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# REDESIGN OF A SPATIAL GEAR PAIR USING CONFIGURATION SPACES

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## ABSTRACT

This paper presents an industrial case study in which a spatial higher pair is redesigned using our configuration space method of kinematic analysis. The task is to remove occasional blocking in an asynchronous reverse gear pair from a car transmission. A systematic kinematic analysis is required because the blocking configurations are unknown and because very few initial configurations cause blocking. We use our configuration space method of kinematic analysis to solve the problem. We determine why the gears block by constructing a series of two-dimensional configuration spaces that model the engagement kinematics. Blocking occurs when two consecutive pairs of teeth make contact during engagement. The gear angles at the contact determine whether or not the gears will block. Our analysis determines that blocking occurs in 4% of the angle space. Fine tuning the gear parameters reduces the range to 0.5%, but cannot eliminate the blocking. Removing every second gear tooth eliminates the blocking. The analysis results are consistent with the experimental data. The case study demonstrates that the configuration space method helps solve industrial problems that are outside the scope of prior work.

**Keywords:** spatial kinematic analysis, gear design, configuration space, parametric design.

## INTRODUCTION

This paper presents an industrial case study in which we redesign a spatial higher pair using our configuration space method of kinematic analysis. The task is to remove occasional blocking in an asynchronous reverse gear pair from a car transmission. A systematic kinematic analysis is required to understand the cause of blocking. Prior analysis methods are inapplicable because the gears have complex shapes and multiple kinematic functions. The configuration space method overcomes these difficulties and allows us to find and remove the blocking.

Kinematic analysis of a pair of parts determines the coupling between their motions due to contacts between their surface features. When two features touch, the relative velocity at the contact point must be tangent to both features. This constraint can be expressed as an algebraic equation in the part configurations whose solution varies smoothly as the contact point moves along the features. When the contact point shifts to another feature, a different contact constraint takes effect, possibly causing a velocity discontinuity. When a driving motion is applied to one part, it drives the other part via a sequence of contacts. Throughout the sequence, the geometry of the touching features determines the coupling between the part velocities. A slightly different initial configuration or driving motion can produce a very different contact sequence. In a complete kinematic analysis, we must compute the possible sequences and classify them by initial configuration and driving motion. The results help designers understand the system function, find and correct design flaws, measure performance, and compare design alternatives.

Spatial higher pairs, such as the asynchronous reverse gear pair, are hard to analyze because they have many surface features, complex contact equations, and many contact sequences. The analyst needs to examine every pair of features to see if they touch in some configuration. Typically, most pairs cannot touch

 

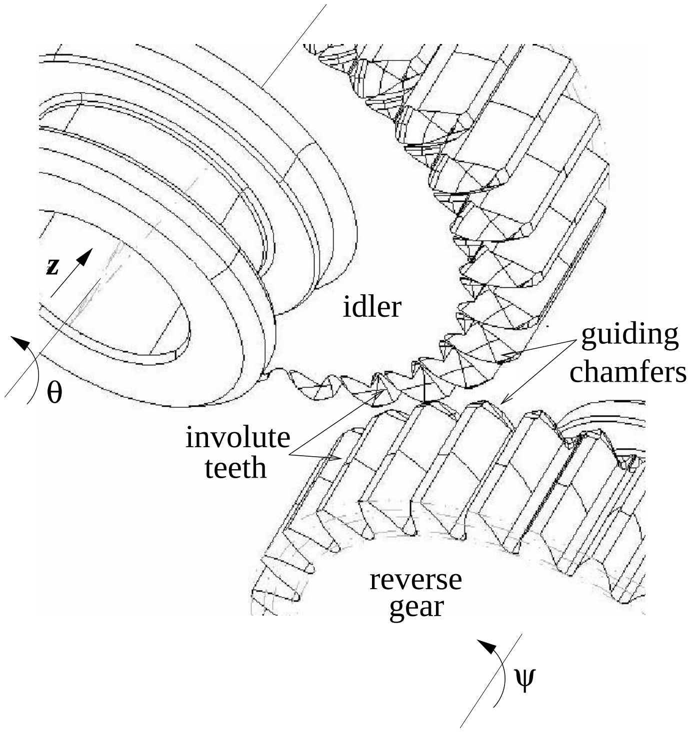


Figure 1. ASYNCHRONOUS REVERSE GEAR PAIR.

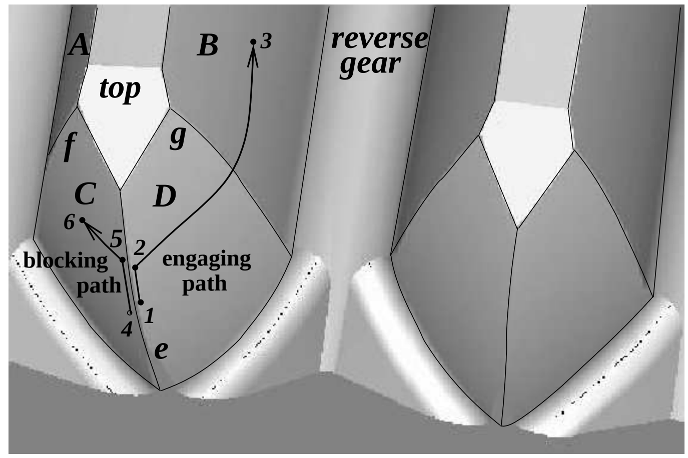


Figure 2. REVERSE GEAR TOOTH GEOMETRY.

because other pairs keep them apart. The remaining pairs generate contacts that generate contact sequences. In the reverse gears, each part has 20 functional features, which produces 400 feature pairs of which 10 generate contacts. The blocking is due to an unintended contact sequence that is produced by a rare type of initial configuration.

Prior work does not provide analysis tools for general spatial higher pairs. Custom algorithms have been developed for specific gears (Litvin, 1994) and cams (Angeles and Lopez-Cajun, 1991; Angeles and Gonzales-Palacios, 1993) of practical importance. They focus on contacts between pairs of complex surface features, such as involutes and helicoids. The contacts are permanent or change in a known sequence, so contact changes pose no problems. These algorithms are inappropriate in our case because the blocking contact sequence is not known in advance. Other research focuses on efficient simulation algorithms for multi-body systems, which are systems whose parts are connected by lower pairs and by simple higher pairs with permanent contacts (Schiehlen, 1990; Erdman, 1993; Haug, 1989). These algorithms can be extended to higher pairs using collision detection (Lin et al., 1996). However, simulation does not provide a comprehensive kinematic analysis because it assumes a single initial configuration and driving motion. A comprehensive analysis is required in our problem because the blocking configurations are unknown and because very few motions cause blocking.

We developed a comprehensive kinematic analysis algorithm for planar pairs based on configuration space (Joskowicz and Sacks, 1999). The configuration space of a pair is a geometric representation of its contacts, contact changes, and contact sequences. Designers can evaluate the qualitative kinematic function of the pair by examining its configuration space structure via a graphical interface. They can evaluate the quantitative kinematics by measuring configuration space features. We have demonstrated the effectiveness of this paradigm with industrial case studies (Sacks et al., 1999; Sacks and Allen, 1999; Sacks and Barnes, 2001). Recently, we extended the algorithm to spatial pairs (Drori et al, 1999; Kim et al., 2002). This paper shows the effectiveness of the spatial algorithm on a new industrial case study.

## PROBLEM DESCRIPTION

We now describe the design problem. The asynchronous reverse gear pair consists of two modified spur gears, the idler and the reverse gear, mounted on parallel axes (Figure 1). The reverse gear rotates around its axis and the idler rotates around and translates along its axis. The idler has 31 teeth and the reverse gear has 13 teeth. Their pitch diameters are 65.62mm and 27.52mm. Figure 2 shows the reverse gear tooth geometry; the idler teeth have the same geometry with different dimensions. The tooth has involute sides, *A* and *B*, topped by spherical patches, *C* and *D*, that form a guiding chamfer. The spherical patches meet along a circular arc, *e*, and meet the involutes *A* and *B* along curves *f* and *g*. The tooth top is not functional.

Initially, the idler and the reverse gear are not in contact. As the driver shifts into reverse, the idler translates toward the reverse gear until its lead chamfer touches a reverse gear chamfer (the configuration shown in Figure 1). As the idler continues to translate, the chamfers rotate the gears into alignment. Once the spur gear teeth mesh, the idler stops translating and the gears ro-

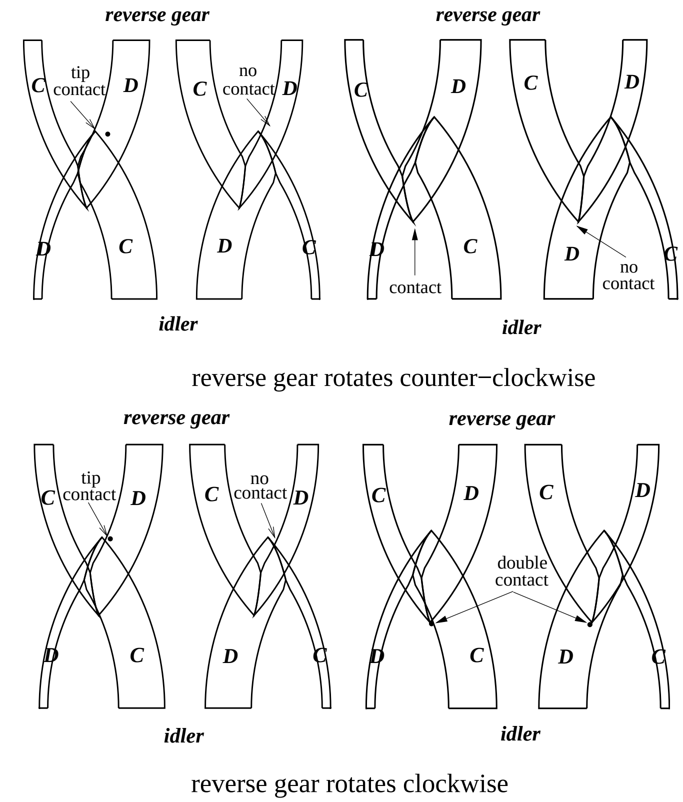


Figure 3. TOP VIEWS OF GEAR TEETH ENGAGING (TOP) AND BLOCKING (BOTTOM). LEFT AND RIGHT SNAPSHOTS ARE STARTING AND ENDING CONFIGURATIONS.

tate in unison. Figure 2 shows the engaging contact point path on a reverse gear tooth. The chamfer arcs *e* touch (point 1) then slide along each other until the *D* patches touch (point 2). The idler slides into full engagement where the involutes *B* mesh (point 3).

After building a prototype pair, the designers observed occasional blocking during engagement. The idler sometimes stops translating before the spur gears mesh due to an interfering contact between an adjacent pair of teeth. The challenges are to characterize the blocking contact sequences, to find the initial configurations that lead to blocking, and to modify the design to eliminate blocking. Figure 2 shows the blocking contact point path. The chamfer arcs *e* touch (point 4) then slide along each other until the *C* patches touch (point 5). The path ends prematurely (point 6) due to an interfering contact between an adjacent pair of teeth. Figure 3 shows top views of the engaging and blocking paths.

## ANALYSIS

We use our configuration space method of kinematic analysis (Joskowicz and Sacks, 1999) to understand the behavior of

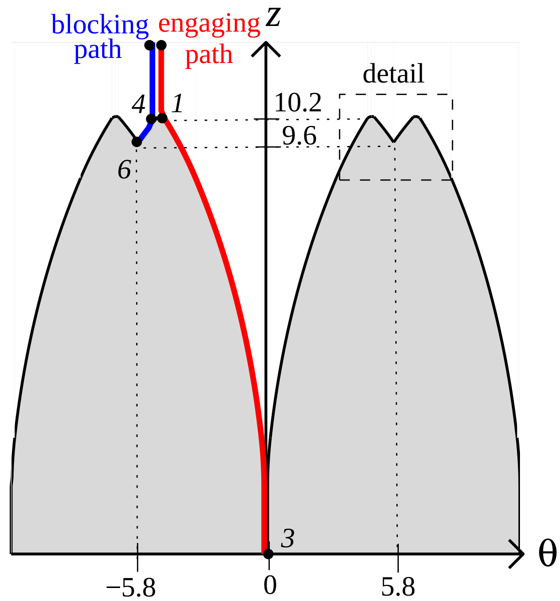


Figure 4. IDLER/REVERSE GEAR CONFIGURATION SPACE CROSS SECTION AT $\psi = 0^\circ$.

the pair. Configuration space is a complete geometric encoding of kinematics that describes contacts, contact changes, and contact sequences. It encodes quantitative information, such as contact constraints and contact change configurations, and qualitative information, such as failure modes.

The configuration space of the idler/reverse gear pair is a three-dimensional manifold whose coordinates are the part degrees of freedom: the idler rotation $\theta$ and its translation $z$, and the reverse gear rotation $\psi$. The configuration space partitions into blocked space where the parts overlap, free space where they do not touch, and contact space where they touch. Free and blocked space are open sets, and contact space is their common boundary. Contact space is a closed, two-dimensional set that partitions into contact patches that represent contacts between pairs of part features. The contact space geometry encodes the operating modes of the pair, such as engagement and blocking.

We have developed a configuration space construction algorithm for spatial pairs with two degrees of freedom (Drori et al, 1999; Kim et al., 2002). It handles part shapes comprised of spherical, cylindrical, and planar patches with line and circle segment boundaries. We use this program to analyze the pair by constructing two-dimensional cross sections along the $\psi$ axis. (We approximate involutes *A* and *B* with planar patches and model the other functional features exactly. The approximation is immaterial to our analysis because blocking does not involve the involutes.) Figure 4 shows the $\psi = 0^\circ$ cross section. The gray area is blocked space, the white area is free space, and the black curves in between are contact space. The contact space consists

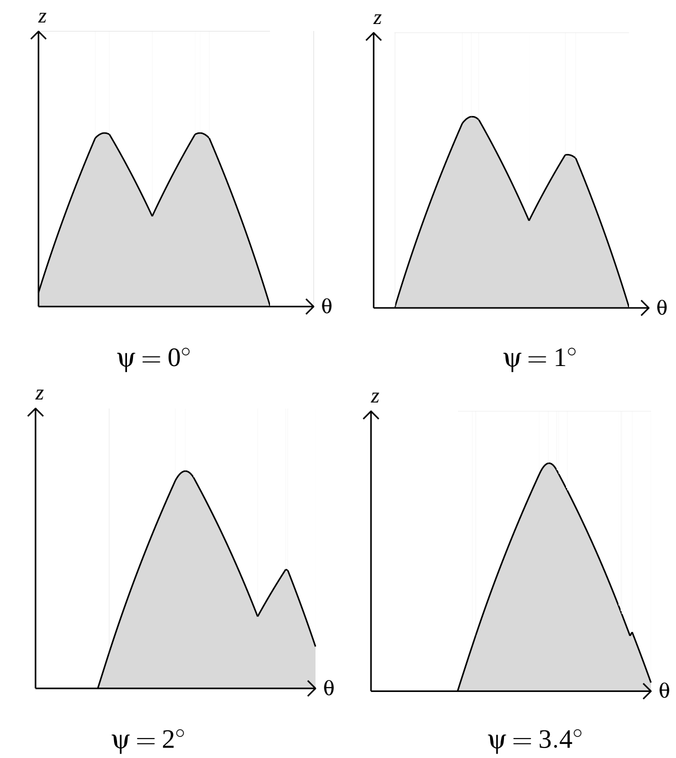

Figure 5. CROSS SECTION DETAILS.

of 31 identical hills, one per idler tooth, two of which are shown. Each hill has two peaks separated by a valley.

The contact sequences of the pair are represented by those paths in the three-dimensional configuration space that never enter blocked space. The driving motion is idler translation in the negative $z$ direction. Each initial configuration leads to a different path. Initially, the idler is away from the reverse gear ($z = 15$mm) and the gear orientations $\theta$ and $\psi$ are arbitrary. The idler translates ($z$ decreases) until contact and then the gears rotate together. The idler engages if it reaches $z = 0$mm and blocks if it stops at a positive $z$ value. Figure 4 shows the projected engaging and blocking paths from Figure 2, including the starting configurations. The configuration numbers correspond to the contact numbers in Figure 2.

Figure 4 suggests that configuration 6 is a blocking configuration. The configuration space shows that $z$ cannot decrease at $\psi = 0°$, hence the reverse gear must rotate for the idler to advance. Since the reverse gear is not driven, the idler must rotate it. This is impossible when the partial derivative of $\psi$ with respect to $z$ is zero on the valley curve at configuration 6. We find that this condition holds within a tolerance by constructing nearby cross sections and comparing the $z$ values at the valley point ($\psi = 0°$ and $\psi = 1°$ in Figure 5).

We use the cross sections to quantify the $\psi$ and $\theta$ values for which blocking occurs. Blocking occurs when $\psi$ produces a valley and $\theta$ lies between its peaks. Due to symmetry, we can perform the calculation for one tooth per gear. This yields $1/31$ of the $\theta$ range, which equals $11.6°$, and 1/13 of the $\psi$ range, which equals $27.7°$. Figure 5 shows that the valley persists slightly beyond $\psi = 3.4°$. It vanishes at $\psi = 3.5°$, and begins at $-3.5°$ by symmetry, so the $\psi$ range is $7°$. The average $\theta$ range is $1.8°$. This yields 4% blocking initial configurations, which is consistent with the experimental data.

## ALGORITHM

This section provides a brief description of our configuration space construction algorithm for spatial pairs (Kim et al., 2002). Readers who are uninterested in the algorithm can skip to the next section. The inputs are the part shapes, motion axes, and motion types (rotation or translation). The output is a partition of free space into connected components whose boundaries are contact curves.

The algorithm consists of three steps. Step 1 constructs contact curves for all pairs of part features (Figure 6a). The contact curves for a pair of features are the configurations in which the features would touch if there were no other features to interfere. Step 2 computes the partition of the configuration space into connected components induced by the contact curves (Figure 6b). The partition is computed via a line sweep and is encoded in the standard winged-edge representation of computational geometry. The elements of the partition are connected components of configuration space bounded by loops of contact curves. Each component lies wholly in free space or wholly in blocked space. Step 3 classifies the components and returns the free space ones. A component is blocked if the parts overlap at a single, arbitrarily chosen interior configuration.

Contact curve construction is the hardest part of configuration space construction. A separate analysis is required for every combination of feature and motion types. There are 72 cases, for example rotating plane/translating cylinder. We analyzed the cases and compiled the results into step 1 of the algorithm. We illustrate the analysis on the contact between the idler and reverse gear sphere $D$ patches.

The idler sphere has center $m$ in the part frame, has radius $r$, and moves along the $z$ axis. The global coordinates of $m$ are $[0,0,z] + R_\theta m$ where $R_\theta$ denotes rotation by $\theta$ around the $z$ axis. The reverse gear sphere has center $n$ in the world frame, has radius $s$, and does not move. Contact occurs when the distance between the centers equals the sum of the radii, that is when $([0,0,z] + R_\theta m - n)^2 = (r+s)^2$ where the first squaring operation denotes the inner product of the parenthesized term with itself. This yields $z^2 + 2(m_z - n_z)z + d^2 - (r+s)^2 = 0$ with $d = R_\theta m - n$, which is a quadratic in $z$ for each $\theta$ value. We solve the quadratic at sample $\theta$ values and link the results into a piecewise linear approximation of the contact curve. The sample spacing is selected

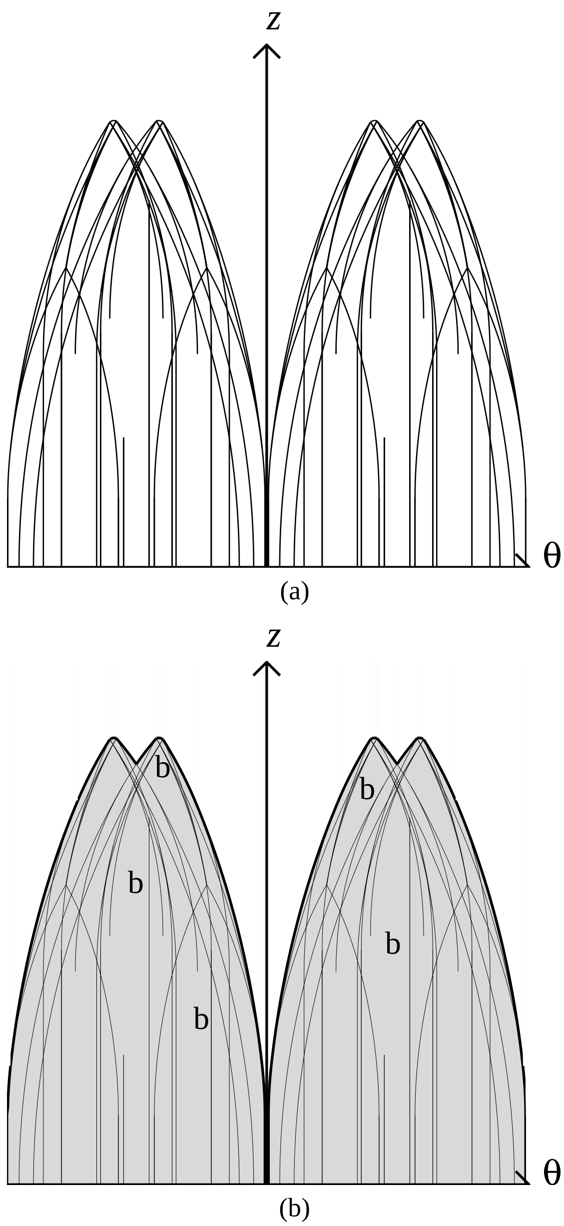

Figure 6. CONFIGURATION SPACE CONSTRUCTION FOR REVERSE GEAR PAIR: (A) CONTACT CURVES; (B) PARTITION WITH "b" ON SELECTED BLOCKED COMPONENTS.

to ensure a specified accuracy.

The approximate curve represents contacts between the two spheres that underlie the $D$ patches. The contact curve is the subset of the approximate curve where the contact point lies on the $D$ patches, not just on the sphere. We test this condition for each curve point, discard the ones that fail, and link the rest into contact curve segments. The contact point at curve point $(\theta_0, z_0)$ is $n + \frac{s}{r+s}\left([0,0,z] + R_{\theta_0}m - n\right)$. Patch $D$ is formed by cutting the sphere with the plane $y = y_m$ where it meets the tooth top, the plane $y = 0$ where it meets patch $C$, and the plane that approximates involute $B$. The contact point lies on $D$ if it lies on the positive sides of these planes.

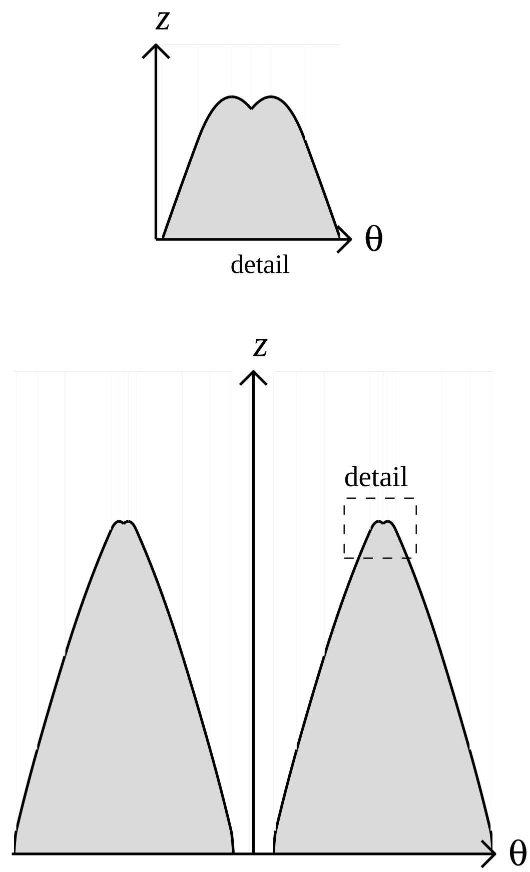

Figure 7. CONFIGURATION SPACE AFTER PART DIMENSION OPTIMIZATION.

## SOLUTION

We first changed the part dimensions to reduce blocking. We extensively sampled the space of part dimensions and analyzed the resulting designs with our configuration space method as described above. We reduced the blocking from 4% to 0.5%. The $\psi$ range is 1.6° and the average $\theta$ range is 1°. Figure 7 shows the configuration space at $\psi = 0°$. The peaks are closer together, but the valley persists. We then found that we can eliminate blocking by removing every second idler guiding chamfer. Figure 8 shows the $\psi = 0°$ configuration space slice for the modified design with the original part dimensions. One peak has disappeared, so there is no valley and thus no blocking. This analysis of the two redesigns is consistent with experimental data.

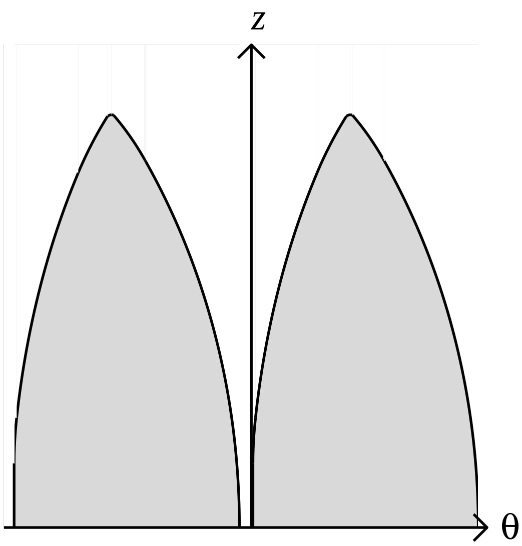


Figure 8. CONFIGURATION SPACE AFTER ALTERNATE CHAMFER REMOVAL.

## CONCLUSION

We have presented an industrial case study of spatial higher pair design using our configuration space method. We analyzed an asynchronous reverse gear pair to understand and correct infrequent blocking. A systematic kinematic analysis is required because the blocking configurations are unknown and because very few initial configurations cause blocking. Simulation and random sampling would require a prohibitive number of samples to find the blocking configurations. The configuration space method provides a global description of kinematic function. The blocking configurations appear explicitly and can be quantified. The method allows the designer to evaluate many design alternatives quickly and comprehensively. We greatly reduced blocking by changing part dimensions then eliminated it with a nonparametric design modification. The case study demonstrates that our method helps solve industrial problems that are outside the scope of prior work.

We see several extensions to the configuration space method. One task is to construct three-dimensional configuration spaces for spatial pairs with parallel motion axes based on our algorithm for planar pairs with three degrees of freedom (Sacks and Bajaj, 1997; Sacks, 1998). Another task is to perform kinematic tolerance analysis to compute the contact space variation for given part tolerances based on our algorithm for planar pairs (Sacks and Joskowicz, 1997; Sacks and Joskowicz, 1998). A third task is to facilitate synthesis by extending our parameter space search algorithm (Kyung and Sacks, 2001) from planar to spatial pairs.

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