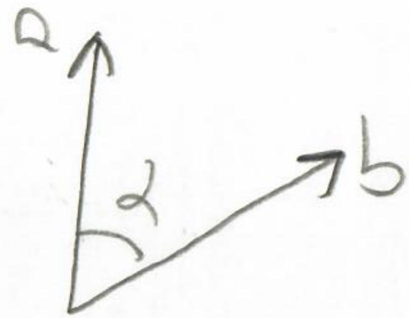
point $P = (p_x, p_y, p_z)$

vector $V = (v_x, v_y, v_z)$

dot product:



$a \cdot b = \|a\| \, \|b\| \cos\alpha$

length

$\|a\| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

$a' \cdot b' = \cos\alpha$

norm $(a) = \dfrac{a}{\|a\|}$ $\hookrightarrow$ $\|a'\| = 1$

$a' \nearrow$

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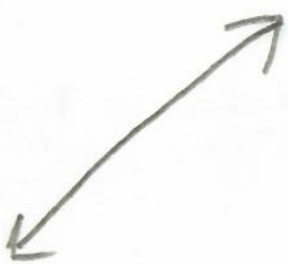
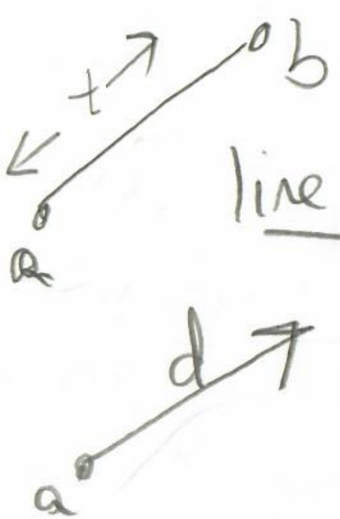
cross product

$c = a \times b = (a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x)$

$\|c\| = \|a\| \|b\| \sin \alpha$



line $y = mx + b \mid ax + by + c = 0$



line segment $l(t) = a(1-t) + bt \quad t \in [0,1]$

ray $r(t) = a + td \quad t \in [-\infty, +\infty]$

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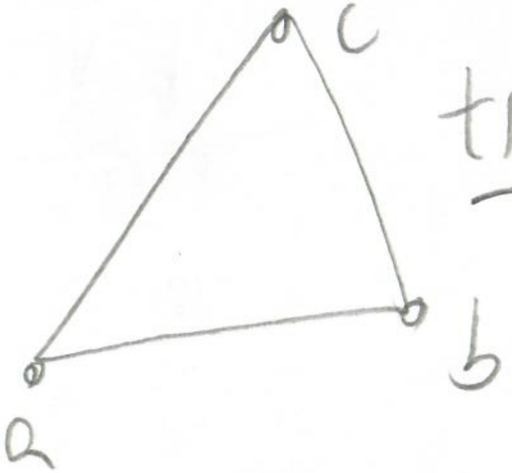
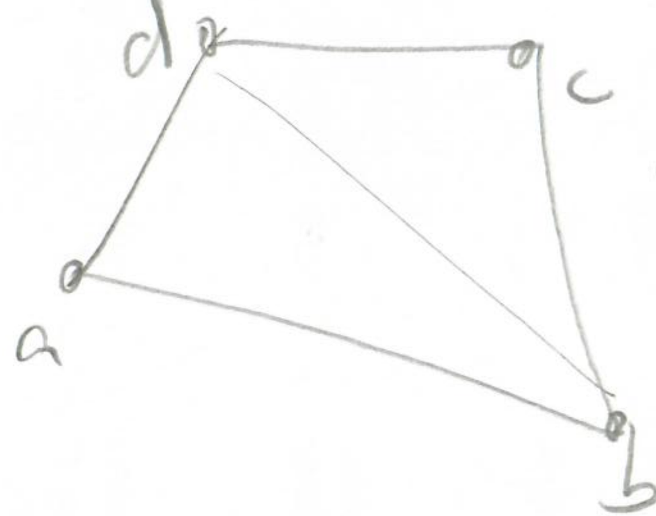
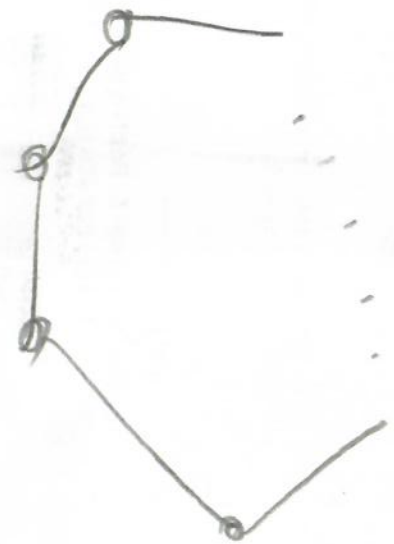


<u>triangle</u>
(flat)



<u>quad</u>
(~~flat~~)



<u>polygon</u>

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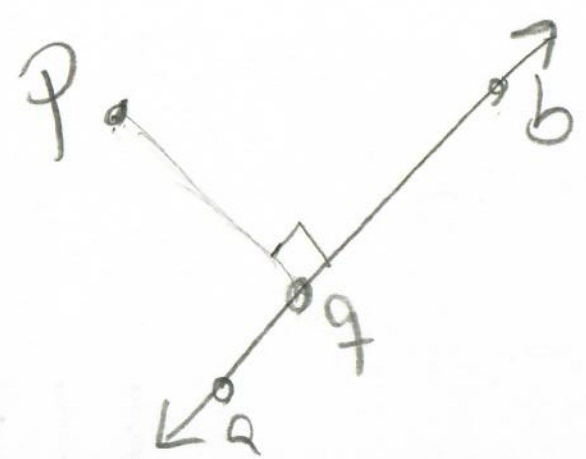
Geometric Operations I



distance of point to line? (2D)

$$(p-q)\cdot(b-a)=0$$

$$q=a+t(b-a)$$

$$(p-(a+t(b-a)))\cdot(b-a)=0$$

$$p\cdot b-p\cdot a-b\cdot(a+t(b-a))+a\cdot(a+t(b-a))=0$$

$t=$ ________

$q=$ ____

distance = len$(p-q)$

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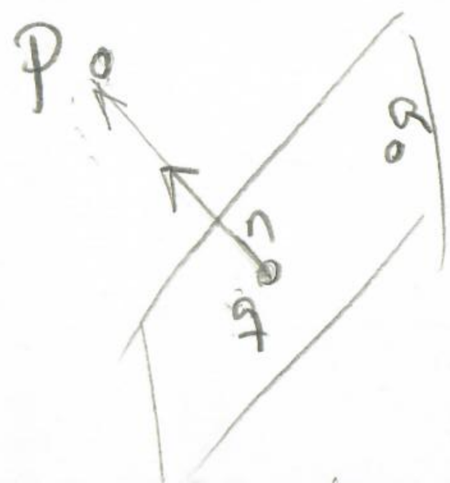
Geometric Operations II



"$n \cdot (q - a) = 0$"

distance of point to plane?

$q = p + tn$

$n \cdot (q - a) = 0$

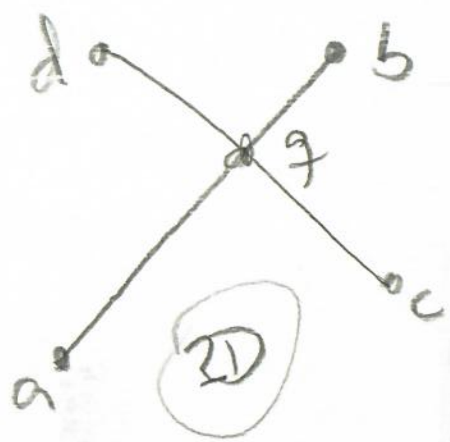
$n \cdot ((p + tn) - a) = 0$

$t = \_\_\_$ //

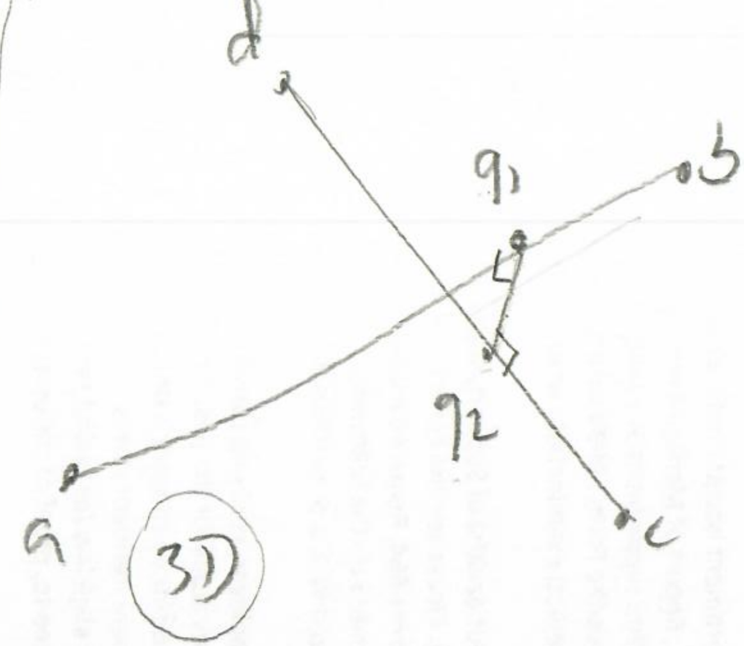
distance $(p - q) = \_\_\_$ //

$q = \_\_\_$ //

Geometric Operations III



$$q = a + t(b-a)$$
$$q = c + s(d-c)$$



$$(q_2 - q_1) \cdot (b-a) = 0$$
$$(q_2 - q_1) \cdot (d-c) = 0$$

$$q_1 = a + t(b-a)$$
$$q_2 = c + s(d-c)$$

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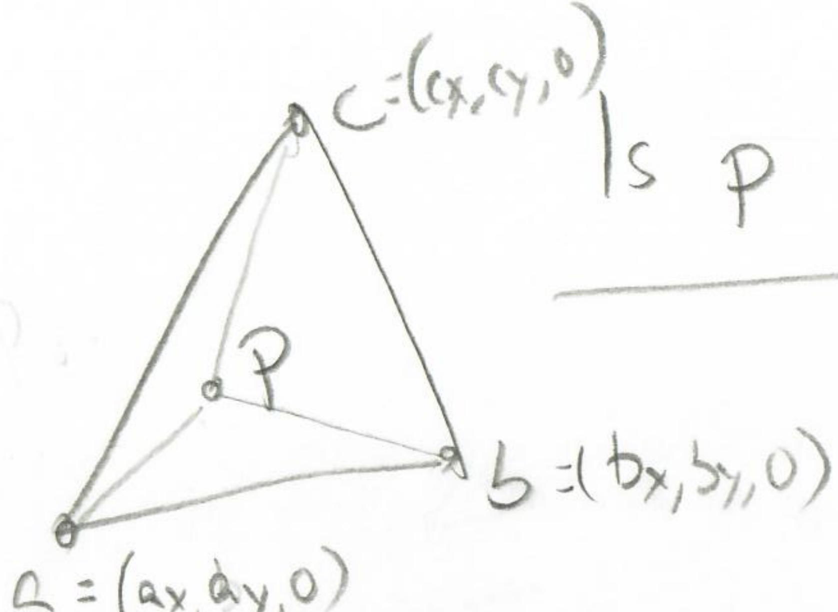
Geometric Operations IV

(Method I) "Winding Rule"

Is p inside Δabc?



$(b-a) \times (p-a) = V_1$

$(c-b) \times (p-b) = V_2$

$(a-c) \times (p-c) = V_3$

$(p-a) \times (b-a) = V_1'$

$(p-b) \times (c-b) = V_2'$

$(p-c) \times (a-c) = V_3'$

z is negative

$V_1.z \geq 0$, $V_2.z \geq 0$, $V_3.z \geq 0$ then inside

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