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ROBUST KINEMATIC DESIGN OF PLANAR ASSEMBLIES

Keywords: Computer-aided mechanical design, synthesis, tolerancing.

Abstract

This paper describes a framework and algorithms for robust kinematic design of mechanical assemblies. The framework, developed over a period of ten years, is based on the configuration space method and supports automatic modeling, nominal and toleranced analysis, and part tolerance envelope computation of open and closed loop planar mechanisms with multiple, changing contacts. It also supports nominal and toleranced kinematic synthesis of planar pairs based on a parametric part model. The tools help designers select nominal parameter values, identify failure modes, and optimize nominal values and tolerance allocation. We illustrate the concept of robust design with a realistic design scenario and show how our tools support the kinematic design process. We briefly describe the configuration space approach and the algorithms we have developed for nominal and toleranced kinematic mechanism analysis, tolerance parts envelopes computation, and robust kinematic synthesis of higher pairs.

1. INTRODUCTION

Kinematic design is the task of devising a system of mechanical parts that implements specified motion transformations. The design must meet its specifications despite part variations due to manufacturing. An optimal design achieves this goal at minimal cost. Kinematic synthesis is central to mechanical design because kinematics largely determines mechanical function.

Kinematic synthesis is an iterative process in which the designer selects a design concept, constructs a parametric model, assigns parameter values, and allocates tolerances (Fig 1). At each step, the designer makes changes, assesses their impact, and decides whether to advance to the next step or to return to a prior step. When a design fails due to part variations, the designer can change the nominal design or tighten the tolerances. Changing the nominal design is often better, since cost increases rapidly as tolerances decrease, but it can be much harder.

Kinematic design is difficult and time consuming. In the conceptual design step, the designer needs to compare competing concepts based on incomplete, high-level characterizations. In the later steps, he has to adapt the chosen concept to comply with numerous, often competing design specifications. The adaptation requires extensive kinematic analysis of many design instances. The analysis is difficult because it involves multiple part contacts that impose nonlinear motion constraints. Some contacts are part of the nominal function, while others arise due to part variation. Both types can introduce failure modes that coexist with or supersede the correct function. Finally, the designer needs to formulate a realistic, application-specific cost function for tolerance allocation.

Software support for kinematic synthesis is limited. There are very few tools for conceptual design. Powerful commercial packages, such as CATIA and IDEAS, support construction and visualization of parametric designs. Kinematic analysis software is limited to multi-body systems: assemblies of parts that interact via a fixed set of feature contacts [6]. Prior research in synthesis provides algorithms for linkages [4] and cams [2,5] but does not address systems with contact changes. Tolerance analysis software is available for individual, user-specified system configurations, but not over a continuous work cycle [1,3,6].

A new methodology, called robust design, has been developed to increase reliability and reduce redesign costs [7]. In robust design, nominal and tolerance changes are evaluated together (dotted box

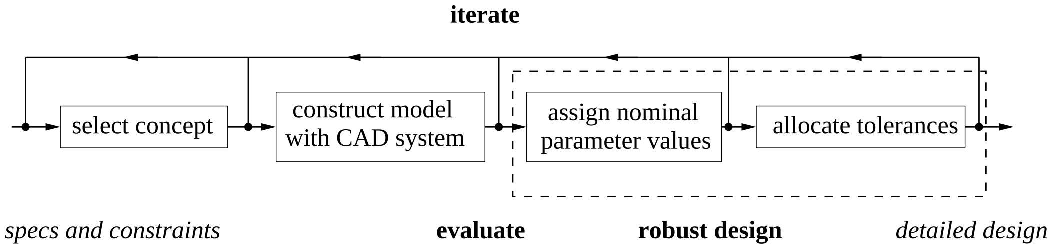


Fig. 1. The kinematic design process and the role of robust design.

in Fig. 1). The nominal design is modified to reduce its sensitivity to part variations. Then tolerances are allocated to guarantee correct function and to minimize cost. Robust design differs from the traditional design paradigm in which failure due to part variations is fixed primarily by tightening tolerances. Robust design is especially relevant to kinematic synthesis because failures due to tolerances are hard to detect and costly to correct.

We have developed over the past ten years a framework and computational tools that support kinematic analysis and synthesis of planar mechanical systems [8-21]. The framework is based on the configuration space method and supports automatic modeling, nominal and toleranced analysis, and part tolerance envelope computation of open and closed loop planar mechanisms with multiple, changing contacts. It also supports nominal and toleranced kinematic synthesis of planar pairs based on a parametric part model. The tools help designers select nominal parameter values, identify failure modes, and optimize nominal values and tolerance allocation.

This paper illustrates the concept of robust design with a realistic design scenario and shows how our tools support the kinematic design process. It briefly describes the configuration space approach and the design tools we have developed for nominal and toleranced kinematic mechanism analysis, tolerance parts envelopes computation, and robust kinematic synthesis of higher pairs.

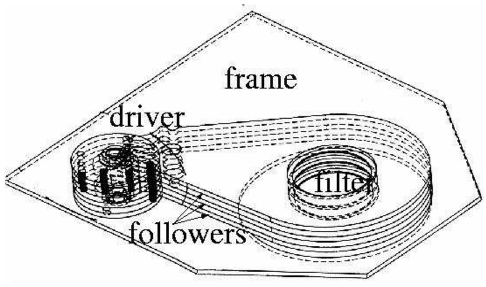
2 DESIGN SCENARIO

We illustrate robust kinematic design and the role of our tools on an optical filter mechanism (Fig. 2). The mechanism consists of a lens, a cam, and three filters mounted on identical followers. The lens is attached to a fixed frame (not shown). The followers are stacked on a shaft and can rotate independently. The cam (external diameter 25mm, height 20mm) consists of three slices that rotate together on a common shaft.

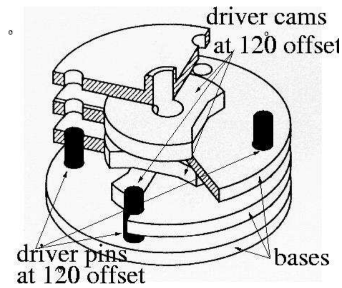
Each cam slice drives the corresponding follower. Fig. 2d shows the top cam slice and its follower. The cam slice consists of a driving pin (diameter 2mm) and a locking arc. When the cam shaft rotates counter clockwise, the pin engages the follower slot and rotates the follower until the filter covers the lens. The other two cam slices are identical, except that they are rotated by 90^{o} and 180^{o}, respectively. In the initial state, the filters are off the lens. When the cam shaft is rotated counter clockwise, the three followers are engaged in sequence. Rotating the cam clockwise resets the filters to the initial state.

The design task is to devise a mechanism to engage and reset the followers in the intended manner. The mechanism must be robust because it will be mounted on a vehicle and must be compact to fit in the allotted space. During conceptual design, the designer chooses a Geneva mechanism with one river and one follower per filter. This concept dictates the functional geometric features: a pin/slot pair for the driving phase and a concentric concave/convexarc pair for the locking phase. The designer creates parametric model of the pair with 25 functional parameters, including the centers of rotation, the pin and locking arc radii, and the slot dimensions.

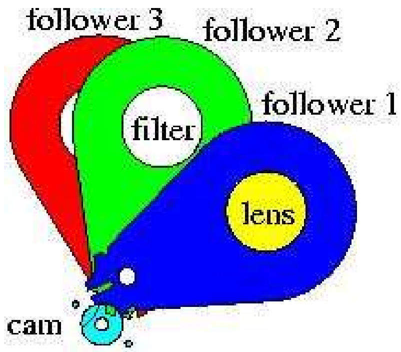
The next step is to assign nominal parameter values that produce the correct function. We perform this step via interactive manipulation of the cam/follower configuration spaces. Configuration space is a complete geometric representation of kinematics that reveals qualitative and quantitative function. We pick initial parameter values, compute the resulting configuration spaces, and evaluate them for correct function.



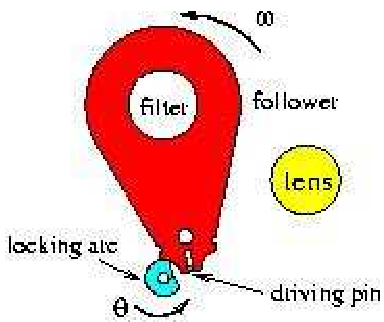
(a) all filters



(b) three-tier driver



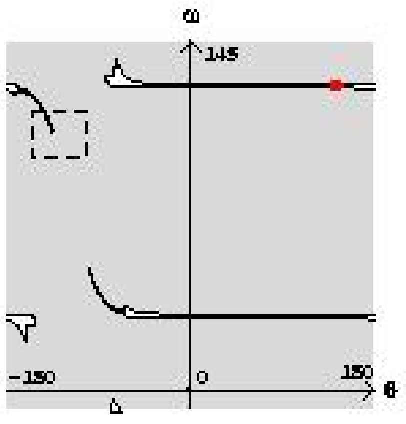
(c) all filters



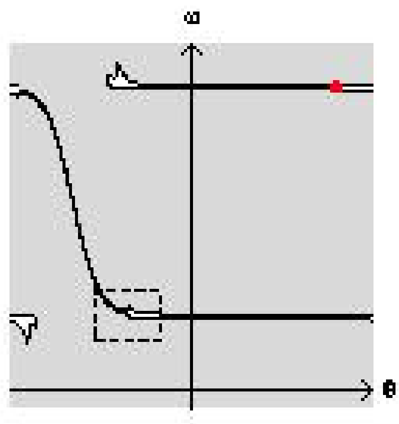
(d) top filter.

Fig 2. Perspective views (up) and top (bottom) views of the optical filter mechanism.

Fig. 3a shows the configuration space of the top cam/follower pair. The coordinates are the part orientation angles. The configuration space wraps around at the top/bottom and left/right boundaries because the coordinates are angles. It is partitioned into free space where the parts do not touch (white area) and blocked space, where they overlap (gray area), separated by contact space where they touch (black curves). The dot marks the displayed configuration in Fig. 2d. The horizontal contact curves correspond to the contact between the cam and the follower locking arcs.The slanted curves correspond to the contact between the cam pin and the driver slot. The gap between the curves represents play. The configuration space shows that the cam blocks when the pin is partially engaged in the follower slot, since the slanted channel consists of two disconnected segments that end at these blocking configurations. Fig. 3b shows a correct configuration space with a single slanted channel that connects the adjacent horizontal channels.

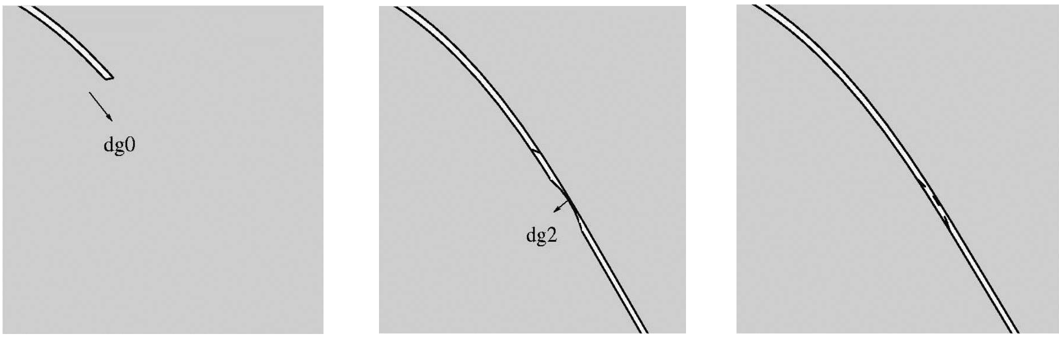


(a) blocked



(b) correct.

Fig. 3: Detail of the configuration spaces for one filter.



(a) blocked space (b) open channel (c) wider midpoint.

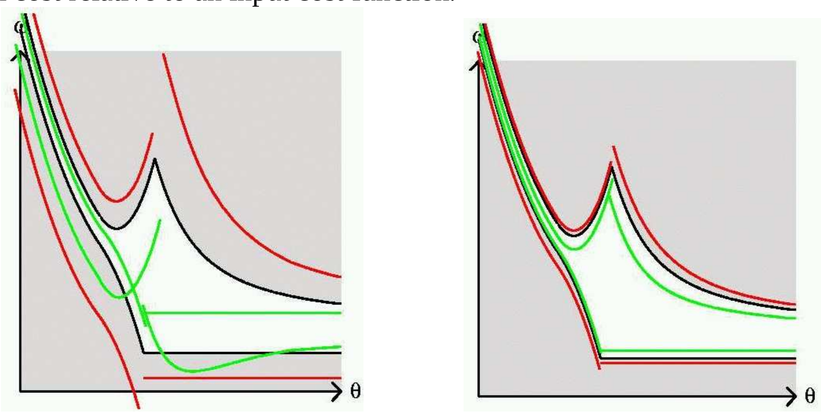
Fig. 4: Parametric modification of the c-space (detail).

We must modify the initial parameter values to merge the two partial channels. We grab the channel bottom with the mouse and drag it down. Fig. 4a shows the configuration space detail in the dashed rectangle in Fig. 3a with dragger *dg*0. The dragging causes the partial channels to meet (Fig. 4b). The program implements dragging by computing parameter values that make the selected contact configuration track the mouse. Although now open, the channel is too narrow at its midpoint, so we widen it with a second dragging operation (Fig. 4c).

The final design step is to assign tolerances to the parameters and to assess their effects. We model kinematic variation by generalizing the configuration space representation to toleranced parts. The contact curves of a pair are parameterized by the touching features, which depend on the tolerance parameters. As the parameters vary around their nominal values, the contact curves vary in a band around the nominal contact space, which we call the contact zone. The contact zone defines the kinematic variation in each contact configuration: every pair that satisfies the part tolerances generates a contact space that lies in the contact zone. Kinematic variations do not occur in free configurations because the parts do not interact.

Fig. 5a shows a detail of the cam/follower contact zone (dashed rectangle in Fig. 3b) in the area where the cam unlocks the follower and the pin is about to enter the follower slot. The contact zone is bounded by the light grey curves. Its width varies with the sensitivity of the nominal contact configuration to the tolerance parameters. The upper and lower zones of the diagonal channel intersect, which implies that there are parameter values in the tolerance intervals that cause blocking.

For robust design, we prefer to remove the blocking by widening the channels, and only if this is impractical, we resort to tightening the tolerance intervals until the zones become disconnected as shown in Fig. 5b. We use our tolerance optimization algorithm to compute intervals that achieve this goal at minimum cost relative to an input cost function.



(a) possible failure (b) robust

Fig. 5: Detail of contact zones

3. TOOLS FOR ROBUST KINEMATIC DESIGN

3.1. Nominal analysis of mechanisms: the configuration space approach

We model nominal kinematic function within the configuration space representation of rigid body interaction. Configuration space is a general representation for systems of rigid parts that is widely used in robot motion planning. We construct a configuration space for each pair of interacting parts in the mechanical system. The configuration space is a manifold with one coordinate per part degree of freedom. Interactions of pairs of fixed-axes planar and spatial parts are modeled with two-dimensional spaces [8,9], whereas interactions between general planar pairs are modeled with three-dimensional spaces [10]. In both cases, points specify the relative configuration (position and orientation) of one part with respect to the other. We perform contact analysis by computing a configuration space for each pair of parts.

Configuration space partitions into three disjoint sets that characterize part interaction: blocked space where the parts overlap, free space where they do not touch, and contact space where they touch without overlap. Blocked space represents unrealizable configurations, free space represents independent part motions, and contact space represents motion constraints due to part contacts. The spaces have useful topological properties. Free and blocked spaces are open sets whose common boundary is contact space. Contact space is a closed set comprised of algebraic patches that represent contacts between pairs of part features. Patch boundary curves represent simultaneous contacts between two pairs of part features.

The configuration space of a pair is a complete representation of the part contacts. Contacts between pairs of features correspond to contact patches (curve segments in two dimensions and surface patches in three). The patch geometry encodes the motion constraint and the patch boundary encodes the contact change conditions. Part motions correspond to paths in configuration space. A path is legal if it lies in free and contact space, but illegal if it intersects blocked space. Contacts occur at configurations where the path crosses from free to contact space, break where it crosses from contact to free space, and change where it crosses between neighboring contact patches.

The configuration space representation generalizes from pairs of parts to systems with more than two parts. A system of n planar parts has a $3n$-dimensional configuration space whose points specify the n part configurations. A system configuration is free when no parts touch, is blocked when two parts overlap, and is in contact when two parts touch and no parts overlap. Computing the complete high-dimensional mechanism configuration space is both impractical and unecessary. Instead, we compute the relevant portion of the system space from the pair spaces.

We have developed a configuration space computation program for planar pairs whose part boundaries consist of line segments and circular arcs [9,10]. These features suffice for most engineering applications with the exception of involute gears and precision cams, which are best handled by specialized methods [2,5]. The program computes an exact representation of contact space: a graph whose nodes represent contact patches and whose arcs represent patch adjacencies. Each node contains a contact function that evaluates to zero on the patch, is positive in nearby free configurations, and is negative in nearby blocked configurations. Each graph arc contains a parametric representation of the boundary curve between its incident patches. After constructing configuration spaces for the pairs in a mechanical system, we analyze the system mechanical function in the system configuration space. We select the relevant pairwise contact equations in the vicinity of the system configuration and derive from them the system function and contact changes.

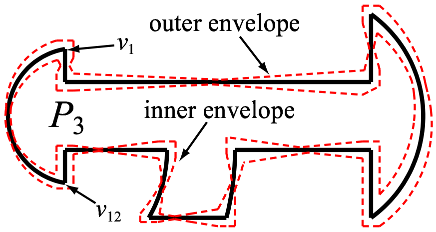
3.3. Tolerance analysis of mechanisms

We model kinematic variation by generalizing configuration spaces to toleranced parts [11-13]. The contact patches of a pair are parameterized by the touching features, which depend on the tolerance parameters. As the parameters vary around their nominal values, the contact patches vary in a

Tolerance specification

$v_1=(110+p_1,169+p_2)$	$v_7=(160+p_8,140-p_7-0.5p_8)$
$v_2=(110-p_1,160+p_3)$	$v_8=(157.5-p_8,120+p_9)$
$v_3=(200+p_4-0.5p_3,160-p_2+p_4)$	$v_9=(134+p_{10},120-0.5p_9)$
$v_4=(200-p_4,180+p_5)$	$v_{10}=(140-p_{10},140+p_{11})$
$v_5=(200-p_5,120+p_6)$	$v_{11}=(110+p_{12},140-0.5p_{11})$
$v_6=(200+p_6,140+p_7)$	$v_{12}=(110+p_2,130-0.6p_1)$

(a)



(b)

Fig. 6. (a) Tolerance specification and (b) envelope of a part. Vertices v_1 to v_{12} are ordered clockwise. Parameters $p_{,...,}p_{12}$ have all nominal values equal to zero. Typically, the vertex functions are derived from the dimensional tolerance specification, either manually or as output from a symbolic geometric constraint solver. Here we chose the parametrization and tolerance intervals that best illustrate the envelopes properties.

band around the nominal contact space, which we call the contact zone. The contact zone defines the kinematic variation in each contact configuration: every pair that satisfies the part tolerances generates a contact space that lies in the contact zone. Kinematic variations do not occur in free configurations because the parts do not interact.

Each contact patch generates a region in the contact zone that represents the kinematic variation in the corresponding feature contact. The region boundaries encode the worst-case kinematic variation over the allowable parameter variations. They are smooth functions of the tolerance parameters and of the part configurations in each region. The variation at boundary configurations is the maximum over the neighboring patch variations. The contact zone regions represent the quantitative kinematic variation, while the relations among regions represent qualitative variations, such as possible jamming, under-cutting, and interference. The contact zone is obtained from the parametric part models and the nominal contact patches.

We have developed an algorithm for kinematic tolerance analysis of planar systems based on these concepts [11-13]. The algorithm constructs a variation model for the system, derives worst-case bounds on the variation, and helps designers find unexpected failure modes, such as jamming and blocking. It constructs a variation model for each interacting pair of parts then derives the overall system variation at a given configuration by composing the pairwise variation models via sensitivity analysis and linear programming. The algorithm analyzes systems with 50 to 100 parameters in under a minute on a PC, which permits interactive tolerancing of detailed functional models [14].

3.4 Tolerance part envelopes

To complement kinematic tolerance analysis and help designers visualize and quantify shape and position variation in Euclidean space, we have developed algorithms to compute the shape variation of individual parts and the relative position variation of parts in an assembly [15-17]. Tolerance envelopes are useful in many design tasks such as quantifying functional errors, identifying unexpected part collisions, and determining device assemblability.

We have developed a framework for modeling parametric variation in planar parts with curved boundaries and for efficiently computing first-order approximations of their worst-case tolerance envelopes. We model part variation with a parametric tolerancing model which is general, reflects current tolerancing practice, incorporates common tolerancing assumptions, and has good computational properties. In this model, part variation is determined by m parameter values $p=(p_1,.., p_m)$, specifying lengths, angles, and radii of part features. The parameters have nominal values and can vary along small tolerance intervals. The coordinates of the part vertices are standard elementary functions of a subset of the m parameters. An instance of the parameter values determines the geometry of the part. Figure 6(a) shows the tolerance specification of a part.

We have derived the geometric properties of the tolerance envelopes and have developed four efficient algorithms for computing first-order linear approximations of the inner and outer part envelopes with successive accuracy. Figure 6(b) shows the tolerance envelope of a part. The algorithms compute the tolerance envelope of the entire part by merging the tolerance envelopes of its segments. They offer clear running time, simplicity, and accuracy advantages over commonly used Monte Carlo and uniform sampling methods. Experimental results on three realistic examples show that the implemented algorithms produce better results in terms of accuracy and running time than Monte Carlo and sampling methods.

3.4. Robust synthesis of higher pairs

Within the configuration space approach to kinematic synthesis [18], we have developled an algorithm that ensures correct kinematic function by synthesizing tolerances that preclude failure modes and that limit motion variation [19-20]. Nominal parameter values are changed when possible and tolerance intervals are shrunken as a last resort. We cannot search the entire parameter space for bad parameter values. Its dimension is prohibitively high because mechanical systems have tens to hundreds of shape and configuration parameters. Tiny steps are required because the kinematic function can vary suddenly or even discontinuously. We limit the search to parameter values that maximize the variation of one or two contacts.

The input to the algorithm is a parametric model of a mechanical system (part profiles and system configuration) with initial tolerance intervals for the parameters. The output is revised tolerances that guarantee correct kinematic function for all system variations. The algorithm consists of a three-step cycle that detects and eliminates incorrect system variations. The first step finds candidate vectors of parameter values whose kinematic variation is maximal. The second step tests the vectors for correct kinematic function. The third step adjusts the tolerances to exclude the vectors with incorrect functions. The cycle repeats until every vector exhibits correct function.

The candidate parameter vectors are selected in two steps. The first step finds sets of parameter values that generate points on the boundaries of the system contact zones. Each set contains the parameters that determine the shape and motion axes of two contacting part features. The set specifies parameter values that maximize the kinematic variation of the contact in one configuration of the nominal work cycle. Two sets are called compatible when they agree on their common parameters, for example $\{x=1,y=2\}$ and $\{y=2,z=3\}$, and in particular if they are disjoint. The union of k compatible sets simultaneously maximizes the kinematic variation of k contacts in k nominal configurations. The second step forms the candidate parameter vectors from unions of compatible sets. These candidates represent limiting cases of contact interactions, which is where failures are most likely to occur and hardest to detect.

Next, the candidate parameter vectors are tested for failure modes and for excessive motion variation. The vectors that fail either test are passed to the tolerance revision module. The failure mode test matches the nominal and candidate contact spaces of the higher pairs. The test succeeds when the two spaces have the same structure: they have the same number of components and each component in the first space matches a unique component in the second space. Two components match when they consist of equivalent curves in the same cyclic order. Two curves are equivalent when they are generated by the same pair of part features. The tolerance revision step revises the current tolerances to exclude the failed parameter vectors. The tolerances define an axis-aligned box in parameter space: the box is centered at u_0 and its width in the k-th dimension is the tolerance interval of the k-th parameter. The failed vectors, ui , lie in this box. The revision excludes them by modifying u_0 when possible and by shrinking the box width otherwise.

We have demonstrated the tolerance synthesis algorithm on three common higher pairs from the engineering literature and on a system comprised of three custom pairs, and on a spatial gear [21]. In each case, the algorithm finds tolerances that correct kinematic problems in the initial tolerances. It never needs to shrink a tolerance interval. The revised nominal values are in the initial intervals. Yet these small changes eliminate hundreds of incorrect kinematic functions to produce robust designs.

4. CONCLUSIONS

Over the past ten years, we have developed a novel framework and algorithms for kinematic analysis and design of planar mechanical system based on the configuration space approach. The tools automate parametric nominal and toleranced kinematic modelling. We have demonstrated the use of these tools on industrial examples, most notably the redesign of a production spatial gear pair. Future work includes incorporating the tools into a CAD system and further developing the technical aspects of the methodology.

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