

# Skylight Approximation for Simulation of Plant Development

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## Abstract

*This paper presents an algorithm for approximation of the skylight of a clear sky with the sun. The skylight and the light reflected from the ground are approximated by a finite set of parallel lights. The long-term simulation is considered by averaging the excitance of the lights over a time period. The main area of interest of the method is considered to be, but is not restricted to, visual simulation of plant development.*

## 1. Introduction

There is an increasing interest in visual simulation of plant development. The algorithms are used in computer graphics for generating realistic models of virtual plants as well as in biology, where computers present important tools for better understanding of the inner mechanisms of plants. As recently found, visual simulation of plant development cannot be correct without considering the external influences. One of the most important factors is the light flux reaching light sensitive modules of the plant (*i.e.*, leaves and buds).

Computer graphics use several types of “well-behaved” light sources *e.g.*, parallel lights, point lights, spot lights, *etc.* These lights are easily maintained namely in the case of rendering. Most of the plants develop in outdoor conditions where both the skylight and sunlight play important roles and where the above mentioned simplifications do not give sufficient results. Skylight has been extensively studied mostly from the viewpoint of rendering. *e.g.*, [7, 11, 13, 14]. The majority of previously published algorithms concerning the simulation of plant development interacting with incoming light use only a very simple skylight: usually nine [12] or thirty two [4] point lights equally spread on a celestial hemisphere emanating with identical intensity.

We present here a model of skylight suitable for simulation of the plant development purposes. The main aim of this paper is the approximation of a quite general model of skylight by a finite set of parallel lights and the exact evaluation of their excitance. The long-term simulation is considered by averaging the excitances over a time span. The skylight function [6], as usually used, is very complex and therefore hard to maintain, whereas the parallel lights can be immediately incorporated into existing methods for calculating the plant-light interactions [1, 2, 4, 5, 9, 12].

This paper is structured as follows. The next section briefly reviews previously published methods and findings. Section 3 deals with light in general, whereas Section 4 deals with the skylight function giving the radiance value for a certain point on the sky dome and the approximation of the skylight is introduced here. Section 5 introduces a notion of average excitance of the light source over a time span. The paper is concluded with algorithm scratch which summarizes the information presented here.

## 2. Previous work

Skylight illumination has a long history in computer graphics, but we have found only very simple models used in visual simulations of plant development.

Chiba and his collaborators [4] approximate the skylight by a fixed number of point light sources equally spread on the celestial sphere. The lights have equal values of excitance assigned *i.e.*, they emanate with the same intensity. The amount of light reaching leaves of the visual plant is determined by a ray-casting algorithm from the leaves to the lights. The same approach is used by Měch and Prusinkiewicz [12].

Greene [9] uses simple skylight representation with the trajectory of the sun. He samples the sky from every bud (which is denoted by the term “voxel space automaton”) and fills the illumination table by values indicating whether

the sample reached the sky or not. The table is then low pass filtered and the brightest area on the sky is found.

A different approach to the estimation of the amount of light incidenting on plant organs has been introduced by myself [2, 3]. This algorithm is based on Z-buffer HSE and uses skylight approximation by a set of parallel lights. The skylight discretization was outlined in [2, 3] and this paper introduces an exact solution.

There is a large number of papers dealing with skylight for rendering purposes *e.g.*, [10, 11, 13, 14]. These methods mostly deal with multiple scattering caused by presence of clouds or dust particles.

### 3. Light

Irradiance (incident radiant flux density) of a differential area  $dA$ , *i.e.*, the power reaching this area, is denoted by  $E$ , and it is equal to (see Figure 1)

$$E = \frac{d\Phi}{dA} = \int_{\Omega} L(dA, \Theta, \phi) V(d\omega, dA) \cos \Theta d\omega, \quad (1)$$

where  $d\omega$  denotes the solid angle,  $L(dA, \Theta, \phi)$  denotes the radiance coming from the  $d\omega$ ,  $N$  is the normal vector to the  $dA$ ,  $\Theta$  is the angle between the direction of the differential solid angle  $d\omega$  and the  $N$ , and finally  $V(d\omega, dA)$  is the visibility term, reflecting the occlusion of the  $dA$  by the other objects in the scene.

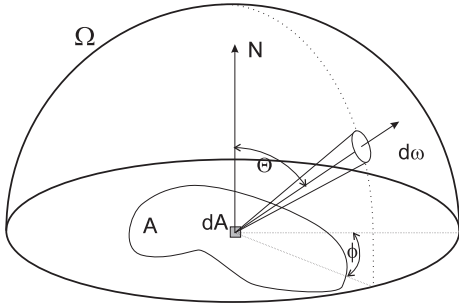


Figure 1. Radiance of  $dA$ .

The units of the irradiance are  $[W m^{-2}]$ . The power reaching the differential area  $dA$  is equal to light flux  $d\Phi$  (radiant power)

$$d\Phi = E dA, \quad (2)$$

and the power reaching the whole area of a plant organ (expressed in  $[W]$ ) is

$$\Phi = \int_A \int_{\Omega} L(dA, \Theta, \phi) V(d\omega, dA) \cos \Theta d\omega dA. \quad (3)$$

The sensitive organs are usually polygonized (discretized) into a set of triangles in visual models of plants. In

this case the evaluation of the outer integral of (3) is simplified and involves only a determination of the visibility term  $V(d\omega, dA)$ . We do not deal with the visibility term here. There are some techniques based on 3D DDA sampling [4] or Z-buffer HSE [2].

Analytic evaluation of the skylight of every point is expensive so we suggest the continuous sky domain  $\Omega$  to be approximated (discretized) by a set of parallel light sources. However, the hemisphere has infinite radius so we can also think about these lights as point lights. In this paper the lights will be considered to be parallel lights.

There are several reasons for the skylight discretization. The main reason is the complexity of evaluating the formula (3). Another reason is that algorithms for determination of the visibility term such as in [1, 2, 4, 12] can very efficiently incorporate this approach. The last reason is that the assumption of the sky approximation fits in well with most of the algorithms for plant-light interaction published [1, 2, 3, 4, 5, 9, 12], so these algorithms can immediately use the approach presented here. The algorithms approximate the skylight by a finite number of lights emanating with the same intensity, whereas our method assigns more exact values to the lights.

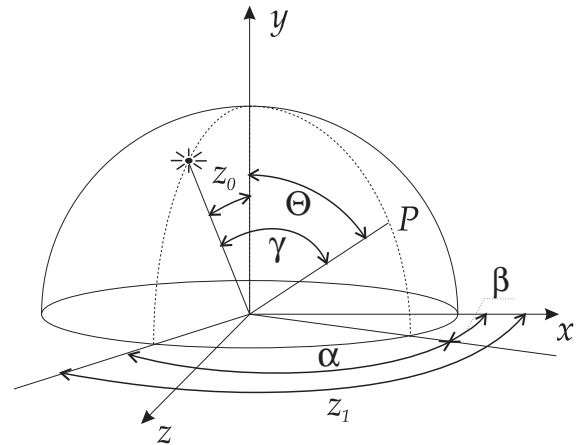


Figure 2. Notion of the angles used in the equation (4).

## 4. Skylight

### 4.1. Continuous function

The sky dome is conceived as a hemisphere of infinite radius. Every point denoted by  $P[\Theta, \gamma]$  (see in Figure 2) at the hemisphere acts on any object in the scene as a parallel light source. The CIE provides the following equation for

radiance of a single point  $P[\Theta, \gamma]$  of the sky hemisphere with the sun contribution [6, page:296]

$$L(\Theta, \gamma) = \quad (4)$$

$$= L_z \frac{(0.91 + 10e^{-3\gamma} + 0.45 \cos^2 \gamma)(1 - e^{-0.32/\cos \Theta})}{0.27(0.91 + 10e^{-3z_0} + 0.45 \cos^2 z_0)}.$$

Every angle is measured from the center of the hemisphere in this equation and  $L_z$  is the radiance of the zenith,  $\gamma$  denotes the angle between the sun and point  $P$ ,  $\Theta$  is the angle between zenith and the point  $P$ ,  $z_0$  is the angle between zenith and the sun, and  $\alpha$  is the projected angle  $\gamma$ . The angle  $\gamma$  can be efficiently computed from  $z_0$ ,  $\Theta$ , and  $\alpha$  using the equation for spherical triangles like

$$\gamma = \arccos(\cos z_0 \cos \Theta + \sin z_0 \sin \Theta \sin \alpha). \quad (5)$$

The equation (4) defines radiance of a clear sky. CIE also proposes an equation for completely overcast sky

$$L(\Theta) = L_z \frac{1 + 2 \cos \Theta}{3},$$

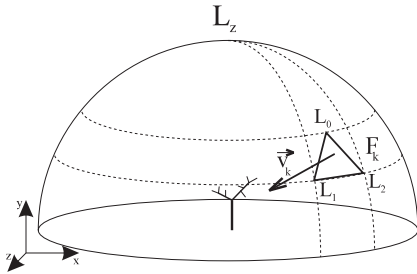
and Tomoyuki *et al.* [14] provides intensity function for partially overcast sky. We will use the equation (4) in the following text, though, we can use any of the mentioned models.

## 4.2. Skylight discretization

We approximate the continuous sky domain  $\Omega$  by  $p$  parallel light sources denoted by  $S_k$ ,  $k = 1, \dots, p$ . This assumption collapses the inner integral of (3) into sum and gives

$$\Phi = \int_A \left[ \sum_{k=1}^p E_k V(\Delta\omega, dA) \cos \Theta \right] dA, \quad (6)$$

where  $E_k$ ,  $k = 1, \dots, p$  denotes the excitance (radiant emitted flux density) of the  $S_k$ .



**Figure 3. One face  $F_k$  approximating certain area of the sky hemisphere.**

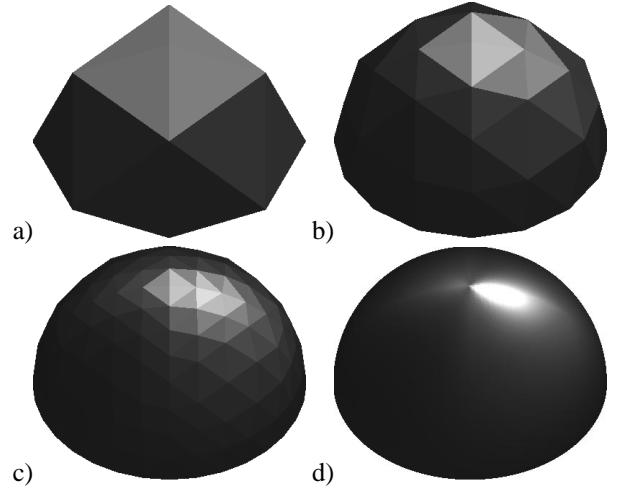
The sky hemisphere is first approximated by a polyhedron consisting of faces denoted by  $F_k$ ,  $k = 1, \dots, p$ . This is achieved by recursive subdivision of a tetrahedron.

The flux densities  $E_k$  are calculated from radiance values denoted by  $L_0$ ,  $L_1$ , and  $L_2$  in the corners of the triangle  $F_k$  (see Figure 3).

$$E_k = \frac{1}{3} \sum_{i=0}^2 L_i. \quad (7)$$

The faces  $F_k$  are substituted with parallel light sources  $S_k$ , keeping the orientation of the face  $F_k$ . These lights shine in direction pointing to the center of the sky hemisphere and, because of the properties of the parallel lights [6, page:25], the irradiance  $E$  of a differential area  $dA$  is equal to  $E = E_k \cos \Theta$ , and the light flux (2) becomes

$$d\Phi = E_k \cos \Theta dA. \quad (8)$$



**Figure 4. Successive approximations of the sky hemisphere a) 16, b) 64, c) 256, and d) 65536 lights). The skylight is calculated at 14 p.m. and the sun is 75° above the horizon.**

## 4.3. Light reflected from the ground

During the simulation of plant development the light reflected from the ground also plays its role. Chiba *et al.*, [4] suggested using the whole sky sphere instead of the sky hemisphere. We have found that doubling the amount of lights for this reason is wasteful, because it takes computational time lasting twice as long and bringing negligible results. Instead, in our model the ground is approximated by one

parallel light source denoted by  $S_{p+1}$  oriented in the direction of zenith, with excittance denoted by  $E_{p+1}$  and

$$E_{p+1} = b \sum_{k=1}^p E_k. \quad (9)$$

The  $b$  denotes albedo [8, page:592] of the ground. This value should differ with material; we use an *ad hoc* value  $b = 0.1$ .

The approximation is then a two step method. First, the flux densities  $E_k$  of the lights approximating the sky are calculated from (4) and (7), then the flux density of the light  $S_{p+1}$  approximating the ground is calculated from (9).

## 5. Time

The equation (4) supposes the sky to be a static object. However, for the simulation of plant development we should consider long-term observations.

The position of the sun on the sky hemisphere is denoted by an ordered double  $[z_0, z_1]$  in Figure 2. In the static case the radiance (4) of a point on the sky dome  $L(\Theta, \gamma)$  is only a function of the actual position of the sun *i.e.*, of the coordinates  $\Theta$  and  $\gamma$ . However, for the long-term simulation we must take into account the movement of the sun on the sky, formally, we specify the coordinates of the sun as a time dependent:  $[z_0(t), z_1(t)]$ . Therefore, the radiance of a point  $P$  on the sky is also dependent on the time *i.e.*,  $L(\Theta, \gamma, t)$ . Because  $\alpha = z_1 - \beta$ , (*c.f.*, Figure 2) equation (5) can be rewritten as

$$\gamma(t) = \arccos(\cos z_0(t) \cos \Theta + \sin z_0(t) \sin \Theta \sin(z_1(t) - \beta)),$$

and (4) becomes

$$L(\Theta, \gamma, t) = \quad (10)$$

$$= L_z \frac{(0.91 + 10e^{-3\gamma(t)} + 0.45 \cos^2 \gamma(t))(1 - e^{-0.32/\cos \Theta})}{0.27(0.91 + 10e^{-3\gamma(t)} + 0.45 \cos^2 z_0(t))}.$$

Substituting the time dependent radiance of the point  $P$  into the equation (7) we obtain an immediate value of the flux density  $E_k(t)$  for every light source  $S_k$ .

However, the immediate value of radiance is not useful for simulation of the plant development over a very long time interval. If we compare the growth speed of an apical meristem with the rate of changes of the sun on the sky we see that such precise simulation is useless.

Therefore, we define the *average radiance* of a point  $P$  over the time span  $[t_0, t_1]$  and we denote this value by  $\bar{L}$ . The  $\bar{L}$  is

$$\bar{L}(\Theta, \gamma, t_0, t_1) = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} L(\Theta, \gamma, t) dt \quad (11)$$

The period  $[t_0, t_1]$  depends on simulated species and ranges from several hours to a whole year. Although it is possible to evaluate this integral analytically, in our implementation we compute this value by substituting the  $dt$  by  $\Delta t$  and summing the values over the  $t_0 \leq t \leq t_1$  for every  $S_k$  at once.

Substituting average values of radiance (11) into (7) we obtain average excittance value of the light sources  $S_k$ . We denote these values by  $\bar{E}_k$ . If we use very long-term observation, the lights which are visited by the sun will form a sun trajectory. On the other hand in the short period only the lights which include the sun position will have excittance increased much more than the others as shown in Figure 5.

We conclude this section with the algorithm for sky discretization.

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### input:

number of the faces approximating the sky:  $p$ ,  
start and end time of the simulation:  $t_0, t_1$

precision:  $n$

### output:

$p + 1$  lights with average excittance  $\bar{E}_k$  assigned

( $p$  lights approximate the sky, the  $(p + 1)$ -st is the ground)

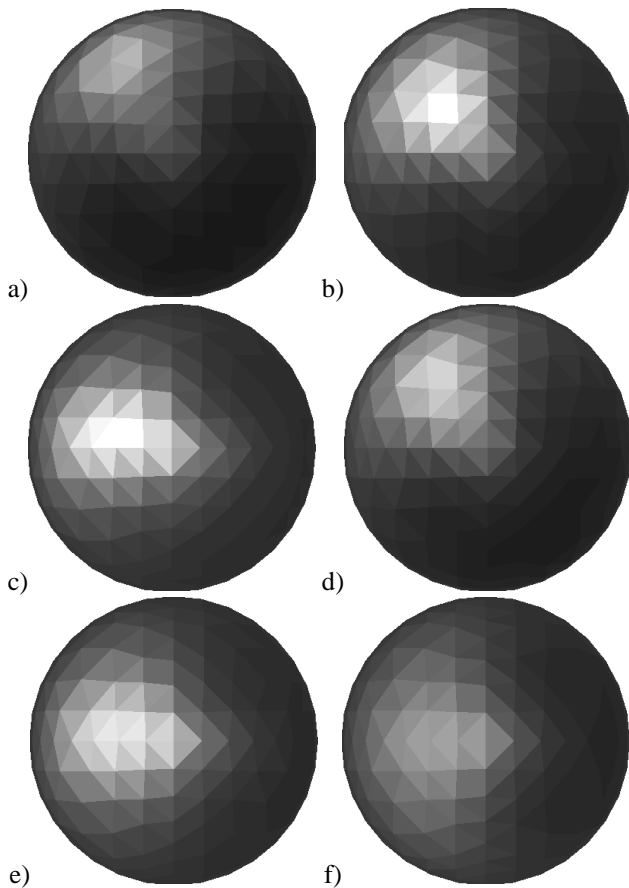
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- 1: subdivide the sky hemisphere into  $p$  faces  $F_k$
  - 2: **for** every face  $F_k$  **do**
  - 3:    $\Delta t = (t_1 - t_0)/n$
  - 4:    $t = t_0$
  - 5:   **while** ( $t < t_1$ )
  - 6:     for each corner of the face  $F_k$  ( $i = 0, 1, 2$ )
  - 7:       compute  $L_i(\Theta, \gamma, t)$  from (10)
  - 8:        $L_i = L_i + L_i(\Theta, \gamma, t)$
  - 9:        $t = t + \Delta t$
  - 9:   **end of while**
  - 10:    $\bar{L}_i = L_i/n$  (\* average radiance over  $[t, t + \Delta t]$  \*)
  - 11:    $E_k = (\sum_i \bar{L}_i)/3$  (\* radiosity of  $S_k$  from (7) \*)
  - 12: **end of for**
  - 13:  $E_{p+1} = (b \sum_k E_k)$  (\* ground light (9) \*)
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The algorithm calculates light condition over time interval  $[t_0, t_1]$  in  $n$  steps. Lines 5–10 calculate integral (11). We use  $n = 100$  in this numerical integration. We have measured convergence of the integral (11) as a function of  $\Delta t$  and we have found the error less than 1% from the exact values. Line 11 corresponds to equation (7) where the light flux density of  $S_k$  is calculated. The ground light (9) is calculated at line 13.

## 6. Summary

The algorithm presented in this paper discretizes the continuous and complex skylight by a finite set of parallel light



**Figure 5. The figures present skylight at a) 9 a.m, b) 10 a.m, and c) noon, while the others are calculated as an average from d) 9 a.m to 10 a.m. e) 10 a.m to 15 p.m, and f) the whole day i.e. 8 a.m to 6 p.m. The sky is approximated by 256 lights.**

sources. This paper introduces skylight approximation allowing long-term observations. This is because the movement of the sun in the sky is also considered. We introduce an average radiance value over a desired time span. This allows us to simulate plant development in long-term observations, or this allows efficient plant rendering during a short time period.

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