Towards robust kinematic synthesis of mechanical systems

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Abstract: We describe our research in kinematic synthesis of planar mechanical systems based on configuration space manipulation. We present a design scenario that illustrates our methodology and describe two algorithms that support robust parametric design. The first algorithm helps designers select nominal parameter values and identify failure modes. The second algorithm helps optimize tolerance allocation. These are the first general algorithms for these tasks, as prior work is limited to lower pairs and to a few custom higher pairs.

Keywords: parametric design, robust design, tolerancing, kinematics, configuration space.

1. INTRODUCTION

We describe our research in robust kinematic synthesis of mechanical systems. Kinematic synthesis is the task of devising a system of mechanical parts that implements specified motion transformations. The design must meet its specifications despite part variations due to manufacturing. An optimal design achieves this goal at minimal cost. Kinematic synthesis is central to mechanical design because kinematics largely determines mechanical function.

Kinematic synthesis is an iterative process in which the designer selects a design concept, constructs a parametric model, assigns parameter values, and allocates tolerances. At each step, the designer makes changes, assesses their impact, and decides whether to advance to the next step or to return to a prior step. When a design fails due to part variations, the designer can change the nominal design or tighten the tolerances. Changing the nominal design is often better, since cost increases rapidly as tolerances decrease, but can be much harder.

Kinematic synthesis is difficult and time consuming. In the conceptual design step, the designer needs to compare competing concepts based on incomplete, high-level characterizations. In the later steps, he has to adapt the chosen concept to comply with numerous, often competing design specifications. The adaptation requires extensive kinematic analysis of many design instances. The analysis is difficult because it involves multiple part contacts that impose nonlinear motion constraints. Some contacts are part of the nominal function, while others arise due to part variation. Both types can introduce failure modes that coexist with or supersede the correct function. Finally, the designer needs to formulate a realistic, application-specific cost function for tolerance allocation.

Software support for kinematic synthesis is limited. There are very few tools for conceptual design. Powerful commercial packages, such as CATIA and IDEAS, support construction and visualization of parametric designs. Kinematic analysis software is limited to multi-body systems: assemblies of parts that interact via a fixed set of feature contacts [Schiehlen, 90]. Prior research in synthesis provides algorithms for linkages [Erdman, 93], [Ramaswamy, 93] and cams [Angeles and Lopez-Cajun, 91], [Gonzales-Palacios and Angeles, 93] but does not address systems with contact changes. Tolerance analysis software is available for individual, user-specified system configurations, but not over a continuous work cycle [Chase et al, 97], [Solomons et al, 97], [Ballot and Bourdet, 97].

A new methodology, called robust design, has been developed to increase reliability and reduce redesign costs. In robust design, nominal and tolerance changes are evaluated together. The nominal design is modified to reduce its sensitivity to part variations. Then tolerances are allocated to guarantee correct function and to minimize cost. Robust design differs from the traditional design paradigm in which failure due to part variations is fixed primarily by tightening tolerances. Robust design is especially relevant to kinematic synthesis because failures due to tolerances are hard to detect and costly to correct [Schultheiss and Hinze, 99].

In this paper, we describe two algorithms that support kinematic synthesis of planar mechanical systems and illustrate the algorithms on a robust design scenario. A system is comprised of kinematic pairs with multiple contacts that form one or more open chains or closed loops. Each part has one degree of freedom: translation along a fixed axis or rotation around a fixed point. The first algorithm helps designers select nominal parameter values and identify failure modes [Kyung and Sacks, 2001]. The second algorithm helps optimize tolerance allocation. These are the first general algorithms for these tasks; prior work in the field is limited to lower pairs and to a few custom higher pairs. The algorithms build upon our kinematic analysis algorithm for the specified class of mechanical systems [Sacks and Joskowicz, 95].

2. DESIGN SCENARIO

We illustrate our kinematic synthesis algorithms on an optical filter mechanism from Israel Aircraft Industries (Figure 1a). The mechanism consists of a lens, a cam, and three filters mounted on identical followers. The lens is attached to a fixed frame (not shown). The followers are stacked on a shaft and can rotate independently. The cam consists of three slices that rotate together on a common shaft. Each cam slice drives the corresponding follower. Figure 1b shows the top cam slice and its follower. The cam slice consists of a driving pin and a locking arc. When the cam shaft rotates counterclockwise, the

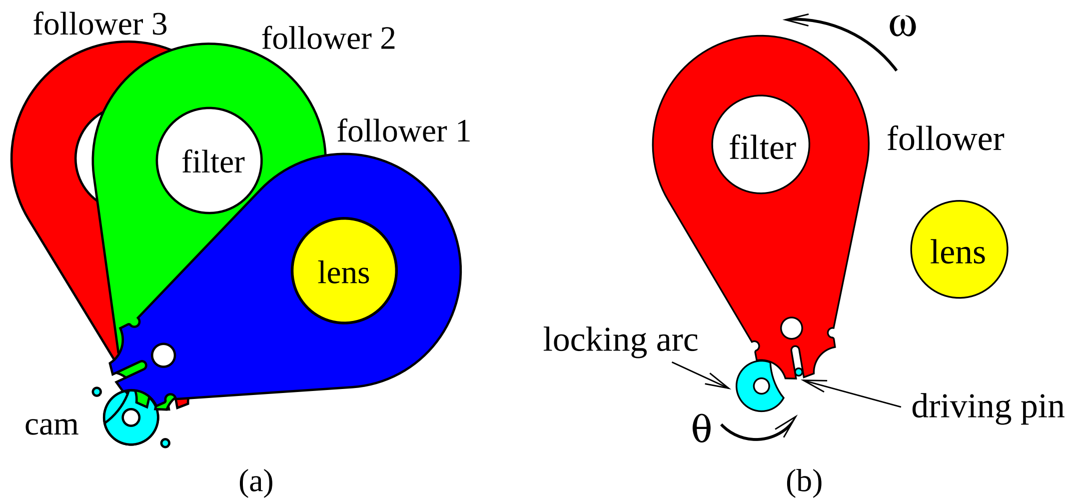


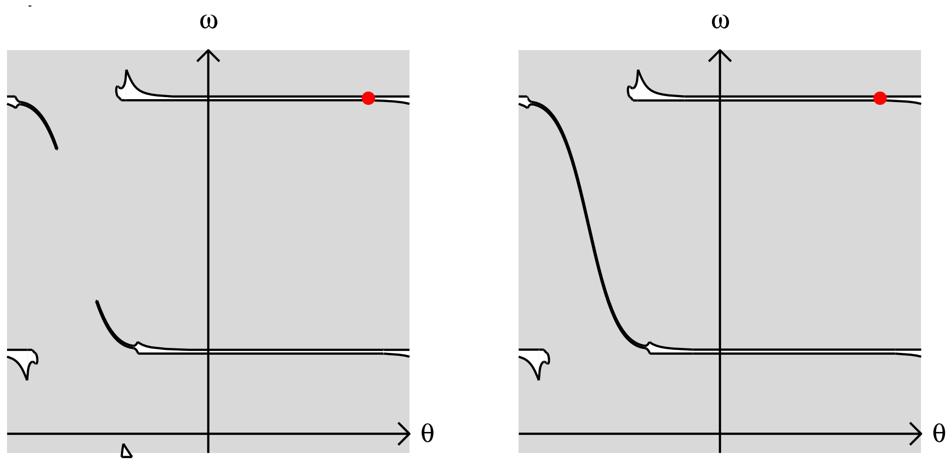
Figure 1: *Optical filter mechanism: (a) all filters, (b) top filter.*

pin engages the follower slot and rotates the follower until the filter covers the lens. The other two cam slices are identical, except that they are rotated by 90 and 180 degrees respectively. In the initial state, the filters are off the lens. When the cam shaft is rotated counterclockwise, the three followers are engaged in sequence. Rotating the cam clockwise resets the filters to the initial state.

The design task is to devise a mechanism to engage and reset the followers in the intended manner. The mechanism must be robust because it will be mounted on a vehicle and must be compact to fit in the alloted space. During conceptual design, the designer chooses a Geneva mechanism with one driver and one follower per filter. This concept dictates the functional geometric features: a pin/slot pair for the driving phase and a concentric concave/convex arc pair for the locking phase. The designer creates a parametric model of the pair with 25 functional parameters, including the centers of rotation, the pin and locking arc radii, and the slot dimensions.

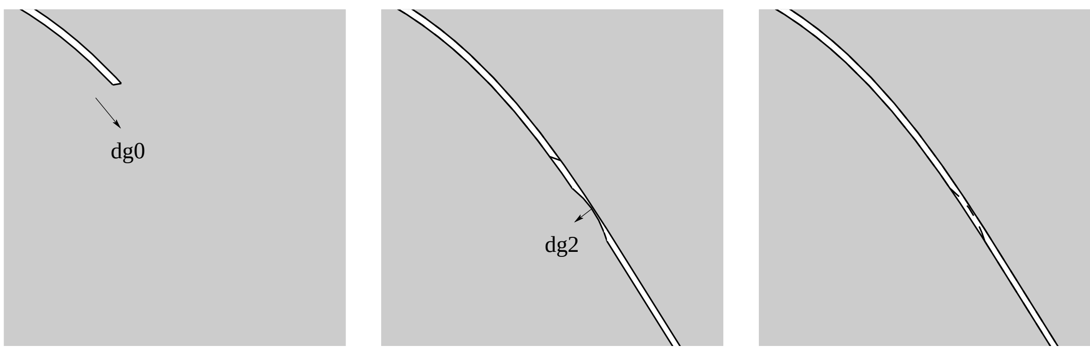
The next step is to assign nominal parameter values that produce correct function. We perform this step via interactive manipulation of the cam/follower configuration spaces. Configuration space is a complete geometric representation of kinematics that reveals qualitative and quantitative function. We pick initial parameter values, compute the resulting configuration spaces, and evaluate them for correct function.

Figure 2a shows the configuration space of the top cam/follower pair. The coordinates are the part orientation angles. The configuration space wraps around at the top/bottom and left/right boundaries because the coordinates are angles. It is partitioned into free space where the parts do not touch (white area) and blocked space, where they overlap (gray area), separated by contact space where they touch (black curves). The dot marks the displayed configuration in Figure 1b. The horizontal contact curves correspond to the contact between the cam and the follower locking arcs. The slanted curves correspond to the contact between the cam pin and the driver slot. The gap between the curves represents



(a) (b)

Figure 2: *Configuration spaces for one filter: (a) blocked, (b) correct.*



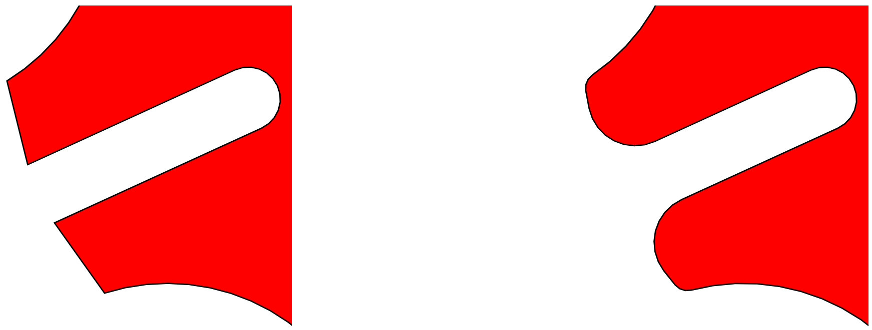
(a) (b) (c)

Figure 3: *Parametric modification of c-space: (a) detail of initial blocked space, (b) open channel, (c) wider midpoint.*

play. The configuration space shows that the cam blocks when the pin is partially engaged in the follower slot, since the slanted channel consists of two disjoint segments that end at these blocking configurations. Figure 2b shows a correct configuration space with a single slanted channel that connects the adjacent horizontal channels.

We must modify the initial parameter values to merge the two partial channels. We grab the channel bottom with the mouse and drag down (Figure 3a). The dragging causes the partial channels to meet (Figure 3b). The program implements dragging by computing parameter values that make the selected contact configuration track the mouse. Although now open, the channel is too narrow at its midpoint, so we widen it with a second dragging operation (Figure 3c).

Now that the nominal function is correct, we modify the parametric design by replacing the sharp corners of the follower slot with fillets (Figure 4). We assign values to the



(a) (b)

Figure 4: *Adding follower fillets: (a) before and (b) after.*

new parameters as before. The configuration space is only slightly different.

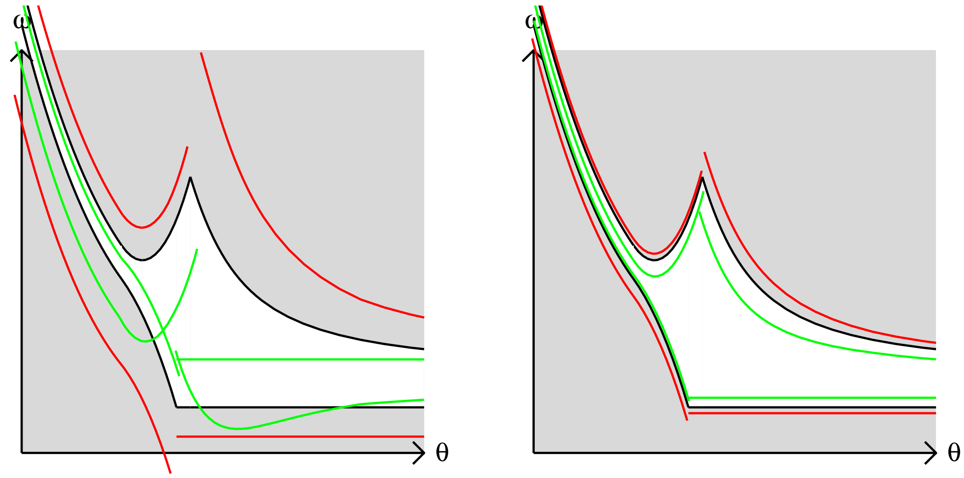
The final design step is to assign tolerances to the parameters and to assess their effects. We model kinematic variation by generalizing the configuration space representation to toleranced parts. The contact curves of a pair are parameterized by the touching features, which depend on the tolerance parameters. As the parameters vary around their nominal values, the contact curves vary in a band around the nominal contact space, which we call the contact zone. The contact zone defines the kinematic variation in each contact configuration: every pair that satisfies the part tolerances generates a contact space that lies in the contact zone. Kinematic variations do not occur in free configurations because the parts do not interact.

Figure 5a shows a detail of the cam/follower contact zone in the area where the cam unlocks the follower and the pin is about to enter the follower slot. The contact zone is bounded by the light grey curves. Its width varies with the sensitivity of the nominal contact configuration to the tolerance parameters. The upper and lower zones of the diagonal channel intersect, which implies that there are parameter values in the tolerance intervals that cause blocking.

In robust design, we prefer to remove the blocking by widening the channels, as described above. If this is impractical, we can tighten the tolerance intervals until the zones become disjoint, as shown in Figure 5b. We use our tolerance optimization algorithm to compute intervals that achieve this goal at minimum cost relative to an input cost function.

3. PARAMETRIC DESIGN ALGORITHM

We briefly describe the parametric design algorithm. For a detailed description, see [Kyung and Sacks, 2001]. The designer inputs a parametric model of a mechanical system and specifies initial parameter values. The synthesis program computes and displays configuration spaces for the kinematic pairs. These spaces encode the initial kinematics: feature contacts appear as contact curves and contact changes appear as curve adjacencies.



(a) (b)

Figure 5: *Detail of contact zones: (a) possible failure, (b) robust.*

Design objectives are expressed as changes in the contact curve geometry. The designer inputs the objectives with the mouse and the program achieves them by changing the design parameters.

The parameter update algorithm computes design parameter values that achieve specified kinematic changes. The input is a set of draggers (dgo and dg2 in Figure 3). Each dragger consists of a contact curve, a start point $\mathbf{q}_0$ on the curve, and a goal point $\mathbf{q}_0 + \delta\mathbf{q}$. Contact curves have the form $C(\mathbf{q}, \mathbf{p}) = 0$ where $\mathbf{q}$ is the two configuration space coordinates (θ, ω in our example) and $\mathbf{p}$ is the vector of design parameters. The start point, $\mathbf{q}_0$, satisfies $C(\mathbf{q}_0, \mathbf{p}_0) = 0$ with $\mathbf{p}_0$ the initial parameter values. The program computes a parameter update $\delta\mathbf{p}$ for which the curve goes through the goal point, $C(\mathbf{q}_0 + \delta\mathbf{q}, \mathbf{p}_0 + \delta\mathbf{p}) = 0$.

The contact equations are solved numerically because a closed-form solution is impractical. The program performs a sequence of small parameter updates governed by the linearized contact equation

$$\frac{\partial C}{\partial \mathbf{q}}(\mathbf{q}_0, \mathbf{p}_0)\delta\mathbf{q} + \frac{\partial C}{\partial \mathbf{p}}(\mathbf{q}_0, \mathbf{p}_0)\delta\mathbf{p} = 0.$$

There is one equation per dragger. The equations are normally under constrained because a typical number of draggers is less than five, while a typical number of design parameters is twenty. But they are over constrained when the draggers are inconsistent or when there are more draggers than parameters. We compute an exact, minimum-norm solution if possible and a least-squares solution otherwise, using singular value decomposition.

The program monitors updates for unintended kinematic changes. Contact curves that were not selected by the designer will often change shape because they share parameters with the selected curves. A sufficiently large change can cause a pair of disjoint curves to intersect or vice a versa. These events can cause structural changes in the system kinematics, such as jamming. The program detects these changes and modifies the parameter update to prevent them.

Our algorithm builds on prior work [Caine, 93], who designs planar part feeders via configuration space manipulation. The kinematic function is represented by a partial part/feeder configuration space. The designer requests a single change in the configuration space and the program changes the feeder geometry accordingly. The models are non-parametric and the modifications are heuristic. Structural changes are not addressed.

4. TOLERANCE ALLOCATION ALGORITHM

Our tolerance allocation algorithm uses constrained optimization to compute tolerance ranges for the design parameters that ensure correct kinematic function at minimum cost. The part shapes and motion axes are parameterized by a vector $\mathbf{p}$ whose nominal value is $\overline{\mathbf{p}}$. Parameter p_i is constrained to the interval $[\overline{p}_i - l_i, \overline{p}_i + u_i]$ with l_i and u_i non-negative. The l_i and u_i are the variables in the optimization problem. The constraints are bounds on the kinematic variation at a sequence of nominal system configurations. Minimum and maximum values for the variables can also be specified.

The kinematic constraints at a single configuration translate into linear inequalities among the variables. We formulate the constraint on a variable z in terms of the maximum increase in its value, δz^+, and the maximum decrease in its value, δz^-, due to variation in $\mathbf{p}$. Following standard tolerancing practice, we use the linear approximations

$$\delta z^+ = \sum_i \frac{\partial z}{\partial p_i} w_i \text{ with } w_i = \begin{cases} u_i & \text{if } \partial z/\partial p_i > 0 \\ -l_i & \text{otherwise.} \end{cases}$$

and

$$\delta z^- = -\sum_i \frac{\partial z}{\partial p_i} w_i \text{ with } w_i = \begin{cases} -l_i & \text{if } \partial z/\partial p_i > 0 \\ u_i & \text{otherwise.} \end{cases}$$

The signs are chosen so that both quantities are non-negative: each term is a non-negative number times a non-negative variable. The kinematic constraints are $\delta z^+ \leq s$ and $\delta z^- \leq t$ with s and t the input bounds on the kinematic variation.

In our design scenario, we bound the kinematic variation at the center of the diagonal channel. We pick configurations $c = (-1.47, 0.519)$ on the top curve and $d = (-1.47, 0.506)$ on the bottom curve. The nominal play is $0.519 - 0.506 = 0.013$, which means that the follower can rotate by 0.013 radians when the cam angle is 1.47 radians. The actual play equals $c^- - d^+$ with c^- the worst-case decrease in the top curve and d^+ the maximum increase in the lower curve. We specify the kinematic constraints $c^- \leq 0.05$ and $d^+ \leq 0.005$ to ensure a minimal play of 0.003 radians. The resulting linear equations are

$$\begin{aligned} c^- = u_1 + l_2 + 0.91u_3 + 0.5u_4 + 0.87l_5 &\leq 0.005 \\ d^+ = l_1 + u_2 + 0.91u_3 + 0.5u_4 + 0.88l_5 &\leq 0.005 \end{aligned}$$

with p_1 (and l_l, u_1) the pin radius, p_2 the slot width, p_3 the distance from the pin center to its center of rotation, p_4, and (p_4, p_5) the (x, y) coordinates of the follower center of rotation relative to the cam center of rotation.

We have developed an algorithm for computing the derivatives of the coordinates with respect to the parameters [Sacks and Joskowicz, 97] The first step is to formulate a parametric contact equation $y = f(x, \mathbf{p})$ for each pair of parts that is in contact. This is done by querying the configuration spaces of the pairs, as explained in our prior work. The second step is to differentiate the contact equations with respect to x and **p**. The final step is to compute the derivatives of the coordinates by the chain rule of calculus. The computation starts from the driving coordinates, which have zero kinematic variation by definition, and propagates from part to part via contact equations.

Choosing an objective function is a modeling decision based on specific product costs. Many options, linear and nonlinear, appear in the tolerancing literature. The research challenge is to find a function that reflects kinematic costs adequately for design optimization, yet yields a tractable optimization problem. The simplest option is to assign linear cost functions $w_i = a_i - b_i l_i - c_i u_i$ with $a_i, b_i, c_i \geq 0$ and to minimize the total cost $\sum_i w_i$. The signs are chosen so the cost of a parameter decreases as its lower and upper variations increase.

In our example, we set $a_i = 10$, $b_i = 1$, and $c_i = 1$ and obtained optimal tolerances of $l_2 = u_2 = 0.005$ and the other variables equal to zero. This is unrealistic because zero tolerances are unrealizable. The problem is that the linear cost function diverges from the true cost. We can improve the model by specifying a minimum value of 0.001 for every variable. The optimal tolerances are $l_2 = u_2 = 0.0017$ and the other variables equal to this minimum value. We see that the slot width dominates the cost of tolerancing for channel play.

5. CONCLUSIONS

We have presented two algorithms for robust kinematic synthesis of planar mechanical systems with one degree of freedom per part. The first algorithm supports parameter value selection for a nominal design. The second supports optimal tolerance allocation subject to kinematic constraints. Our next step is to apply these algorithms to large-scale problems in automotive transmission design and in other application areas. Our first research goal is to extend the algorithms to general planar systems with three degrees of freedom per part. The research challenge is structual change detection in three-dimensional configuration spaces. Developing an effective user interface is a significant technical challenge. Our second research goal is to handle spatial systems with one degree of freedom per part. The planar algorithms are applicable, but we need to derive parametric contact functions for all pairs of spatial features, such as planes, spheres, cylinders, and spatial curves.

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