Modelling the Appearance and Behaviour of Urban Spaces

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Abstract
Urban spaces consist of a complex collection of buildings, parcels, blocks and neighbourhoods interconnected by streets. Accurately modelling both the appearance and the behaviour of dense urban spaces is a significant challenge. The recent surge in urban data and its availability via the Internet has fomented a significant amount of research in computer graphics and in a number of applications in urban planning, emergency management and visualization. In this paper, we seek to provide an overview of methods spanning computer graphics and related fields involved in this goal. Our paper reports the most prominent methods in urban modelling and rendering, urban visualization and urban simulation models. A reader will be well versed in the key problems and current solution methods.

Keywords: urban modelling, procedural modelling, visualization, computer graphics

ACM CCS: I.3.3 [Computer Graphics]: Picture/Image Generation; I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling; I.3.7 [Computer Graphics]: Three-dimensional Graphics and Realism; I.6.3 [Simulation and Modeling]: Applications

1. Introduction

Modelling the appearance and behaviour of urban spaces is a great challenge. An urban space is a complex collection of architectural structures arranged into buildings, parcels, blocks and neighbourhoods interconnected by streets. Understanding, describing and predicting the appearance (e.g. creating 2D/3D geometric models) and behaviour (e.g. simulating urban development over time) of cities is useful in a growing number of applications. Traditionally, modelling urban spaces has been a rather manual task that consumes significant amounts of resources. With the growing requirements of quantity and quality in urban content, there is an imperative need for alternative solutions that allow for fast, semiautomatic urban modelling.

Urban modelling methods are important in a growing number of applications. Some of them are

- Mapping and visualization: Reconstructing existing urban spaces for mapping and navigation tools, visualizing previously existing cities for which only partial data exists and allowing architects to visualize a new city.

- Entertainment: Fast generation of detailed digital content for populating urban areas in video games and movies.

- Emergency response: Creating models to train emergency response personnel in current and speculative urban layouts, including planning evacuation routes for various catastrophes, and suggesting emergency deployments of resources.

- Urban planning: Predicting outcomes of land use policies and their effect on existing neighbourhoods, and creating hypothetical views of an urban space after applying development and growth algorithms.
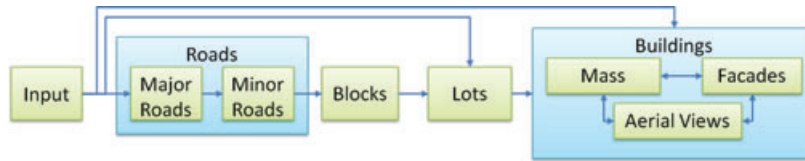
Figure 1: *General pipeline for modelling urban spaces.*

1.1. Challenges

Urban spaces are difficult to model because the underlying structure is determined by a very large number of hard-to-quantify variables including land policies, market behaviour, transportation infrastructure, governmental plans and population changes. Moreover, dense urban environments are particularly complex to model because they are simultaneously dense and large, spanning from a few to hundreds of square kilometres. Although works in computer vision and photogrammetry have focused on the challenge of acquiring and reconstructing existing urban spaces (e.g. from LIDAR data, from aerial and terrestrial imagery [HYN03]), in this report we focus on the challenges of simulating, modelling and rendering digital content of new urban spaces. As opposed to shorter term simulations of traffic flow and crowds in 3D environments, our use of the term simulation refers to imitating long term behaviours of an urban space such as urban development, the result of land use policies and the influence of the transportation network.

On one hand, the challenging problem of the 3D modelling and rendering of urban spaces is being tackled in computer graphics. We present a summary of efforts for automatic generation of visually appealing synthetic and real-world urban scenarios and for their efficient rendering. On the other hand, urban simulation and visualization are used by urban planners as a tool for decision-making regarding land use policies in current and projected urban areas. Such urban simulators generate large amounts of data that needs to be interpreted by decision makers. The challenging problem of visualizing such data is addressed by several works that we also present. However, the above-mentioned efforts have been largely independent, with computer graphics researchers focusing on complex and visually appealing 3D models, whereas urban planners focus on accurate urban dynamics and behaviourally-validated simulations. Thus, we also provide a short insight into a few new methods that address the challenge of bridging the gap between these two groups of approaches.

1.2. Table of contents

The contents of this paper are motivated by the recent proliferation of urban modelling publications in some of the top conferences and journals in computer graphics and related areas. We expect an even faster expansion of the field will take place within the following few years. Furthermore, the need for an integration of the multi-disciplinary efforts towards the simulation, modelling and rendering of urban spaces is clear. Bearing this in mind, we believe that an article documenting the recent advancements and the forecasted and desired future work in urban modelling will be valuable to the research community. Thus, we have brought together researchers in urban modelling to produce a survey of the most prominent methods available and, consequently, to encourage other researchers to pursue further integrated research.

The literature review we provide includes works in the following major areas:

- urban modelling methods,
- rendering acceleration techniques and
- urban simulation and visualization algorithms.

2. Modelling of Urban Spaces

In this section, we focus on recently developed methods for creating models of urban spaces. Although the categorization of such varied methods as those presented in this paper is debatable, we have opted to group the methods based on (i) the part of the urban modelling pipeline (Figure 1) that they address and (ii) the dimension of the geometric entities that they produce. Table 1 presents a summary of the methods mentioned in this section. The urban modelling pipeline roughly consists of producing a road network, subdividing the blocks extracted from the road network into lots, and generating a building inside each lot. Different inputs to this process have been used including target architectural designs, example 3D models or imagery, socioeconomic data sets and tensor fields. As an introductory framework for the presentation of urban modelling methods, general concepts behind procedural urban modelling are discussed in Section 2.1. Next, we present several techniques for modelling road networks and layouts in Section 2.2, which generally synthesize 2D geometric entities and aerial imagery. Finally, several methods for procedural generation of 3D building mass and facades are presented in Section 2.3.

2.1. Procedural modelling

The modelling of urban structures has been performed using several approaches including procedural modelling, synthesis methods and other semi-automatic creation mechanisms.

Table 1: *Urban modelling methods with respective input and output data, grouped according to the main urban modelling problem they address.*

Method	Input	Output
Layout modelling		
Hertzman *et al.* (2001)	Small aerial image, small map, large map	Aerial image for the large map
Parish, Müller (2001)	User painted maps	Street network, facade shader
Aliaga *et al.* (2008)	Example urban layout and aerial imagery	New road network and imagery with similar style
Chen *et al.* (2008)	Interactive editing of a tensor field	Road networks
Weber *et al.* (2009)	Simulation parameters, user interaction	Road network, parcels, land use information
Building modelling		
Hahn *et al.* (2006)	Building exterior, generation rules	3D geometry of building interior
Liu *et al.* (2006)	Architectural mesh	Quad dominant mesh, offset mesh
Müller *et al.* (2006)	Shape grammar rules	3D building and facade model
Aliaga *et al.* (2007)	Facade images	3D building similar to example facade images
Pottman *et al.* (2007)	Architectural surface	Beam and node layout, offset mesh
Merrell, Manocha (2008)	3D polygonal object	Larger collection of similar objects
Cabral *et al.* (2009)	Mesh and textures, user interaction	Reshaped mesh and textures
Facade modelling		
Legakis *et al.* (2001)	3D polygonal mesh	Detailed 3D texture on the mesh
Wonka *et al.* (2003)	Shape grammar rules	3D facade model
Haveman (2005)	GML rules, 3D geometry	Procedurally generated amplified geometry
Marvie *et al.* (2005)	L-system rules, functions	3D facade models, 3D building models
Müller *et al.* (2007)	Facade image, some user interaction	3D facade model, grammar for the facade
Finkenzeller (2008)	Scripts, user interaction	3D facade models
Lipp *et al.* (2008)	Interactive editing of shape grammar rules	3D building and facade model
Xiao *et al.* (2008)	Multiple facade images, user interaction	3D facade models, textures

Figure 2: *Procedurally generated urban models [PM01].*

Procedural modelling has been used to automate the generation of complex urban structures, including buildings and houses, to produce digital content from a relatively simple set of parameters and rules (Figure 2). Synthesis methods have extended the concept of texture synthesis to 3D. Moreover, several forms of interactive editing have been proposed to further extend the aforementioned approaches and increase ease of use.

Procedural architectural modelling can use one of several production systems such as Semi-Thue processes [DSWD94], Chomsky grammars [Sip96], graph grammars [HER99], shape grammars [Sti75], attributed grammars [Knu68], L-systems [PL91] or set grammars [WWSR03]. One aspect that guides the choice of production system is expressiveness. The first fundamental question is therefore: *how many different buildings can be modelled*? If expressiveness is the main criterium it would be possible to just start modelling using C++ or Turing machines. This would enable us to compute all types of architecture and modelling would also be very flexible. However, there is also the question of efficiency: *how efficiently can a designer work with the framework*? This second question makes simplifications and limitations more attractive. Otherwise, the complexity of the procedural model can quickly get out of hand and the design often becomes inconsistent.

In architecture, Stiny pioneered the idea of shape grammars [Sti75, Sti80]. These shape grammars were successfully used for the construction and analysis of architectural design [DF81, Dua02, Fle87, KE81, SM78]. The original formulation of the shape grammar operates directly on an arrangement of labelled lines and points. Rules of the derivation can basically be modelled by drawing lines and points and labelling them. In practice this leads to a derivation problem, because at every step of the iteration there are typically many different transformation under which a rule can
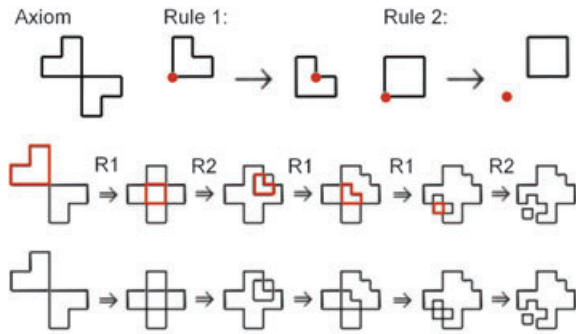
Figure 3: *Shape Grammar. This figure shows an example of a traditional shape grammar. The shape grammar operates on arrangements of lines. The first row of the figure shows the two rules of the shape grammar. The red dot is used to denote a reference coordinate system, so that the translation of the shapes used in the rule is defined. The second and the third row show a derivation of a new shape (right) starting from an axiom (the initial shape) shown to the left. Two rules are given: rule one and rule two. Which rule was chosen for a derivation step is written over the arrow: R1 means rule one and R2 means rule two. The red lines in the middle row highlight the subshape that is selected for replacement by the grammar.*

be applied and additionally there are many different rules to choose from. Classical shape grammars therefore have a missing piece that prevents an automatic derivation: a control mechanism that selects which rule to apply and under what transformation. In Figure 3, we show an example of a shape grammar and one example derivation.

To make the shape grammar concept more applicable in computer graphics, Wonka *et al.* [WWSR03] and Müller *et al.* [MWH*06] introduced a framework that includes rules to replace shapes with zero, one or multiple other shapes, as well as mechanisms to specify an automatic rule derivation. Only the automatic rule derivation enables large-scale procedural modelling. The framework by Müller *et al.* [MWH*06] was also further developed in the commercial software *CityEngine* [Pro08]. The original grammar by Wonka *et al.* [WWSR03] was focused mainly on size independent design rules of facades using splitting operations. The splitting operation allows breaking down elementary shapes (such as cubes and cylinders) by cutting the elementary shapes along splitting planes. The size independent design rules allow the designer to specify how the location of a splitting plane should change when the size of the elementary shape changes and how many splitting planes should be used for a shape of a certain size. In Figure 4, we show an example of the original split grammar proposed by Wonka *et al.* Other ideas presented in the original paper are several mechanisms for rule selection to ensure consistency while allowing interesting random variations.
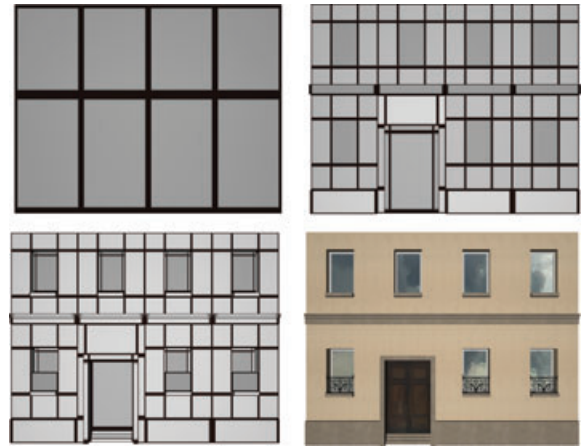

Figure 4: *Split Grammar [WWSR03]. An example derivation of a small facade using splitting rules. Note that the final model is three dimensional.*

2.2. Road network and layout modelling

The modelling of roads and layouts has focused mostly on creating plausible aerial images and street networks for urban spaces. Similar to model synthesis, the creation of aerial imagery builds off the concept of texture synthesis (e.g. [WLKT09]) but in addition to the synthesis of pixel-data, associated vector data is also synthesized. The design of the streets themselves attempts to mimic the visual style of the street networks in real-world urban spaces.

Hertzmann *et al.* [HJO*01] introduced a two-phase design framework for image processing that can be directly applied to texture synthesis by example and is applicable to urban aerial imagery. In the first phase, a pair of images, with one image purported to be a filtered version of the other, is presented as training data. In the second phase, the learned filter is applied to some new target image to create an analogous filtered result. This method supports a wide variety of image filter effects including traditional image filters, super-resolution, improved texture synthesis and texture-by-numbers. In the last of these applications, realistic scenes composed of a variety of textures are created using a painting interface. New imagery is synthesized by applying the statistics of a labelled example image to a newly labelled image. An interesting example of this works consists of synthesizing an aerial view of a city, as it was shown by the authors in their work (Figure 5).

More recently, synthesis approaches have been adapted to exploit the typical organization of an urban space. Traditional texture synthesis is not aware of the unique geometrical structure of an urban space (e.g. streets, parcels, building footprints). Although the texture distortion can be measured and perhaps minimized (e.g. [SSGH01]), the resulting imagery is not consistent and valid in the sense of possessing a plausible network of streets, parcels and building footprints.
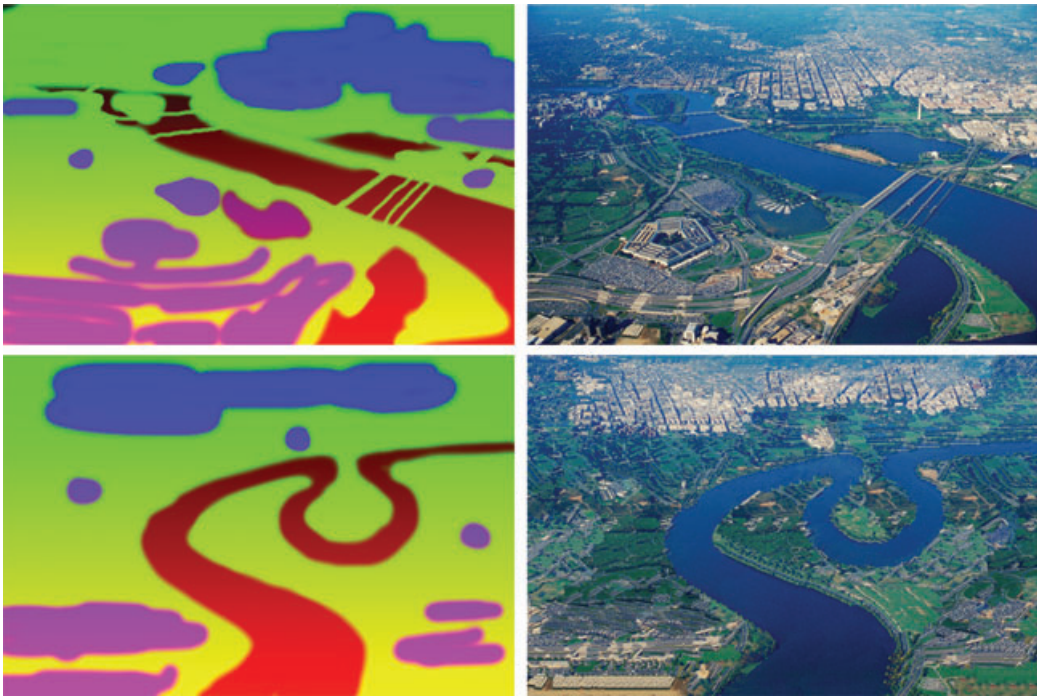
Figure 5: *Image Analogies* *[HJO*01]. Use of image analogies for synthesizing novel aerial views of urban spaces by example. Ordinary texture synthesis cannot reproduce the terrain in the photograph because it is not stationary: far elements are different from near elements. The images are courtesy of Aaron Hertzmann, University of Toronto, Canada, and the Corbis database.*
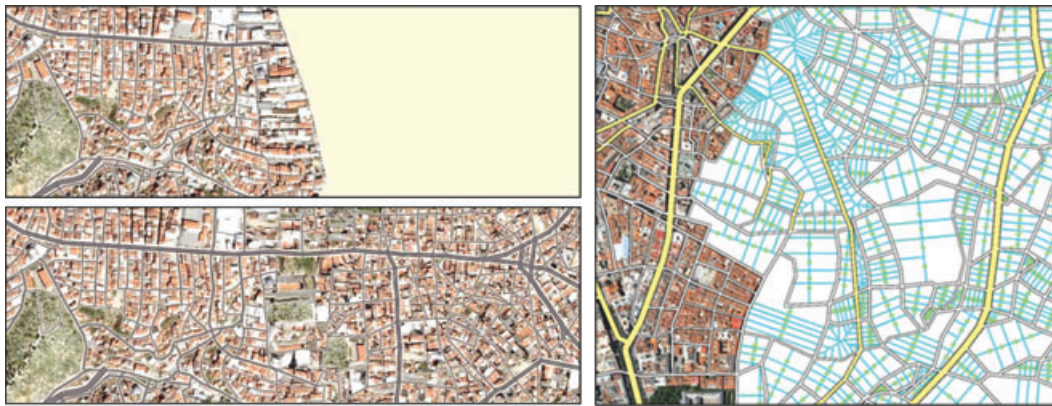

Figure 6: *Example-based urban layout synthesis* *[AVB08]. A new urban layout is generated by extracting and reproducing the structural attributes of the example fragment and reusing aerial-view imagery.*

Aliaga *et al.* [AVB08] propose a method for example-based synthesis of urban layouts that is aware of the structure of an urban space. Their method uses as input a set of example urban layout fragments and simultaneously performs both a structure-based synthesis and an image-based synthesis to generate a complete urban layout with a plausible street network and with aerial-view imagery (Figure 6). Structure and image data from real-world cities are used by the synthesis algorithm to provide several high-level operations that can be used to interactively generate complex layouts by example. The user can create new urban layouts by a sequence of operations such as join, expand and blend without being concerned about low-level structural details.

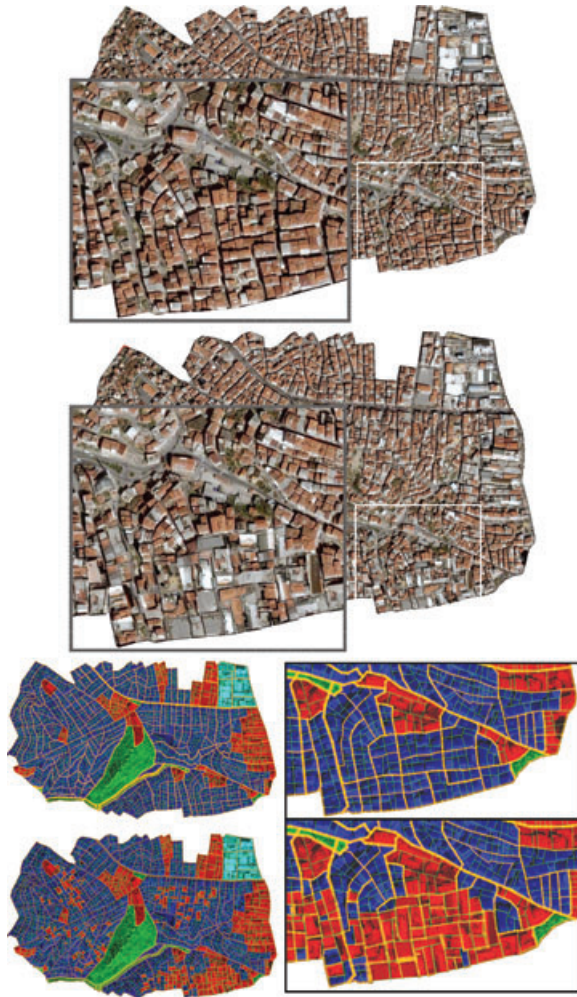In related work, the same authors propose a method for interactive reconfiguration of urban layouts [ABVA08]

Figure 7: *Interactive reconfiguration of urban layouts [ABVA08]. Satellite images of an original and modified urban layout in which a residential zone has been converted to an industrial zone (top and middle). The process consists of recomputing the topology of the affected area to accommodate parcels of a new zoning type, and copying selected tiles from the industrial zone (red) of the city to the previously residential (blue) area (bottom).*

Figure 8: *Interactive procedural street modelling [CEW*08]. The concepts of flow fields and tensor fields are used to model the street layout of a city.*

(Figure 7). In that paper, the image of the urban layout can be changed, but the editing system is aware of urban structure. In particular, the method takes as input the vector data of the streets, blocks and parcels of the urban space, together with aerial-view images of the same space, and considers the connectivity and zoning of the parcels and streets. Several editing operations, such as expand, scale, replace and move, are supported. The urban layout is decomposed into a collection of adjacent tiles, separated by road or parcel boundaries. The specified transformation of the layout is performed by distributing the resulting global deformation among all the
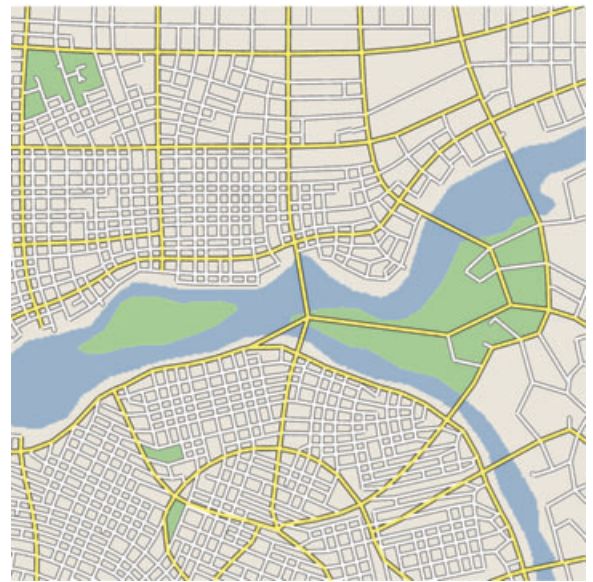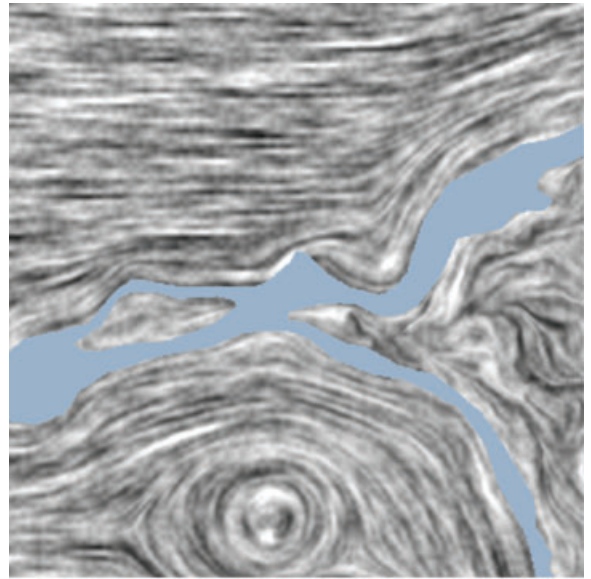
tiles, although preserving their connectivity and minimizing their individual distortion—similar to texture distortion minimization during the texture mapping process for complex objects.

Chen *et al.* [CEW*08] employ the concept of flow fields and tensor fields, as developed in some example-based texture synthesis methods (e.g. [KEBK05]), but rather use it to model the street layout of a city (Figure 8). They build on the observation that for many street patterns there exist
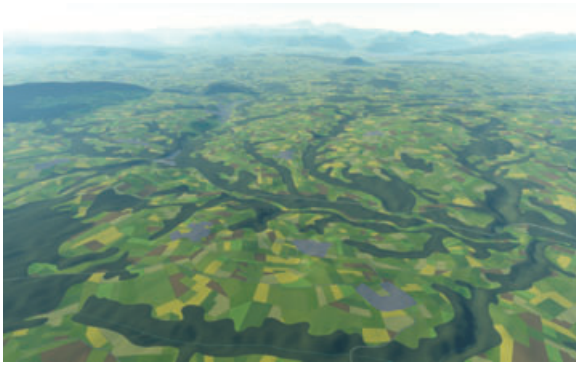
Figure 9: *Real-time rendering and editing of vector-based terrains [BN08]. Large terrains are populated with detailed features such as roads, rivers, lakes and fields in real time. The images are courtesy of Eric Bruneton, INRIA, France.*

two dominant directions due to the need for efficient use of space. Interestingly, tensor fields give rise to two sets of hyper-streamlines: one follows the major eigenvector field and the other the minor eigenvector field. In their paper, Chen *et al.* introduce a modelling pipeline that consists of a tensor field modelling stage and a street graph generation stage. The tensor field modelling stage uses several modelling operations, including hierarchical editing, noise-based tensor field modification, smoothing, a brush interface and the computation of tensor fields from topographical maps. The street graph generation extends existing streamline tracing algorithms to create a street graph. A visually plausible street graph has some constraints on the proximity of streets and on the number of dead ends.

In a related effort to synthesize detailed geometric features of a terrain, Bruneton and Neyret [BN08] addressed the problem of conforming a terrain shape to a set of vector features including roads, rivers, lakes and fields. Their approach is to combine a digital elevation model with layered GIS vector data to get precise feature boundaries and to enforce the constraints of the vector data on the terrain. GPU-friendly data structures and algorithms are proposed to allow for real-time editing and rendering of large terrains (Figure 9).

2.3. Building modelling

Several works have specifically addressed the problem of generating 3D building models. Most of these methods use shape grammars for generating building mass and facades, and propose a variety of tools for efficient user interaction and controllability of the output geometry.

Müller *et al.* [MWH*06] built upon splitting rules, but they added several other components. First, they included shape operations for mass modelling (a kind of rough 3D sketch) by extending turtle commands used in L-systems [PL91]. The

Figure 10: *Rome Reborn. Two renderings of the ancient Rome reconstruction consisting of more than 7000 procedurally generated domestic buildings. The landmarks such as the Colosseum (top) and the Circus Maximus (bottom) have been modeled manually. The images are courtesy of Bernard Frisher, IATH and Procedural Inc.*

mass models are typically created by compositing several elementary shapes. This mass modelling is fairly intuitive and mirrors the actual design process used in architecture. Secondly, they introduced context sensitive rules for the coordinated derivation of different building masses. As a result, a wide variety of buildings can be generated procedurally. In Figure 10, we show renderings of the virtual reconstruction of the city of ancient Rome. Figure 11 shows an extrapolation of New York city 250 years into the future, inspired by the 1997 motion picture The Fifth Element.

Lipp *et al.* [LWW08] introduced the idea of interactive grammar editing. Instead of writing rules with a text editor, the framework of Lipp *et al.* makes it possible to design and edit rules entirely using a graphical user interface. This extension should make procedural modelling more accessible to a larger audience.
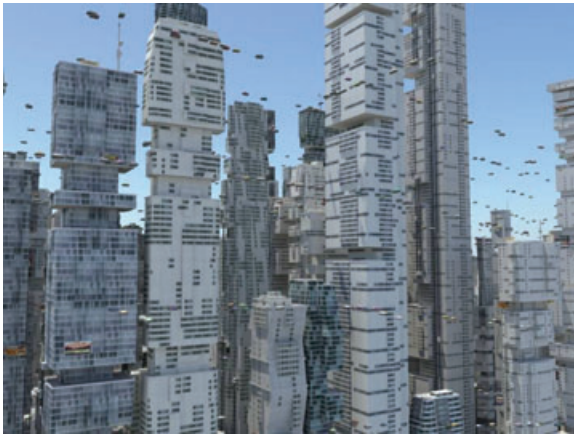
Figure 11: *New York City 2259. Procedurally generated buildings from the New York city model 250 years into the future. The images are courtesy of Procedural Inc.*

Recently, the idea of merging concepts from computer vision and procedural modelling has had some success. Müller *et al.* [MZWG07] use the idea of splitting rules to create a top-down framework for facade analysis using a single orthographic image as input. First, important symmetry offsets are found and the corresponding pixels are collapsed onto each other until a small *irreducible facade* image is computed. Therein global optimization is used to find splitting lines to define rows and columns of facade elements. Further subdivision is done similar to the splitting rules introduced by Müller *et al.* [MWH*06] and finally shape grammar rule parameters can be extracted. This makes it possible to compute variations of the reconstructed building. Figure 12 shows an example of a reconstructed building facade from a single aerial image.

A related approach by Xiao *et al.* [XFT*08] also uses subdivision similar to how architecture is modeled with a grammar. In contrast to grammar-based modelling they use subdivision to oversegment the model so they also make use of merge operations. This work applies vision-based technology to reconstruct 3D facade models of high visual quality from multiple ground-level street-view images (Figure 13). Their method uses images captured along streets and relies on structure from motion to automatically recover camera positions and point clouds. The facade is initially considered as a flat rectangular plane with an associated image composited from multiple photos and is further decomposed and augmented using the 3D point cloud information. A key ingredient to creating the high-quality models are interactive tools that allow the user to refine the model.

Aliaga *et al.* [ARB07] proposed a method to construct a grammar from photographed and subdivided buildings, enabling the rapid sketching of novel architectural structures in the style of the original. Using data from several captured

Figure 12: *Image-based procedural modelling of facades [MZWG07]. A building facade image that is used as input to the algorithm (top). A wireframe overlaid over the original image (middle). The resulting 3D model rendered using relighting and shadow maps (bottom).*

models, novel buildings can be designed very quickly and rendered photorealistically or non-photorealistically (e.g. pen-and-ink) but always in a style comparable to the original structures. Further, occlusion removal and colour equalization algorithms make it possible to use highly occluded buildings in varying lighting conditions (Figure 14).

With regards to model synthesis, Merrell [Mer07] introduced a representative method for example-based 3D model synthesis. This approach can be used to create symmetric models, models that change over time, and models that fit soft constraints. A restriction of this first method is that the input objects must fit on an axis-aligned grid. In a second method [MM08], the connectivity between the adjacent boundary

Figure 13: *Image-based facade modelling [XFT*08]. 3D facades are reconstructed from multiple ground-level streetview images. The images are courtesy of Tan Ping, National University of Singapore, Singapore.*

Figure 15: *Continuous Model Synthesis [MM08]. All these building models are automatically generated from a single example model. Different textures are applied to the buildings, but the shape of each building resembles the shape of the input. The images are courtesy of Paul Merrell, University of North Carolina, USA.*

can be computed to assign brick patterns to building surfaces. A great example for a general and powerful modelling language is generative mesh modelling using GML introduced by Havemann in his Ph.D. thesis [Hav05]. GML allows the specification of commands that can refine and define a mesh.

There have been several other papers that propose alternative procedural modelling methods for architecture. Greuter *et al.* [GPSL03] present a method for generation of pseudo-infinite cities, in which all geometrical components of the city are generated in real time as they are encountered by the user. Marvie *et al.* [MPB05] propose some extensions to L-systems to make them more suitable for architectural modelling. Finkenzeller [Fin08] and Birch *et al.* [BBJ*01] introduce an interactive procedural modelling framework. Hahn *et al.* [HBW06] present a solution focusing on building interiors. Cabral *et al.* [CLDD09] present an approach for modelling architectural scenes by reshaping and combining existing textured models, where the manipulation of the geometry and texture are tightly coupled.

Another completely different method of computational design is necessary when the objective is to compute interesting free form surfaces that are popular in modern glass buildings that try to impress with geometric complexity. Liu *et al.* [LPW*06] use sequential quadratic programming to compute a quad dominant panel layout on a surface. This idea was extended by Pottmann *et al.* [PLW*07] and [PSB*08] to include more general layouts. Another interesting work is the computation of beam layouts by Smith *et al.* [SHOW02] for truss structures.

Figure 14: *Style grammars for visualization of architecture [ARB07]. The user creates and subdivides an initial model of the building (top left). Repetitive patterns of the building features are automatically found and a representative grammar is constructed (top right). The user can then view the captured model (bottom left) and change the model on the fly producing new models (bottom right).*

features of the input model is exploited to overcome the previous limitation, and models with arbitrary orientations are computed which have similar connected features and resemble the sample models (Figure 15).

Although the previous papers focused more on high-level modelling primitives and on designing the overall architectural structure, there are strategies better suited for modelling details and for modelling more general designs. A notable example for detail modelling are cellular textures [LDG01] that
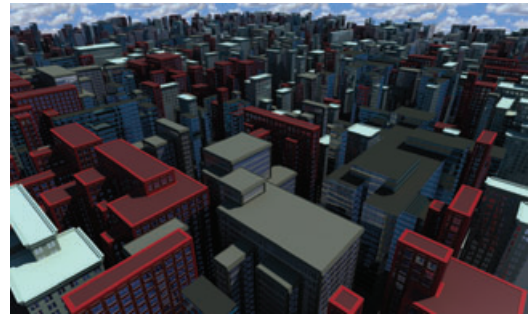
3. Rendering Acceleration Techniques

We briefly review methods for rendering acceleration of large urban environments divided into the following categories: mesh data structures, visibility culling and simplification.

Modern graphics hardware requires that the models are processed in a specific format. Typically, indexed data structures are used to cache per-vertex computations and geometry has to be batched together in larger data structures, as the rendering of individual triangles is inefficient. In addition, rendering needs to minimize changes such as switching of textures and shader programs. Hoppe introduced one of the first algorithms to reorder triangles to efficiently utilize the vertex cache [Hop99]. Other interesting ideas are to extend this concept to order the geometry to be additionally aware of pixel-level occlusion culling [SNB07], or to create cache-oblivious mesh layouts [YLPM05]. In general, these basic optimizations are beneficial for all interactive applications. In the context of urban environments, these data-structure optimizations are beneficial for finely tesselated architectural details, but not for urban mass models with a few planar polygons and most details stored in texture maps.

Besides the well established and easily implementable techniques of back face culling and view frustum culling, occlusion culling can provide speed-ups of several orders of magnitude for most large and dense urban scenes. This is because an urban environment usually lies on a locally flat surface and the nearby urban structures easily fill the field-of-view and prevent observing distant structures. Occlusion culling either can rely on a precomputation or can be computed online. To precompute visibility, the navigable space is broken down into smaller volumetric view cells. Researchers have tackled the question of how to efficiently compute visibility for volumetric view cells and proposed multiple algorithms [SDDS00, DDTP00, WWS00, LSCO03, NS04, WWZ*06]. The advantage of precomputation is that occlusion culling needs very little runtime overhead and that the results of precomputation can be used for other preprocessing algorithms, such as level-of-detail selection and rendering time estimation. The disadvantage are the complexity of the algorithms and the long preprocessing times. Online visibility typically computes visibility for each frame of an interactive simulation from the current view point. In a seminal paper Greene *et al.* [GKM93] introduce the *hierarchical z-buffer* algorithm. The algorithm allows the identification of large parts of the scene that are guaranteed to be occluded and that is especially suited to rasterization based rendering. Currently, a complete implementation of the hierarchical z-buffer is not supported by graphics hardware, but the gap is closing. Today's graphics hardware includes several useful features for occlusion culling, including pixel level occlusion culling to avoid expensive shading operations and a hierarchical z-buffer with a few levels (e.g. three) to discard smaller tiles of pixels during rasterization. The book by Möller and Haines has a good overview of the current state of the art [MH02]. Another important feature are occlusion queries that can test bounding volumes for occlusion. A major challenge in this context is to hide the latency of the occlusion queries and to utilize temporal coherence [BWPP04, GBK06, MBW08]. In all cases, when the viewpoint is far above the city occlusion culling has often little impact and other strategies need to be used that focus on simplifying the environment.

An interesting challenge is to compute geometric levels-of-detail (LOD) for urban models. There are many strong methods for general mesh and object simplification. A good introduction is the book by Luebke *et al.* [LRC*02]. Two fundamental early papers introduce operations to collapse edges [Hop96, GH97] in triangle meshes. Although these methods work well for objects that contain smooth surfaces modeled with many triangles, there are substantial obstacles when applying them to urban environments. Leaves in vegetation and planar structures with many sharp corners are common, resulting in discontinuities that cause many problems for traditional level-of-detail techniques. Some special purpose methods have been invented for building footprints. For example, Chang *et al.* [CBZ*08] describes a clustering based simplification method for urban spaces inspired by Kevin Lynch's Image of the City book, but in general a high-quality automatic simplification of urban environments is an unsolved problem. The main aspect of urban environments that can benefit from level-of-detail is the terrain. Early approaches, for example [LKR*96], built on the assumption that triangle rendering is expensive and therefore the algorithms were quite sophisticated. Current algorithms try to incorporate the fact that larger batches of triangles need to be rendered at once and focus on algorithm simplicity [LH04] and large memory management [GMC*06]. For architecture and vegetation, simple alternatives to automatic simplification are to create multiple versions of each building or tree procedurally [MWH*06] or by hand. Further, procedural modelling can help simplification by providing semantic data. For example, it is very helpful to know what the actual facade planes are and what geometry belongs to one facade. Finally, because man-made structures are hard to simplify with geometric methods ([WFM01]), another option is to use image-based approaches. Several recent papers try to explore a hybrid rendering architecture where rasterization provides the rough object outlines and the fragment shader implements a ray tracer to generate details [POJ05]. An adaption to urban environments are block maps to ray cast collections of buildings [CDG*07] or facade displacement maps to ray cast facade details [AYRW09]. More general image-based techniques simplify scene parts and replace them by images often called impostors. An online version of this algorithm idea can dynamically create and store image-based representations in a hierarchical data structure [SLS*96, SS96], and offline versions can obtain guaranteed frame rates [AL99, JWSP05] and be integrated with level of detail and occlusion culling [ACW*99]. The advantages of image-based simplification is that it is significantly more robust than geometric simplification and it also works for difficult cases such as many disconnected plant leafs. The disadvantage of impostors are the high storage requirements, some challenges in recomputing the shading of the computed representations and the quick generation of high-quality representations. In
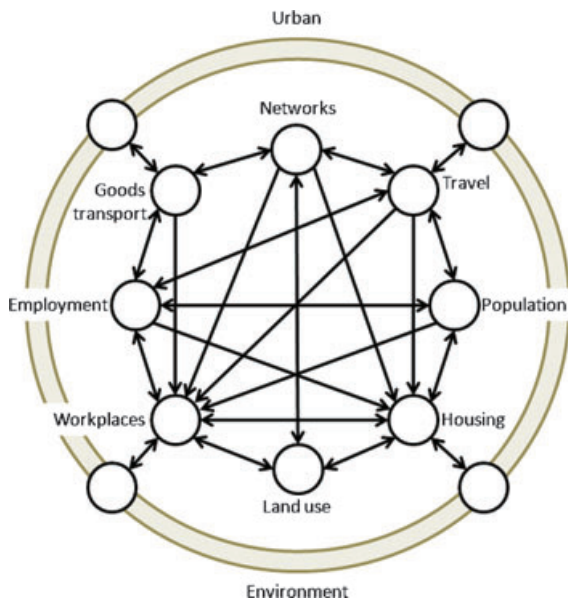
Figure 16: *Diagram of the interactions in urban simulation models (recreated from [Weg94]).*

summary, there is no simple solution to simplifying all aspects of urban environments.

4. Urban Simulation and Visualization Algorithms

Urban simulation models and the visualization of computed data sets are used to help regional planning agencies evaluate alternative transportation investments, land use regulations and environmental protection policies. The simulation models typically output massive spatially distributed data about several variables, including number of inhabitants, land prices and traffic. Urban simulation systems (e.g. [WBN*03]) generate predictions of real estate development, prices and location choices of households and firms at fine-grained levels of geography such as grid cells or parcels, over entire metropolitan areas and over planning horizons of up to 30 years. The amount of data generated by such a microscopic model over a long forecasting horizon and a large scale is overwhelming for users to easily interpret. Visualization techniques are essential to be able to render useful information from the mass of data generated by such simulations.

4.1. Urban simulation

Urban simulation refers to the use of behavioural or process modelling of the the spatial patterns of urban economic agents and objects such as jobs, population, housing and land use. In Figure 16, we present a generalized framework for describing the behaviours that urban simulation models attempt to represent in varying degrees of comprehensiveness

and with differing approaches to temporal abstraction and detail of agents and location [Weg94, Weg04]. It highlights the interaction among household and business agents that are locating in housing and non-residential buildings, and on the differing time scales of the evolution of buildings, transportation networks, urban form and travel that connects the agents within the urban system.

Three dominant paradigms for creating urban simulation models have appeared in the field. Early models attempting to represent emergent dynamics adopted cellular automata (CA) as the modelling framework [TO01]. One of the most widely known is the Urban Growth Model [Cla98]. It has been applied to long-term changes in land cover patterns classified from remote sensing data [AkWS08]. Unfortunately, this modelling approach only simulates the conversion of non-urban land to urban use, based on the characteristics of cells and their immediate spatial context, and does not address changes to the built environment or its occupants, or the travel that connects agents.

Agent-based models (ABM) have extended the CA framework to include mobile, interacting agents in an urban spatial context. This work has focused on examining cities as self-organizing complex systems, and solutions have been designed to explore the emergent properties of agents with relatively simple behavioural rules embedded by the modeller [Por00]. However, relatively little attention has been paid to issues of validating models using observed data or trends, and as with CA models, most ABM urban simulation models have behaviour that is influenced only by localized context.

An alternative approach to urban simulation has emerged from a combination of urban economic analysis with statistical modelling of choices made by agents in the urban environment, such as households choosing residential locations. This work builds on the pioneering work of McFadden on Random Utility Theory [Mcf74] and the development of discrete choice models, for which he recently won the Nobel Prize in Economics. Research using this approach diverges on the dimensions of temporal representation and level of aggregation. Aggregate models represent agents by grouping them into types, and locations into large zones, whereas microsimulation models represent individual agents such as households and jobs, and objects such as buildings and parcels. Similarly, there are contrasting approaches to the representation of time, with earlier research focusing on equilibrium in a set of equations of locating agents and buildings, and later work exploiting a dynamic representation that uses explicit chronological time and incorporates path-dependence. Examples of the aggregate and equilibrium approach to urban simulation include spatial interaction models [Put91] and equilibrium discrete choice models [AK96, Mar96].

An example of more recent work in this area using a dynamic microsimulation approach is UrbanSim [Wad02,

dPPW07], which simulates the choices of individual households, businesses and parcel landowners and developers interacting in urban real estate markets [WUFL07]. This approach works with individual agents as is done in the ABM, with very small cells as in the CA approach, or with buildings and parcels. But it differs from these approaches by integrating discrete choice methods, an explicit representation of real estate markets, and statistical methods to estimate model parameters and to calibrate uncertainty in the model system [ŠRW07].

4.2. Urban visualization

Visualization and computer graphics have played an integral part in the development and use of urban simulations of several types. A number of works have focused in developing novel visualization techniques for better understanding the results of urban simulation models.

Several groups of population with different levels of expertise in handling urban simulation data are normally interested in these results, including urban planners, policy makers, the public or even the modellers running the simulation. On one hand, traditional information visualization techniques have focused in handling large urban simulation data sets and making their analysis more intuitive to urban planners. On the other hand, recent research works have proposed an interdisciplinary collaboration between computer graphics, visualization and urban modelling to produce new visualization techniques of urban simulation data sets. These techniques aim to facilitate the presentation and increase the impact of urban simulation data to different population sectors.

Traditional visualization approaches generally make use of techniques including choroplethic (coloured) maps generated by exporting simulation results, summarized by a zonal geography, to a Geographical Information System (GIS) for rendering; other variants include animations generated by rendering a series of such 2D maps in a loop, viewing different time slices or quantities and 3D renderings of simulation results by extrusion of polygonal forms to indicate density, or by spatial smoothing in the form of contour or terrain maps with the elevation representing some quantity of interest.

Batty [Bat92] first introduced various approaches that relate urban modelling, GIS and computer graphics. The same author later described the impact of virtual reality and 3D visualization to GIS and he has demonstrated this on a variety of complex examples [BC97]. More recently, Batty presented a comprehensive view of urban dynamics in the context of complexity theory [Bat07].

Although there has been a large amount of work on GIS, very little research has been done evaluating the usefulness of other types of visualizations for this domain. A study by Pinnel et al. [PDBB00] examines various visualization types and attempts to find appropriate visual representations for urban modelling tasks. The types of visualizations considered include graphs, pie chart, 2D and 3D maps, symbol charts and bubble charts. They cross-reference each of these types with the encodings that can be effectively utilized (e.g. colour intensity, bars, area/height, marker size, marker shape). Their study concludes that for urban planning and analysis, map type visualizations provide the necessary geographical information, although for quantitative tasks bar charts and summaries better present the needed information.

A widely used urban visualization technique is cartograms which use map shape warping to visualize relationships and values of urban and geospatial datasets (e.g. [KNP04]). The core idea behind cartograms is to distort a map by resizing its regions according to a statistical parameter, but in a way that keeps the map recognizable.

Chang et al. [CWK*07] propose an aggregation method that combines buildings and city blocks into legible clusters. Their goal is to visualize an urban model in a focus dependent and multi-resolution fashion, although retaining the legibility of the city. In their approach, the 3D model view and the data view are integrated so that relationships between the geospatial information of the urban model and the related urban data (e.g. census information) can be more intuitively identified. Although the user-study that they conducted showed that some features introduced by their system enhanced the user's ability to better understand an urban model, they also noted that creating legible cities for users of all backgrounds is not a trivial task and would require knowledge of the user's perspective of the city before creating the clusters.

Dykes and Brunsdon [DB07] introduced a series of geographically weighted (gw) interactive graphics to explore spatial relationships between geographic processes. These techniques include standard colour maps, maps of gw-means, gw-residual maps and a localized version of the box-and-whisker plot. The techniques introduced reveal information about geographically weighted statistics at several scales concurrently.

Roman et al. [RGL04] presented an interactive system for constructing multi-perspective images from sideways-looking video captured from a moving vehicle. The input to their system is a set of video frames with known camera pose. Their system automatically computes an additional cross-slits camera between every pair of adjacent user-specified cameras leading to a smooth interpolation of viewpoint in the final multi-perspective image. Multi-perspective image of a whole city block can be created in a few minutes. The goal of this work is to simultaneously view real-world urban scenes that cannot be captured in a single photograph, rather than to visualize the simulation data of an urban space. New techniques could be explored that combine a multi-perspective approach for data visualization.

To date, simulation systems such as UrbanSim have been relatively limited in their scope of visualization, in spite of
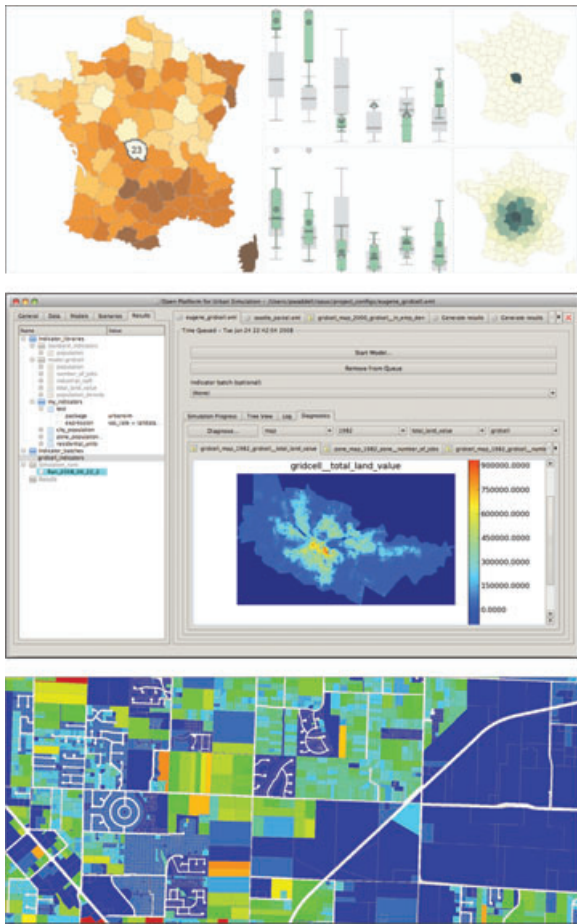
Figure 17: *Traditional Urban Visualization. (Left) Multivariate geographically weighted boxplots are one of the data visualization tools for scale-varying exploratory analysis presented by Dykes and Brunsdon [DB07]. (Middle) A screen snapshot from the Indicator system supported by UrbanSim [SB07]. The program is a web-based interface for visualizing and browsing indicator results in the form of tables, charts, or maps. The figure shows a choroplethic visualization of the land value in the city of Eugene, Oregon. (Right) A standard choropleth map traditionally used to visualize the results of an urban simulation.*

providing a sophisticated economic and behavioural simulation engine to model the location and travel choices of millions of agents in the system. A typical scenario is that manual post-processing of simulation results must be done by a model user to extract summary indicators from the results, export them from the simulation environment into a GIS system, establish relational joins of the indicators to existing GIS layers and then manually rendering thematic or choroplethic maps to render the spatial variation in the resulting indicators (Figure 17). As used in the planning literature,

an indicator is a variable that conveys information on the condition or trend of one or more attributes of the system considered. The work of Schwartzmann and Borning [SB07] developed a web-based indicator system for UrbanSim and evaluated design techniques including value sensitive design, paper prototyping and frequent user testing.

Virtual reality visualization techniques have also been explored for urban modelling and urban planning. For example, Drettakis *et al.* [DRRT07] presents one of such techniques applied to a small-scale real-world scenario. The paper concludes that appropriate levels of realism such as spatialized 3D sound, high-detail vegetation and shadows and crowds, enable better appreciation of overall ambience of the virtual environments, perception of space and physical objects as well as the sense of scale. Thus, using virtual environments for the visualization of large-scale urban simulations is a desirable line of future work with significant challenges to overcome.

4.3. Bridging the gap between simulation and visualization

This process of simulation and then visualization has many limitations, not the least of which is the level of effort. As a result, too little visualization is actually done in practice, and this leads to diminished access to the simulation results, and reduced diagnostic capacity to determine when there are problems in the simulation. One option being recently explored is to more tightly integrate visualization efforts with the simulation process, achieve a beneficial and symbiotic relationship and ultimately lead us to a more desired and integrated approach to urban modelling.

A recent approach for larger scale problems is that of Weber *et al.* [WMWG09] who proposed to combine procedural modelling techniques with urban simulation to obtain three-dimensional models that change over time (Figure 18). The system includes a street expansion algorithm to place new streets, a land use simulation and a traffic simulation as major building blocks. Although much simpler than full-featured urban simulators, the simulation is interactive and the user can make modifications during the simulation, such as controlling the major growth areas, editing road segments, editing land use and changing parameters used to control the simulation. The goal of this work is to provide a generic framework that can be configured for different types of land use categories.

Another recent approach proposed by Vanegas *et al.* [VABW09] uses urban layouts for the visualization of urban simulation results. This work builds upon existing visualization techniques of urban simulations and extends them by automatically inferring new urban layouts for any time step of the simulation sequence, by considering both the values of the state variables of the simulation model, and the original street network, parcels, and aerial imagery of the simulated
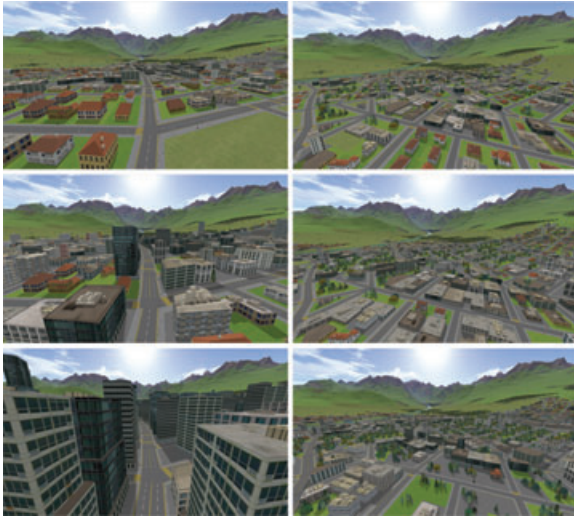
Figure 18: *Interactive Geometric Simulation of 4D Cities [WMWG09]. Two time series are shown in the columns. The left column shows the transition from low density to high density in the city centre. The right column shows a transition of a city based on sustainable development with sufficient green areas.*

city (Figure 19). The inference algorithms gather stochastic data of the original urban layout and use the simulation state values to obtain a plausible urban layout, consisting of new parcels, streets and imagery (e.g. vector and image data). Altogether, this approach allows for traditional visualizations as well as that of new content. It has been applied to visualize a 16,300 km^2 urban space.

Both of these works make the initial strides towards the ultimate goal of a more integrated approach of urban modelling, simulation and visualization.

5. Conclusions, Challenges and Open Problems

Providing realistic and plausible models of dense urban spaces is a challenge that requires knowledge from several disciplines. The pursuit of the accurate modelling of urban spaces is of significant interest today to urban planners, to emergency management and to visualization efforts. In recent years, the data collected via several forms of acquisition is available, through the Internet, to a widespread audience and has fomented significant activity and applications. Several geometric modelling methods have focused on urban spaces to improve their flexibility and efficiency. Simultaneously, simulation models are becoming increasingly sophisticated and better able to represent the complex processes occurring

Figure 19: *Visualization of Simulated Urban Spaces [VABW09]. (Left column) An overview of the simulated region (borrowed from Google Maps) along with the parcel geometry inferred from the urban simulation data for a subset of that region. (Middle column) A close up of a part of the city where new developments are predicted by the simulation after a 30-year period, and their geometry inferred by our system. (Right column) A close up of a different part in which parcel subdivision was indicated by the simulation.*

in urban spaces. These simulations serve to better understand urban spaces and to help guide 3D modelling and rendering efforts to produce more realistic and interactive imagery.

In this article, we have attempted to help guide future efforts in urban modelling to have a better understanding of the multiple aspects of this challenge. We look forward to more holistic approaches and to multi-disciplinary collaborations.

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