

Permeable and Absorbent Materials in Fluid Simulations

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Abstract

The interaction between fluids and materials is an important part of fluid simulations. Many research efforts have been performed to incorporate the coupling between solid objects and fluids. However, the interaction of fluids with permeable materials has been neglected. We present a solution to fluid simulations that allows for the simulation of porous and absorbent materials. Our extension is based on well known physical laws that describe the interaction of fluid with permeable objects. These laws can easily be integrated into any semi-Lagrangian based solver which enable fluid simulations to have effects such as water flowing through soil or a towel soaking up spilled liquid on a kitchen counter.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Physically Based Modelling I.3.7 [Computer Graphics]: Animation

1. Introduction

Despite the tremendous progress in fluid simulations, an important problem of including porous materials in a simulation has not been addressed. Animators may wish to include effects such as water pouring through a bed of sand or a sponge soaking up a puddle, but these simulations are not currently possible since fluid solvers only take into account non-permeable objects. To create the desired effect, animators must hand tweak their scenes to work around the limitation of liquid being restricted from entering an object. Recent animated movies, such as Shrek and Ratatouille, have had scenes where a method to physically simulate fluid and permeable material interaction would have made the animation process easier. Our method extends Semi-Lagrangian based fluid solvers (such as those presented in [Sta99] and [FF01]) to allow permeable and absorbent materials in fluid simulations by integrating a simple and elegant extension to the pressure solver.

2. Permeability

The flow of water in a material is described by the equation known as Darcy's Law. This law was originally formulated based on observation, but later was derived from the Navier-Stokes equations. Since Darcy's Law is a simplified version

of the equations used in our fluid solver, we can seamlessly integrate permeable materials into our solver by introducing a new variable called *permeability*, denoted by κ . Permeability is described as the ability of a material to transport liquid. The equation for three-dimensional flow through a permeable material has the form:

$$q = \frac{-\kappa}{\mu}(\nabla p - \rho \mathbf{g}) \quad (1)$$

Where q is the flux [m/s], κ is the tensor permeability of the material [m²], μ is the dynamic viscosity of the fluid [Pa·s], ∇p is the pressure gradient vector of the fluid, ρ is the density of the fluid, and \mathbf{g} is gravity.

We extend the fluid solver's velocity projection step to take into account a material's permeability. When constructing the right-hand side of the equation, we consider the permeability of the cells that share a grid face. These permeability values are multiplied together with the face velocity to obtain the velocity that is to be used on the right-hand side of the Navier-Stokes incompressibility equation. As an example, $u_{i+1/2,j,k}$ becomes:

$$u'_{i+1/2,j,k} = u_{i+1/2,j,k} \kappa_{i,j,k} \kappa_{i+1,j,k} \quad (2)$$

3. Porosity

Another parameter is included to represent a material's *porosity*, denoted by ϕ [Bat67]. This parameter is the ratio of the material's void space, V_V , and its total space, V_T :

$$\phi = \frac{V_V}{V_T} \quad (3)$$

Porosity is used to determine the maximum amount of liquid that the material can hold at a particular voxel in space. Though porosity and permeability are often related, some materials do not follow this property. A material such as clay may have many holes where water can propagate, which gives it a high porosity. But those holes may be poorly connected, resulting in low permeability.

The porosity is considered when the fluid levels of the simulation are updated. The maximum amount of fluid allowed at a given cell, \mathcal{F}_c , can be expressed as a product of porosity and the maximum amount of fluid in air, \mathcal{F}_a .

$$\mathcal{F}_c = \phi \mathcal{F}_a \quad (4)$$

When we update the fluid levels, we check to see if the new fluid level exceeds \mathcal{F}_c . It is possible for a cell to become overly saturated and steps need to be taken to correct this. One possible method is to leave behind some of the fluid along the path of advection.

4. Capillary Action

We can simulate the phenomena known as *capillary action* through the addition of another parameter. Capillary action is the ability of a material to draw liquid into itself and retain it, which will allow the simulation of sponges and clothes. The force from capillary action, \mathbf{F}_c , is related to total body force on a fluid \mathbf{F} , the fluid's mass density, ρ , and gravity, \mathbf{g} .

$$\mathbf{F} = \rho \mathbf{g} + \mathbf{F}_c \quad (5)$$

$$\mathbf{F}_c = -\nabla \Pi \quad (6)$$

Equations (5) and (6) are the basis for the integration of capillary action into our fluid solver. The capillary body force is obtained from the gradient of the capillary potentials of the materials, Π . The capillary potential is what will allow an object to suck in and retain a fluid [Hil06].

We extend this force computation by including local forces created by the capillary potential of the material. The capillary action force at a cell face is calculated by taking the gradient of the capillary potentials at the center of the cells that share the cell face.

$$\mathbf{F}_{\mathbf{c}_{i+1/2,j,k}} = \mathbf{F}_{\mathbf{c}_{i+1,j,k}} - \mathbf{F}_{\mathbf{c}_{i,j,k}} \quad (7)$$

When liquid is on the surface of an absorbent material, the difference between the material's capillary potential (some value greater than zero) and the air's capillary potential

(zero) results in a force that pulls the liquid into the material. Another desirable property of this method is that when a liquid is inside of a material, the capillary action force prevents the liquid from leaving.

5. Results

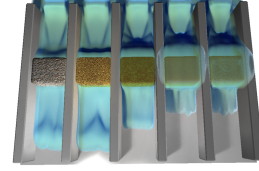


Figure 1: Simulation of fluid propagating through different material types. From left to right: Gravel (high permeability and porosity), Sand (high permeability, low porosity), Soil (semi-permeable and porous), Clay (low permeability, high porosity), Shale (low permeability and porosity)

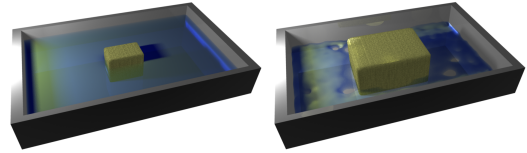


Figure 2: A sponge is placed in water and is allowed to grow as it absorbs water.

The addition of the three new parameters allows the physically accurate simulation of permeable objects. Figure 1 depicts how varying the parameters can produce realistic results for numerous material types. In figure 2, we place a sponge in water. The sponge grows as it absorbs more and more liquid. The growth of the sponge is determined by the amount of liquid that enters it.

References

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