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# On validating STEP product data exchange

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**Abstract**

Product data exchange requires exchanging geometrical shape data that may have to be represented differently in the sending and in the receiving system. Since the translation process thereby entailed can lead to errors, emerging exchange standards include numeric invariants computed from the shape. Agreement of these invariants in the sending and receiving system are then used to increase confidence in the translation process. In this paper we show that there are many classes of noncongruent solids for which all the invariants of volume, surface area, moments and products of inertia agree. Thus these quantities alone are insufficient to exclude many translation errors. We also consider whether the examples are realistic and find that in today's constraint-based shape constructions the examples are not unlikely to occur.


*Keywords:* Geometry data exchange; STEP; Product data exchange; Shape invariants; Inertial properties

## 1. Introduction

Product data exchange is of growing importance in manufacturing. This importance is driven by the cost and effort to exchange data between different CAD systems and is unavoidable because of the underlying globalization of the supply chain. STEP standards for data exchange have been devised and continue to evolve to respond to this need. Of particular concern in practice is the exchange of geometrical data [1]. In part, the difficulty of exchanging geometry is due to proprietary technology that is used to devise special surfaces such as sweeps or to represent with acceptable accuracy surface intersections as well as proprietary geometrical constraint solvers. But the difficulty is also due to the fact that different geometry kernels work with different representations. Devising accurate conversions between those representations remains, in some cases, a research issue.

Against this backdrop of geometrical data exchange problems, STEP standards include the use of numeric invariants of volume and surface area to increase the confidence in the shape translation. While it is obvious that there are many shapes that are not congruent, yet have the same volume and surface area, it is not known whether the same can be said about two shapes that have the same volume and surface area, as well as the same moments and products of inertia. In this note we prove that there are two simple convex shapes that are not congruent, yet have the same volume, surface area, moments and products of inertia. While the inclusion of the latter invariants is helpful, it is therefore not sufficient to distinguish between noncongruent shapes.

This result is strengthened in a number of ways. Integral properties of point symmetrical functions, for which $f(x, y, z) = f(-x, -y, -z)$, are of no help. Restricting to convex bodies also is of no help. Considering the invariants in a coordinate system at the centroid is of no help. Finally, the constructions that demonstrate these properties point to the distinct possibility that translation errors due to constraint solving variants are especially vulnerable.

Although our focus in this note is on shape data exchange, we note that related questions arise in the semantics of CAD. Briefly, how do we associate with a mathematical description of a solid $A$ a representation $A'$ that, by efficient computation, can be guaranteed to be *close* in some metric? We shall say more about this question below.

## 2. Definitions and notation

Given a solid object $V$ and a line $L$, the *moment of inertia* about the line $L$ is the integral

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$$I_L = \int r^2 \rho \mathrm{d}V.$$

That is, the volume elements d$V$ are summed weighted by the squared distance from the line. The quantity $\rho$ is the density of the solid. In the following, we assume solids of homogeneous density and thus omit the factor $\rho$.

Assume that the solid $V$ is given with respect to a coordinate system with origin $O$. Then the *product of inertia* in the coordinate system, with respect to the coordinate axis $z$, is the integral

$$J_{xy} = \int xy\rho \mathrm{d}V.$$

That is, the volume elements d$V$ are summed weighted by the product of the coordinates $(x, y)$ of d$V$ and multiplied by the density $\rho$. Again, we assume homogeneous solids and omit $\rho$ in the following. The products of inertia $J_{yz}$ (with respect to the axis $x$) and $J_{xz}$ (with respect to the axis $y$) are defined similarly.

Note that a solid that is symmetrical about the $yz$-plane has zero products of inertia $J_{xz} = J_{xy} = 0$.

Point-symmetry $\mu$ with respect to the origin of the coordinate system is defined by requiring that the point $P = (x, y, z)$ correspond to the point $\mu(P) = (-x, -y, -z)$.

Plane symmetry $h_x$ with respect to the $x = 0$ plane is defined by requiring that the point $P = (x, y, z)$ correspond to the point $h_x(P) = (-x, y, z)$ in a suitable coordinate system. Symmetries with respect to other coordinate planes are defined analogously, namely $h_y(P) = (x, -y, z)$ and $h_z(P) = (x, y, -z)$. Note that if two bodies are symmetrical with respect to a plane, then there is a coordinate system in which this plane is $x = 0$.

We recall the *regularized Boolean set operations* of Constructive Solid Geometry; e.g. [2]. A *solid* $A$ (in the sense of [2,3]) is *regular*, if the closure of the interior of $A$ is equal to $A$. An example of a nonregular solid would be a cube with an attached surface patch (colloquially termed a *dangling face*). To *regularize* a solid take the closure of the interior of the solid. The *regularized union* of two solids $A$ and $B$, denoted $A \cup^* B$, is the closure of the interior of the set-theoretic union $A \cup B$. Similarly, the *regularized intersection* of two solids, $A \cap^* B$, is the closure of the interior of the set-theoretic intersection $A \cap B$, and the regularized difference, $A -^* B$, is the closure of the interior of the set-theoretical difference $A - B$.

## 3. First construction

Consider a cube centred at the origin $O$ of the coordinate system, with vertices at the points with coordinates $(\pm 1, \pm 1, \pm 1)$ (i.e. aligned with the coordinate system). We add to the cube a square pyramid with the four base vertices coincident with the cube's vertices at $(\pm 1, -1, \pm 1)$ and apex $P_0 = (0, -1-u, v)$ where $u = v = 0.4$. Call the resulting solid $C$; see also Fig. 1.

We will construct two convex solids, $A$ and $B$, that are not congruent and demonstrate that they agree in all numerical invariants (which we considered earlier).

To obtain solid $A$, we add to the top face of $C$ a square pyramid with apex $P_1 = (0, 0, 1 + w)$ and choose $w$ arbitrarily so the solid remains convex, say $w = 0.4$. The resulting solid $A$

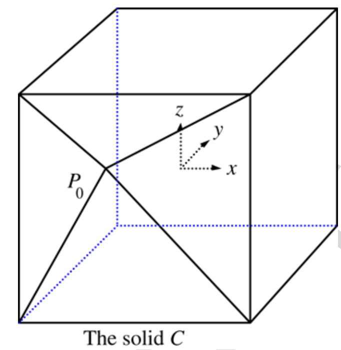


Fig. 1. Core solid $C$ used for constructing solids $A$ and $B$ for Proposition 1.

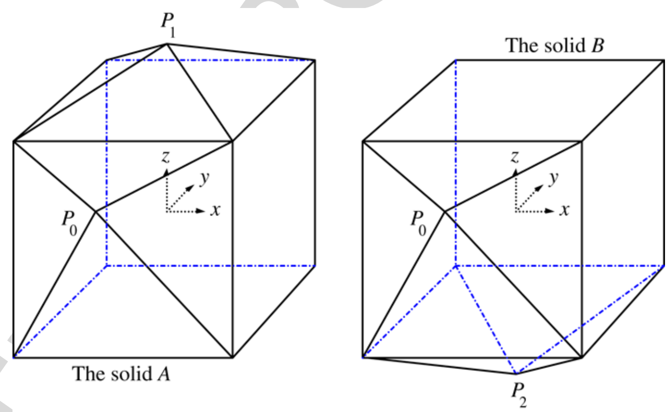


Fig. 2. Solids $A$ and $B$ for Proposition 1.

is shown in Fig. 2 to the left. It is easy to verify that $A$ is convex and that it is symmetrical with respect to the $yz$-plane.

We construct the solid $B$ similarly, by adding to the bottom face of $C$ a square pyramid with apex $P_2 = (0, 0, -1 - w)$ and observe that $B$, too, is convex and symmetrical with respect to the $yz$-plane. The solid $B$ is shown in Fig. 2 to the right.

By the construction, it is obvious that both solids $A$ and $B$ have the same volume and surface area, and that they are not congruent owing to the fact that $u$ and $v$ are positive. We show that they have the same inertial moments.

**Lemma 1.** *Solids $A$ and $B$ have the same moments of inertia about any axis through the origin $O$.*

**Proof.** Let $L$ be any line through the origin and recall that the origin of the coordinate system is at the centroid of the cube. To each volume element d$V$ of solid $A$ that is in the core $C$ of $A$ there is a corresponding volume element of $B$ at the same location. Moreover, to each volume element d$V$ of $A$ that lies in the pyramid with apex $P_1$ over the top face of $C$, there corresponds the volume element $\mu(\mathrm{d}V)$ of solid $B$ that lies in the pyramid with apex $P_2$. Since $L$ contains the origin of the coordinate system, the distance of d$V$ of $A$ to the line $L$ must be equal to the distance to $L$ of the corresponding volume element of $B$. Thus the integral $I_L$ over solid $A$ is equal to the integral $I_L$ over solid $B$. ■

**Lemma 2.** *Solids $A$ and $B$ have the same products of inertia $J_{xy}$, $J_{yz}$, and $J_{zx}$ with respect to any coordinate system with the origin $O$.*



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**Proof.** Consider any coordinate system with origin $O$ and observe that the (regularized) solid

$$X = (A \cup B) -^{*} (A \cap B) = (A \cup B) -^{*} C$$

is point-symmetrical with respect to $O$. That is, we associate with the point $Q = (x, y, z)$ of solid $A$ in the pyramid with apex $P_1$ the point $Q' = (-x, -y, -z)$ in $B$, and with the point $Q$ of $A$ in the core $C$ the same point $Q$ of $B$. Observing this correspondence, it is now obvious that the integrals are pairwise equal: $J_{xy}(A) = J_{xy}(B)$, $J_{yz}(A) = J_{yz}(B)$, and $J_{zx}(A) = J_{zx}(B)$. That is, the corresponding products of inertia are equal. ■

Lemma 2 can be strengthened by dropping the requirement that the coordinate system be aligned with the sides of the cube. This is easy to see: Considering the integral for the products of inertia, we note that the argument of equality rests on the point symmetry $\mu$. As long as the variants have a common core and additions that are point-symmetrical (to each other) with respect to the origin, then the products and moments of inertia must agree. Summarizing, we have the following proposition:

**Proposition 1.** *The solids A and B, as constructed, have identical volume, surface area, moments and products of inertia with respect to any coordinate system with origin at the centroid of the cube of C. They are convex and not congruent.*

Note that both moments and products of inertia are integrals of point-symmetrical functions. That is, the integrand $f$ satisfies $f(x, y, z) = f(-x, -y, -z)$. It is this property on which Proposition 1 depends, and therefore there is the obvious corollary

**Corollary 1.** *The solids A and B, as constructed are convex and not congruent, they have identical volume and surface area. Moreover, with respect to any coordinate system with origin at the centroid of the cube of C the integrals*

$$J_A = \int_A f(P)\mathrm{d}V = \int_B f(P)\mathrm{d}V = J_B$$

*are the same for A and for B provided that* $f(x, y, z) = f(-x, -y, -z)$.

## 4. Centroid-based invariants

In applications one usually determines the inertial properties with respect to the centroid of a body. The constructions of the previous section are not based on local coordinate frames with origin at the centroid of the shape in question. We now give equivalent constructions that use the centroid as the origin. The two variants will again be called $A$ and $B$, but will have a different shape. Dropping the convexity requirement for the moment, we wish to prove the following:

**Proposition 2.** *There are noncongruent polyhedral solids A and B that have identical volume, surface area, moments and products of inertia with respect to any coordinate system through the centroid of the solids.*

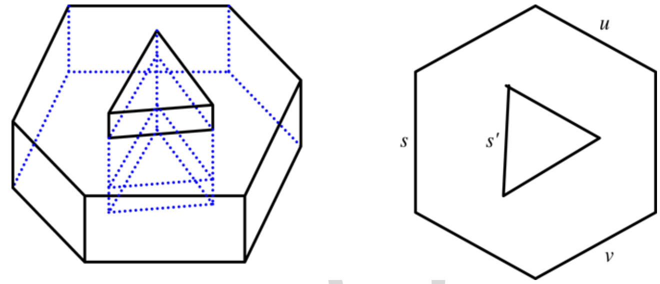


Fig. 3. Core solid $C$ for Proposition 2.

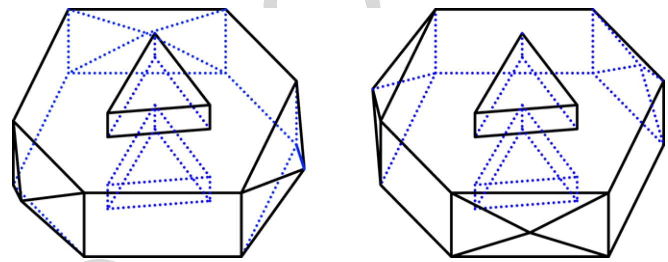

Fig. 4. Solids $A$ and $B$ for Proposition 2.

As before, we can strengthen the statement to achieve equality of the integrals over the solids of point-symmetrical integrands $f$.

We construct a base solid $C$ that is the union of an hexagonal prism and a triangular prism as shown in Fig. 3 (left). The base hexagon is regular and the triangle equilateral. The centres of the two prisms coincide (let us denote them by $O$), but the triangle is rotated by a small angle, say two degrees, so the sides $s$ and $s'$ are not parallel. This reduces the symmetries of the solid $C$. The triangular prism sticks out by the same amount on both sides of the hexagonal prism.

To construct solid $A$, we affix to the three faces of the hexagonal prism (with sides $s$, $u$ and $v$) three pyramids such that the base of each of them is the face of the prism; the pyramids are congruent and their apices lie on lines through the centroid of $C$ perpendicular to the faces in question. The resulting solid $A$ thus has its centroid at the centroid of $C$. Let us denote by $P(A)$ "the total decoration", i. e., the union of the three pyramids. This is shown in Fig. 4 (left). For solid $B$ we attach the same pyramids, only at the three other faces, as shown on the right of Fig. 4 (let us denote by $P(B)$ the union of these new pyramids).

Obviously, $P(B)$ is point-symmetrical to $P(A)$ with respect to $O$. Note, however, that the two solids are not congruent since the rotation of the triangular prism destroyed the rotational symmetry of $C$ under a 60° rotation. However, it is clear that the two solids demonstrate Proposition 2, namely, that they agree as to volume, surface area, and inertial properties with respect to any coordinate system with origin at the centroid. As before, the proof of the inertial invariants rests on the fact that $P(B)$ is point-symmetrical to $P(A)$ with respect to $O$.

The construction of solids $A$ and $B$ involves "decorating" three of the side faces with the same "feature". The choice of a pyramid is not essential. Other shapes could have been chosen,



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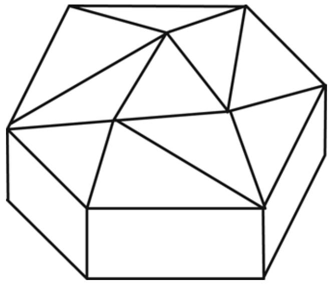

Fig. 5. Core solid $C$ used for Proposition 3. No hidden lines are shown.

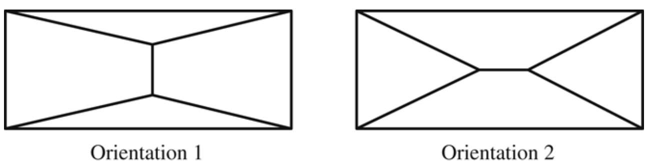


Fig. 6. Face decorations used for Proposition 3.

but they should be such that the first "total decoration" is point-symmetrical to the second one with respect to $O$.

We also note that it is possible to construct convex solids $A$ and $B$ that satisfy Proposition 2. To do so, we first convexify the solid $C$ as shown in Fig. 5. Then, we add to each of the six faces a gadget as shown in Fig. 6. The gadget is constructed placing a short edge above the centre of each lateral face, in lieu of a pyramid vertex, giving it an alternating orientation. Then the convex hull defines the additions to the faces, as shown.

The solids $A$ and $B$ differ only in the orientation decorations. Where the solid $A$ has a gadget in orientation 1, solid $B$ has it in orientation 2, and vice versa. Clearly, the two solids are not congruent, yet "the total decoration" of $A$ is point-symmetrical to "the total decoration" of $B$ with respect to the centroid and $A$ and $B$ agree in the common core $C$. Thus, we have proved

**Proposition 3.** *There are noncongruent, convex polyhedral solids A and B that have identical volume, surface area, moments and products of inertia with respect to any coordinate system with origin at the centroid of the two solids.*

Other classes of examples of noncongruent solids can be constructed that have various properties in common. Among them we can prove

**Proposition 4.** *There are polyhedral solids A and B with the following properties:*

1. *the surfaces of A and B have different genus (in particular, A and B are not congruent);*
2. *A and B have the same volume and surface area;*
3. *A and B have the same moments and products of inertia with respect to any coordinate system with origin at their centroid O;*
4. *for each continuous function* $f\colon \mathbf{R}^3 \to \mathbf{R}$ *such that* $f(P) = f(\mu(P))$ *we have* $\int_A f(P)\mathrm{d}V = \int_B f(P)\mathrm{d}V$.

*Note that moments and products of inertia are special cases of the integrals in property* 4.

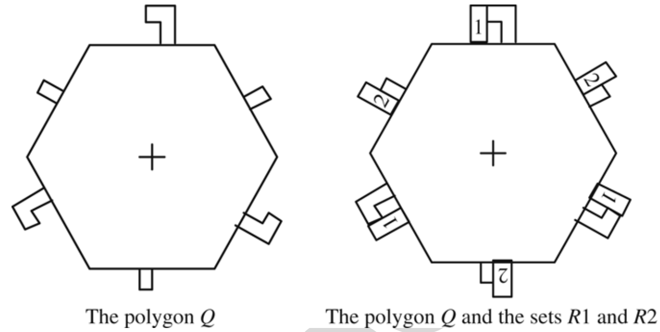


Fig. 7. Base cross-sections construction for solids with identical inertial properties but different genus.

**Proof.** Let us denote by $H$ a regular hexagon, let $O^*$ be the centroid of $H$. Let $\tau$ be a rotation of the plane of $H$ around $O^*$ by the angle of 120°.

It is easy to see that there is a (2-dimensional) polygon $Q$ which is the union of $H$ and six small polygons (see Fig. 7) such that

(1) $\tau(Q) = Q$;
(2) there are two sets $R_1$ and $R_2$ such that
  $R_1$ is the union of three rectangles labelled "1", and $R_2$ is the union of three rectangles labelled "2";
  $\tau(R_1) = R_1$ and $\tau(R_2) = R_2$;
  $R_1$ is symmetric to $R_2$ with respect to $O^*$;
  $Q \cup R_1$ is topologically equivalent (homeomorphic) to a disk with three "holes" but $Q \cup R_2$ is topologically equivalent to a disk;
  $Q \cup R_1$ and $Q \cup R_2$ have the same area and perimeter.

Let us denote by $A$ (respectively, $B$) the prism of height 1 with the base $Q \cup R_1$ (respectively, $Q \cup R_2$) such that centroids of $A$ and $B$ coincide.

It is easy to see that $A$ and $B$ have properties 1–4. ■

## 5. Chirality

We now consider the question of distinguishing parts that are mirror images of each other and yet have the same volume, surface area, and products and moments of inertia. In other words, what is the effect on these properties when the STEP translator commits chirality errors.

Recall that two solids $A$ and $B$ are *congruent* if there is an isometry $F : \mathbf{R}^3 \to \mathbf{R}^3$ such that $F$ preserves the orientation of the space and $F(A) = B$. If we do not require that the isometry preserves orientation, then we call the solids *isometric*. Formally, two solids $A$ and $B$ are *isometric* if there is an isometry $F : \mathbf{R}^3 \to \mathbf{R}^3$ such that $F(A) = B$.

We note that in the formulations of our preceding Propositions 1 through 4, we can replace "congruent" by "isometric". That is, the solids $A$ and $B$ in those propositions are not only not congruent but even are not isometric.

So far our examples are constructed by choosing a suitable base shape and adding features to it in a particular way. These added features have been placed point-symmetrically with respect to a chosen centre, for example, the centroid of the base shape. Now we consider objects that are



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mirror-symmetrical, that is, solids $A$ and $B$ such that $B = \pi(A)$ where $\pi$ is a reflection with respect to a plane. We show that there are objects congruent to them that have equal inertial and volumetric properties as well.

**Proposition 5.** *Let $A$ and $B$ be two solids such that $B$ is symmetrical to $A$ with respect to a plane. Then there is a solid $C$ such that:*

(1) *$C$ is congruent to $B$;*
(2) *the centroid of $A$ coincides with the centroid of $C$— call it $O$;*
(3) *for any straight line $L$ through $O$ the moments of inertia of $A$ and $C$ about $L$ are equal;*
(4) *for any coordinate system with origin $O$ and for any coordinate axis $l$ of it, the products of inertia of $A$ and $C$ with respect to $l$ are equal.*

**Proof.** Let $O$ be the centroid of the solid $A$. Let the coordinate system have origin $O$. Let $\pi : \mathbf{R}^3 \to \mathbf{R}^3$ be a reflection with respect to a plane such that $B = \pi(A)$. Put $C := h_z \circ h_y \circ h_x(A)$. Hence, $C = h_z \circ h_y \circ h_x \circ \pi^{-1}(B)$. Since $h_z \circ h_y \circ h_x \circ \pi^{-1}$ is an isometry of $\mathbf{R}^3$ which does not change the orientation of the space, $C$ is congruent to $B$. But $h_z \circ h_y \circ h_x$ is a point-symmetry with respect to $O$. Thus, $C$ is point-symmetrical to $A$ with respect to $O$. Hence, like earlier, (3) and (4) are true. ∎

The obvious corollary is that two noncongruent tetrahedra that are mirror images of each other satisfy the proposition. They constitute the simplest example we can find of two noncongruent shapes with identical volume, surface area and inertial properties.

Let us call a solid $S$ *unidentifiable* if there is a solid $S^*$ such that:

(1) the solid $S^*$ is not congruent to $S$;
(2) the solids $S$ and $S^*$ have identical volume;
(3) the solids $S$ and $S^*$ have identical surface area;
(4) the centroid of $S^*$ coincides with the centroid of $S$ — call it $O$;
(5) for any straight line $L$ through $O$ the moments of inertia of $S$ and $S^*$ about $L$ are equal;
(6) for any coordinate system with origin $O$ and for any coordinate axis $l$ of it, the products of inertia of $S$ and $S^*$ with respect to $l$ are equal.

So, Proposition 5 states that, if $A$ is a solid such that its mirror image is not congruent to $A$, then the solid $A$ is unidentifiable. In other words, if the translation algorithm constructs a solid that is a mirror image of the original solid, then checking surface area, volume and moments and products of inertia with respect to the centroid is not going to reveal this fact. Thus, left-handed and right-handed bolts, screws and nuts could not be distinguished.

## 6. Discussion

The STEP standards use agreement of volume and surface area as evidence that the translation of a solid of CAD system 1 into a congruent solid of CAD system 2 has a high likelihood of being correct. More sophisticated invariants, such as moments and products of inertia, can be added, since it is intuitively clear that equal volume and surface area are necessary but not sufficient. It should be no surprise that using these additional invariants is also merely a necessary condition, not a sufficient one, but the simplicity of our examples does raise eyebrows.

The particular constructions of our examples of noncongruent solids with identical invariants of volume, surface area and inertial properties, point to a difficulty that is bound to become more prominent once CAD system history and constraint structure is exchanged along with the geometry. It is known that geometrical constraint systems have multiple solutions [4], and that the selection of the particular solution to be used is a highly proprietary and individual strategy of the solver algorithm. So, it is rather possible, if not likely, that CAD system 2, reconstructing a solid imported from CAD system 1, may choose a different solution. In that situation is a distinct possibility that a feature, such as the pyramid we added to solid $C$ in our constructions is instantiated at a different location. In such cases, especially for mix-ups in highly symmetrical parts, grossly noncongruent shapes with equal invariants would be constructed. Thus, there are entire classes of practical, common mechanical parts that have shape variants that are not congruent yet have the same volume, surface area and inertial properties. See also [5] for the related problem of *persistent naming*.

A particular class of noncongruent shapes are ones in which the two parts with equal invariants are such that one of them is simply (congruent to) a mirror image of the other. For instance, left-handed and right-handed screws have equal invariants, with bad consequences.

More can be said about the reliance on the numerical invariants. Since the quantities of volume, surface area, etc. are computed numerically, establishing confidence in the translation will be based on a tolerance within which corresponding invariants are equal to each other. Similarly, a solid $S'$ is an $\varepsilon$-approximation of a solid $S$ if for every point $P'$ of $S'$ there is a point $P$ of $S$ that is closer to $P'$ than $\varepsilon$, and, conversely, for every point $P$ of $S$ there is a point $P'$ of $S'$ that is closer to $P$ than $\varepsilon$. We state the following without proof:

**Corollary 2.** *For each polyhedral solid $S$ (not necessarily convex) and for each real number $\varepsilon > 0$, there is a polyhedral solid $S_\varepsilon$ such that*

(1) *the solid $S_\varepsilon$ is an $\varepsilon$-approximation of $S$; and*
(2) *the solid $S_\varepsilon$ is unidentifiable.*

This adds further uncertainty to the use of numerical invariants for judging whether the translation proceeded flawlessly. These traditional invariants are rather limited.

We noted in the introduction a connection between the question of relying on quantitative shape invariants as correctness indication of a shape translation between two systems, and the problem of shape semantics. Consider the following. The translation of solid $A$ in one CAD system to a solid $B$ in a different CAD system makes only sense if we have a way of establishing a mathematical entity that is



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sufficiently well approximated by each of the two computer representations $A$ and $B$. In seminal papers such as [6–8] that question is examined with a topological perspective in mind. To bring topology into the picture is a good idea: It is possible to approximate an intersection curve of two surfaces, for instance, and get good geometrical approximation but fail in a topological sense. Namely, the approximation may be a knot while the original curve is not [7,9].

Computing mass and inertia properties involves algorithms that sample the solid. That suggests using point sampling directly to gain confidence in the translation process. This approach would not guarantee uncovering small surface cracks, for instance, but does look promising to us, especially when combined with topological methods.

## Acknowledgements

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## References

[1] Srinivasan V. Standardizing the specification, verification and exchange of product geometry: Research, status and trends, manuscript. 2007.
[2] Hoffmann C. Geometric and solid modeling: An introduction. San Mateo (CA): Morgan Kaufmann; 1989 [chapter 2].
[3] Requicha A. Mathematical models of rigid solids. Tech. Memo 28. Production Automation Project. University of Rochester; 1977.
[4] Bouma W, Fudos I, Hoffmann C, Cai J, Paige R. A geometric constraint solver. Comput Aided Des 1995;27:487–501.
[5] Chen X. Representation, evaluation and editing of feature-based and constraint-based design. Ph.D. thesis. Dept of CS, Purdue Univ; 1995.
[6] Bern M, et al. Emerging challenges in computational topology; NSF workshop on comp. top. report. June 1999. http://xxx.lanl.gov/abs/cs/9909001.
[7] Blackmore D, Peters T. Computational topology. In: Perl E, editor. Open problems in topology II. Amsterdam: Elsevier; 2007. p. 493–546.
[8] Blackmore D, Mileyko Y. Computational differential topology. Appl Gen Top 2007;8:35–92.
[9] Amenta N, Peters T, Russell A. Computational topology: Ambient isotopic approximation of 2-manifolds. Theoret Comput Sci 2003;305:3–15.



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