GRAPH CAMERA PROJECTION

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We derive the mapping Q_{k+1} of a point on the image plane of planar pinhole camera $k+1$ (PPC_{k+1}) to PPC_0 by first establishing the mapping R_{k+1} between PPC_{k+1} and PPC_k as shown in Figure 1. Then we show by induction that $Q_{k+1} = R_1 R_2 \ldots R_{k+1}$. The base case is verified as follows:

$$\begin{bmatrix} u_0 \\ v_0 \\ 1 \end{bmatrix} w_0 = R_1 \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}, \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} w_1 = R_2 \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_0 \\ v_0 \\ 1 \end{bmatrix} w_0 = R_1 \frac{1}{w_1} R_2 \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \frac{1}{w_1} R_1 R_2 \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} \tag{A.1}$$

$$\begin{bmatrix} u_0 \\ v_0 \\ 1 \end{bmatrix} w_0 w_1 = R_1 R_2 \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}$$

$$Q_2 = R_1 R_2$$

By the induction hypothesis:

$$Q_k = R_1 R_2 \ldots R_k$$

$$\begin{bmatrix} u_0 \\ v_0 \\ 1 \end{bmatrix} w_0 = R_1 R_2 \ldots R_k \begin{bmatrix} u_k \\ v_k \\ 1 \end{bmatrix} \tag{A.2}$$

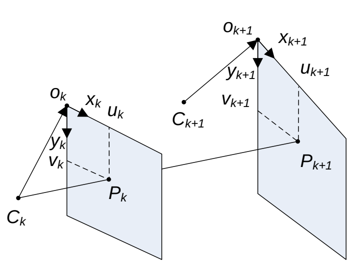
Using the equations in Figure 1 we obtain:

$$\begin{bmatrix} u_k \\ v_k \\ 1 \end{bmatrix} w_k = R_{k+1} \begin{bmatrix} u_{k+1} \\ v_{k+1} \\ 1 \end{bmatrix} \tag{A.3}$$

Combining equations A.2 and A.3 terminates the proof:

$$\begin{bmatrix} u_0 \\ v_0 \\ 1 \end{bmatrix} w_0 = R_1 R_2 \ldots R_k \frac{1}{w_k} R_{k+1} \begin{bmatrix} u_{k+1} \\ v_{k+1} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_0 \\ v_0 \\ 1 \end{bmatrix} w_0 w_k = R_1 R_2 \ldots R_k R_{k+1} \begin{bmatrix} u_{k+1} \\ v_{k+1} \\ 1 \end{bmatrix} \tag{A.4}$$



$$P_{k+1} = \begin{bmatrix} x_{k+1} & y_{k+1} & (C_{k+1} + o_{k+1}) \end{bmatrix} \begin{bmatrix} u_{k+1} \\ v_{k+1} \\ 1 \end{bmatrix}$$

$$P_{k+1} = C_k + \begin{bmatrix} x_k & y_k & o_k \end{bmatrix} \begin{bmatrix} u_k \\ v_k \\ 1 \end{bmatrix} w_k$$

$$R_{k+1} = \begin{bmatrix} x_k & y_k & o_k \end{bmatrix}^{-1} \begin{bmatrix} x_{k+1} & y_{k+1} & (C_{k+1} - C_k + o_{k+1}) \end{bmatrix}$$

$$\begin{bmatrix} u_k \\ v_k \\ 1 \end{bmatrix} w_k = R_{k+1} \begin{bmatrix} u_{k+1} \\ v_{k+1} \\ 1 \end{bmatrix}$$

Figure 1 Derivation of mapping PPC_{k+1} and PPC_k with COP's C_{k+1} and C_k. Vectors *x* and *y* give the row and column direction and are one pixel width and one pixel length long, respectively. Vectors *o* point from the COP to the top left corner of the image. Point P_{k+1} on the image plane of PPC *k+1* is mapped to point P_k on the image plane of PPC *k* through matrix R_{k+1}.