more complex than R-D or boiling
- See “Fast Fluid Dynamics Simulation on the GPU” (Harris, *GPU Gems*, 2004)

Fluid Simulations





Thermodynamics

- Temperature affected by
 - Heat sources
 - Advection
 - Latent heat released / absorbed during condensation / evaporation
- Δ temperature = advection + latent heat release
 - + temperature input



Cloud Dynamics

- 3 components
 - 7 unknowns
- Fluid dynamics
 - Motion of the air
- Thermodynamics
 - Temperature changes
- Water continuity
 - Evaporation,
condensation

Velocity: $\mathbf{u} = (u, v, w)$

Pressure: p

Potential temperature: θ
(see dissertation)

Water vapor mixing ratio: q_v

Liquid water mixing ratio: q_c

Cloud Dynamics





Wave Equation

- Remember heat equation:
 - Rate of change of value proportional to Laplacian
- Wave equation:
 - Rate of change of the rate of change is also proportional to the Laplacian



Wave Equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

where u models the displacement and c is the propagation speed

Water Simulation: Wave Equation



U = value, V = rate of change

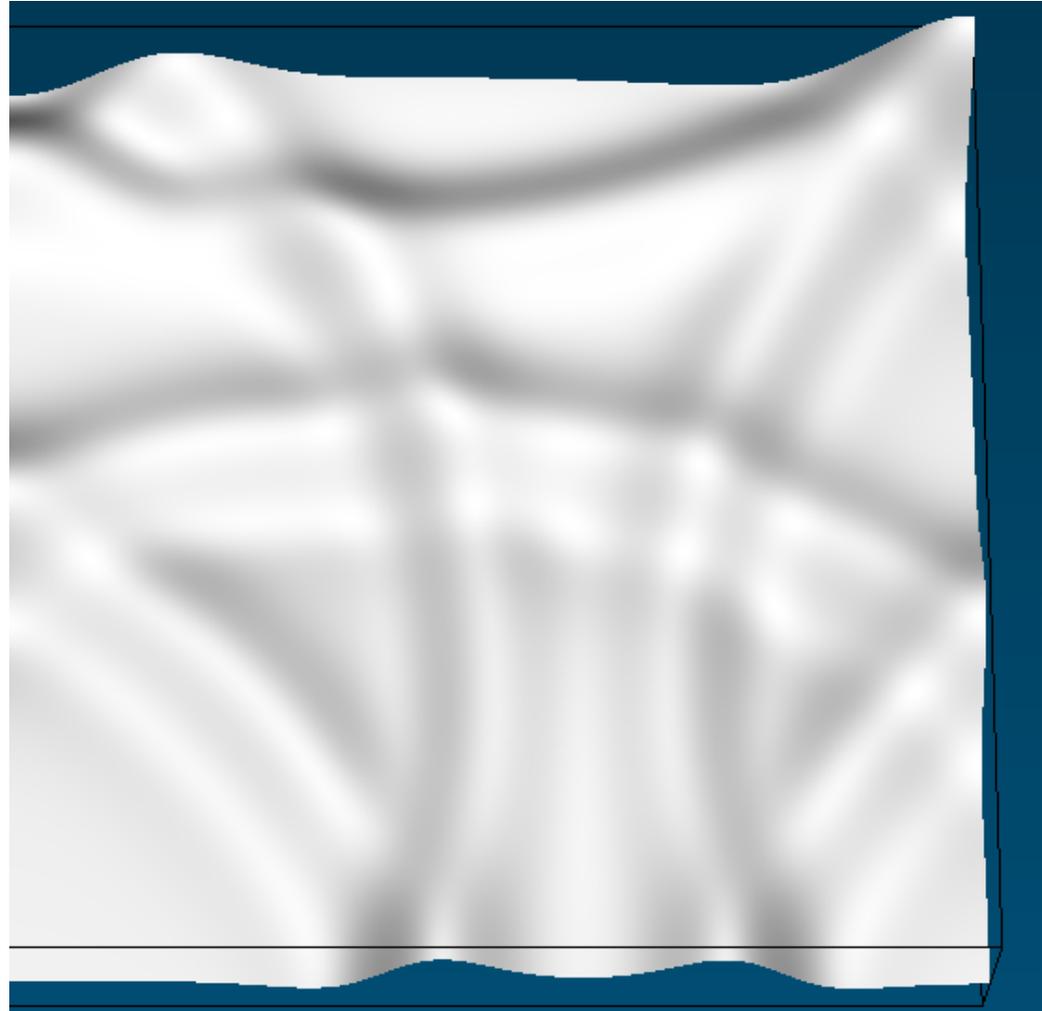
$$\frac{\partial U}{\partial t} = \frac{b}{k} + d\nabla^2 U$$

$$\frac{\partial V}{\partial t} = k\nabla^2 U$$

Water Simulation: Wave Equation



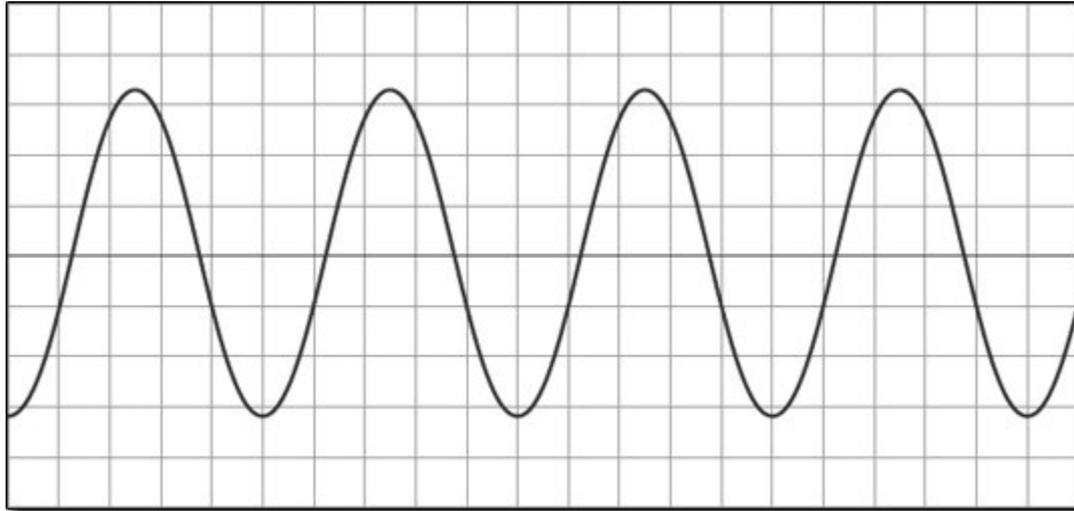
- Demo...



Water Simulation: Sine Waves



$$A \sin(\omega x + t)$$



Water Simulation: Sine Waves



$$A_1 \sin(\omega_1 x + t_1) + A_2 \sin(\omega_2 x + t_2) + \dots$$

Water Simulation: Sine Waves



- Using sine-wave summations:

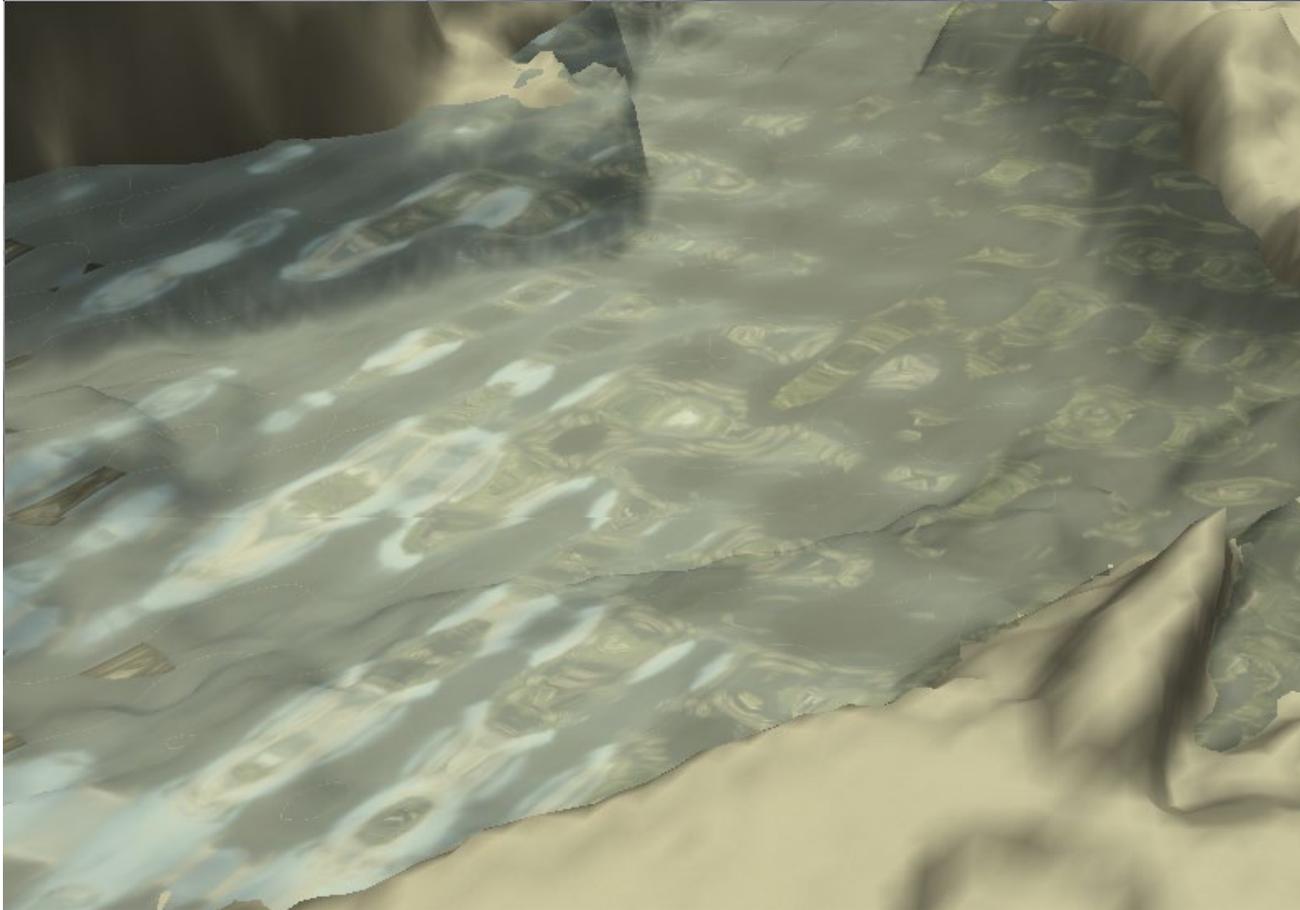
$$H(x, y, t) = \sum A_i \sin(D_i \cdot (x, y) \omega_i + t \phi_i)$$

[use H as height or a pixel intensity]

- Pixel values over time are:

$$P(x, y, t) = (x, y, H(x, y, t))$$

Water Simulation: Sine Waves



(here, pixel normals are computed as well for reflections)



Water: Surface Normals

- Use binormal and tangent:

$$B(x, y, t) = \left(\frac{dx}{dx}, \frac{dy}{dx}, \frac{dH(x, y, t)}{dx} \right) = \left(1, 0, \frac{dH(x, y, t)}{dx} \right)$$

$$T(x, y, t) = \dots = \left(0, 1, \frac{dH(x, y, t)}{dy} \right)$$

- Normal is:

$$N(x, y, t) = B \times T$$

$$N(x, y, t) = \left(-\frac{dH(x, y, t)}{dx}, -\frac{dH(x, y, t)}{dy}, 1 \right)$$



Water Simulation: Gerstner Waves

- These waves also change the x, y of the wave imitating how points at top of wave are squished together and points at bottom are separated

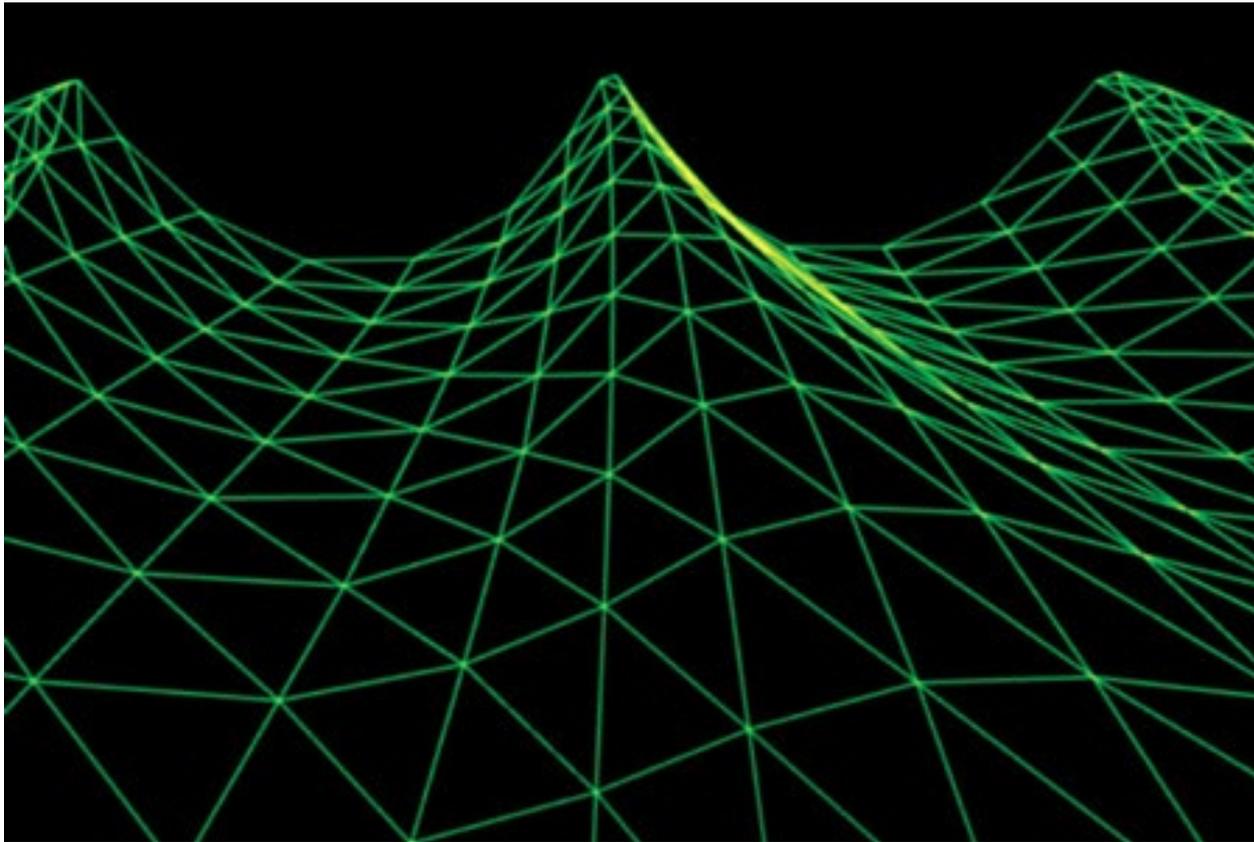
Water Simulation: Gerstner Waves



$$P(x, y, t) = \begin{bmatrix} x + \sum Q_i A_i D_i \cdot x \cos(\omega_i D_i \cdot (x, y) + \phi_i t) \\ y + \sum Q_i A_i D_i \cdot y \cos(\omega_i D_i \cdot (x, y) + \phi_i t) \\ \sum A_i \sin(\omega_i D_i \cdot (x, y) + \phi_i t) \end{bmatrix}$$

where Q_i =sharpness

Water Simulation: Gerstner Waves





Video

- <https://www.youtube.com/watch?v=lqPa389vi4s>