



# Light Transport

CS434

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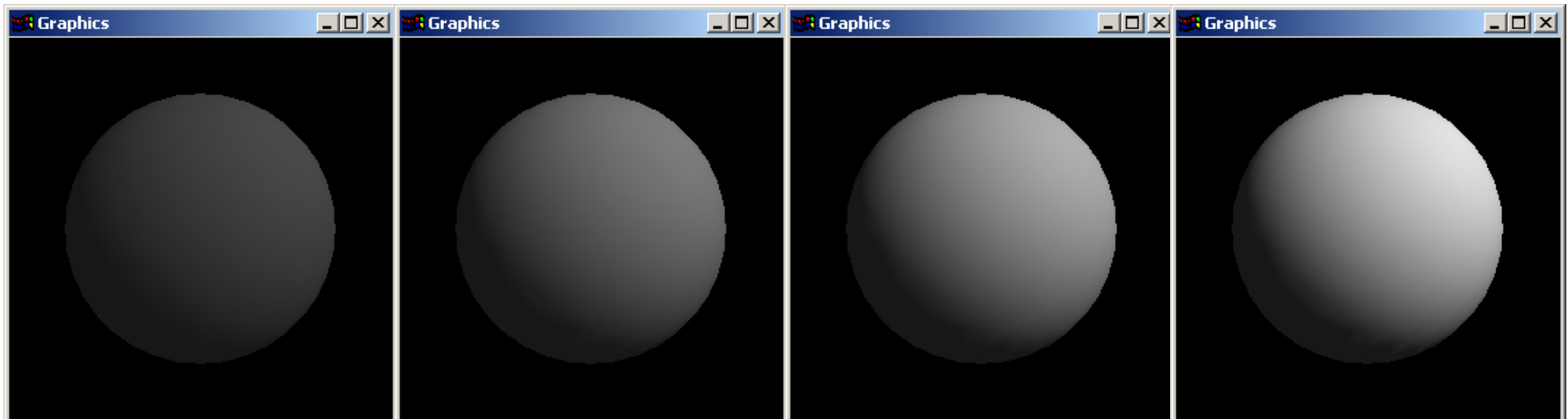
# Topics

- Local and Global Illumination Models
- Helmholtz Reciprocity
- Dual Photography/Light Transport in Real-World



# Diffuse Lighting

- A.k.a. Lambertian illumination
- A fraction of light is radiated in every direction
- Intensity varies with cosine of the angle with normal



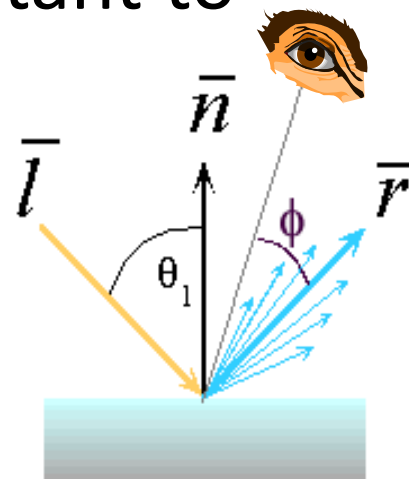


# Specular Lighting

- The most common lighting model was suggested by Phong

$$I_{spec} = \rho_{spec} I_{Light} (\cos \phi)^{n_{shiny}}$$

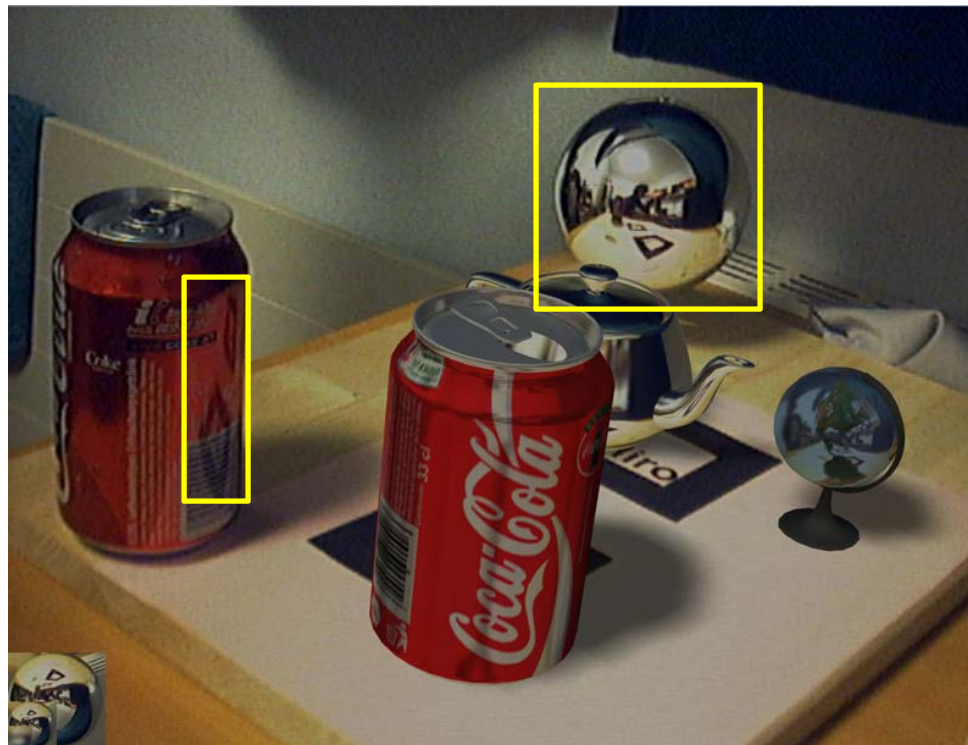
- The  $n_{shiny}$  term is an empirical constant to model the rate of falloff
- The model has no physical basis, but it sort of works



# Example



# Inter-reflections





# Scattering





# Scattering



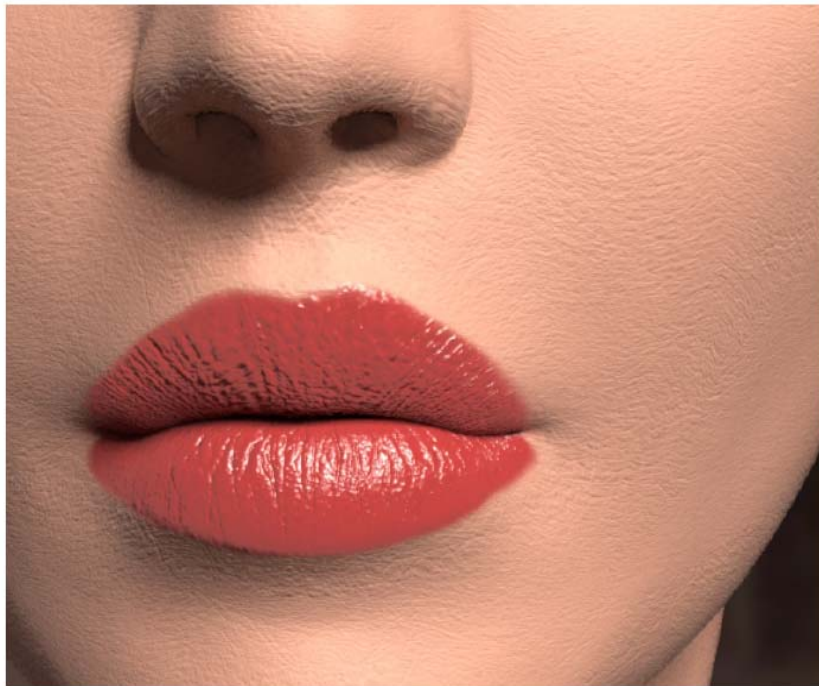
Without (subsurface) scattering



With (subsurface) scattering



# Scattering



BRDF

Without (subsurface) scattering

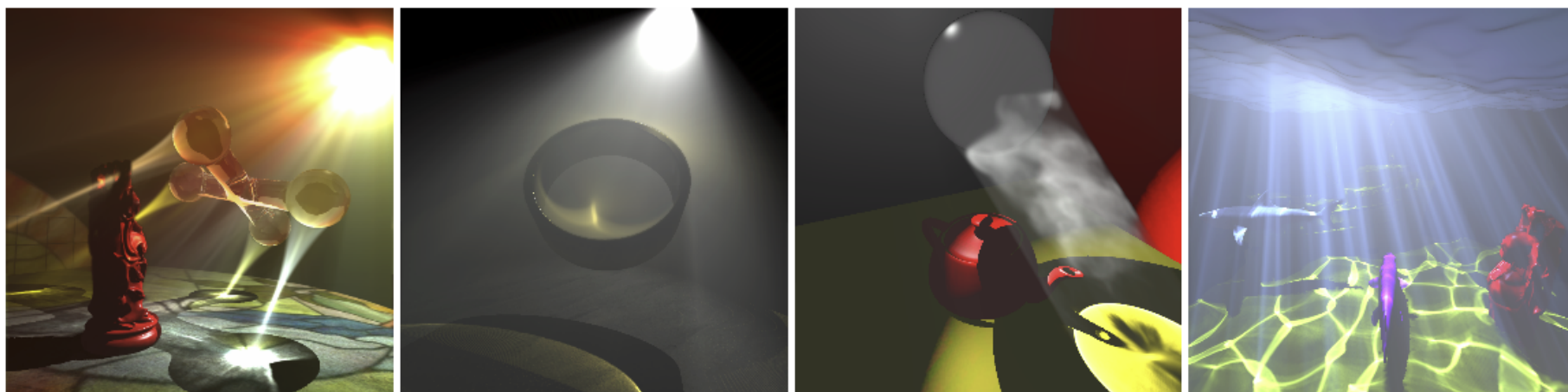


BSSRDF

With (subsurface) scattering



# Scattering



Hu et al. 2010

Scattering through participating media with volume caustics...



# Rendering Equation

(also known as the light-transport equation)

- Illumination can be generalized to

$$I(x, x') = g(x, x') \left[ \varepsilon(x, x') + \int_s \rho(x, x', x'') I(x', x'') dx'' \right]$$

$I$ : illumination at first point from second

$g$ : geometry term for visibility

$\varepsilon$ : emitted light from second point to first

$\rho$ : reflectivity of light from  $x''$  to  $x$  via  $x'$

(note: equation is recursive)

**...but it does not model all illumination effects**



# Conclusion

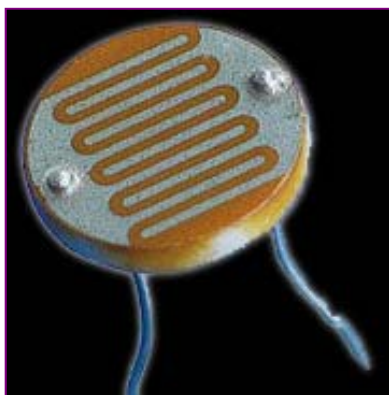
- Modeling illumination is hard
- “Undoing” physically-observed illumination in order to discover the underlying geometry is even harder
- Insight: let’s sample it and “re-apply” it!



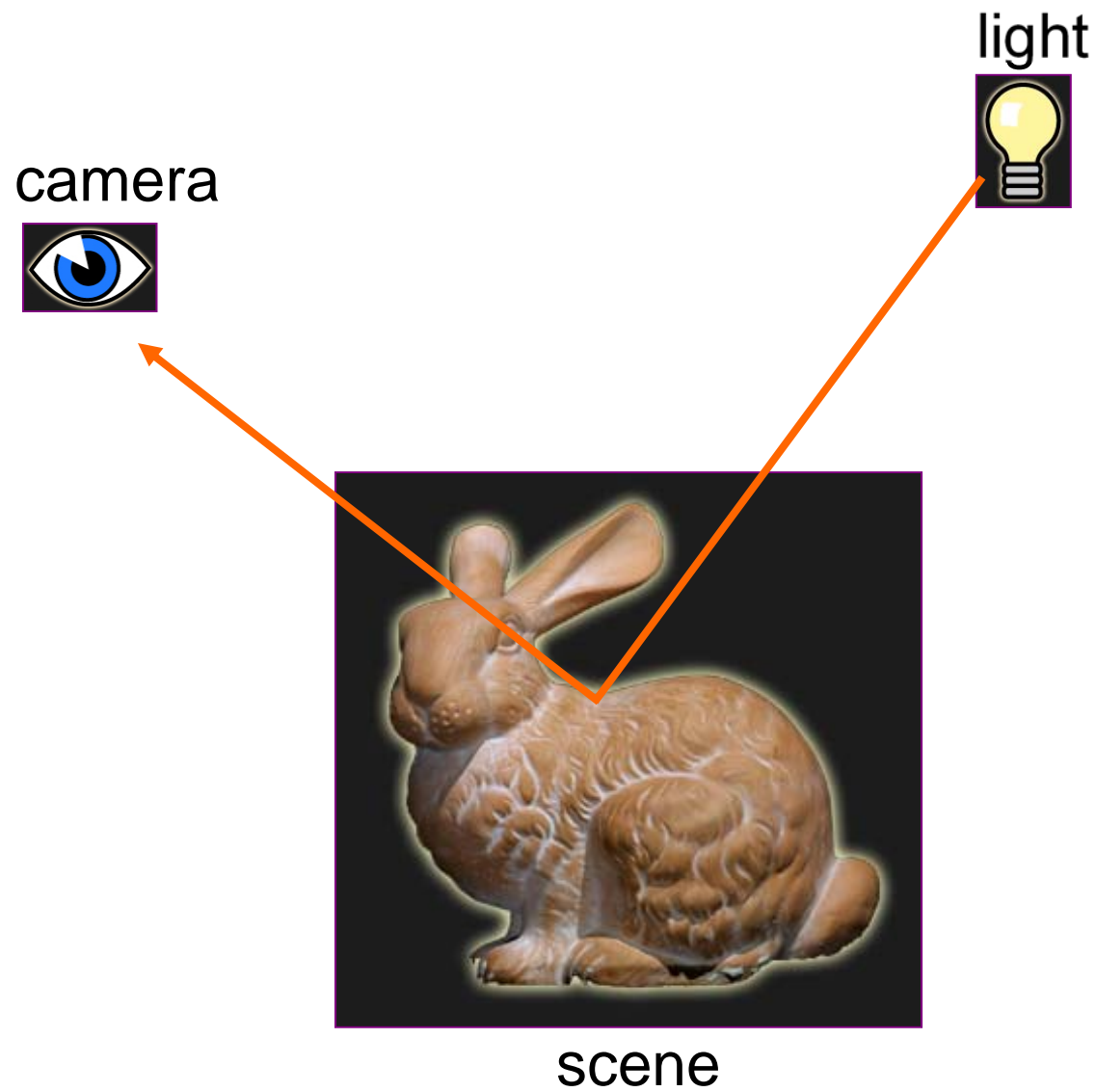
# Dual Photography

Sen et al., SIGGRAPH 2005

(slides courtesy of M. Levoy)

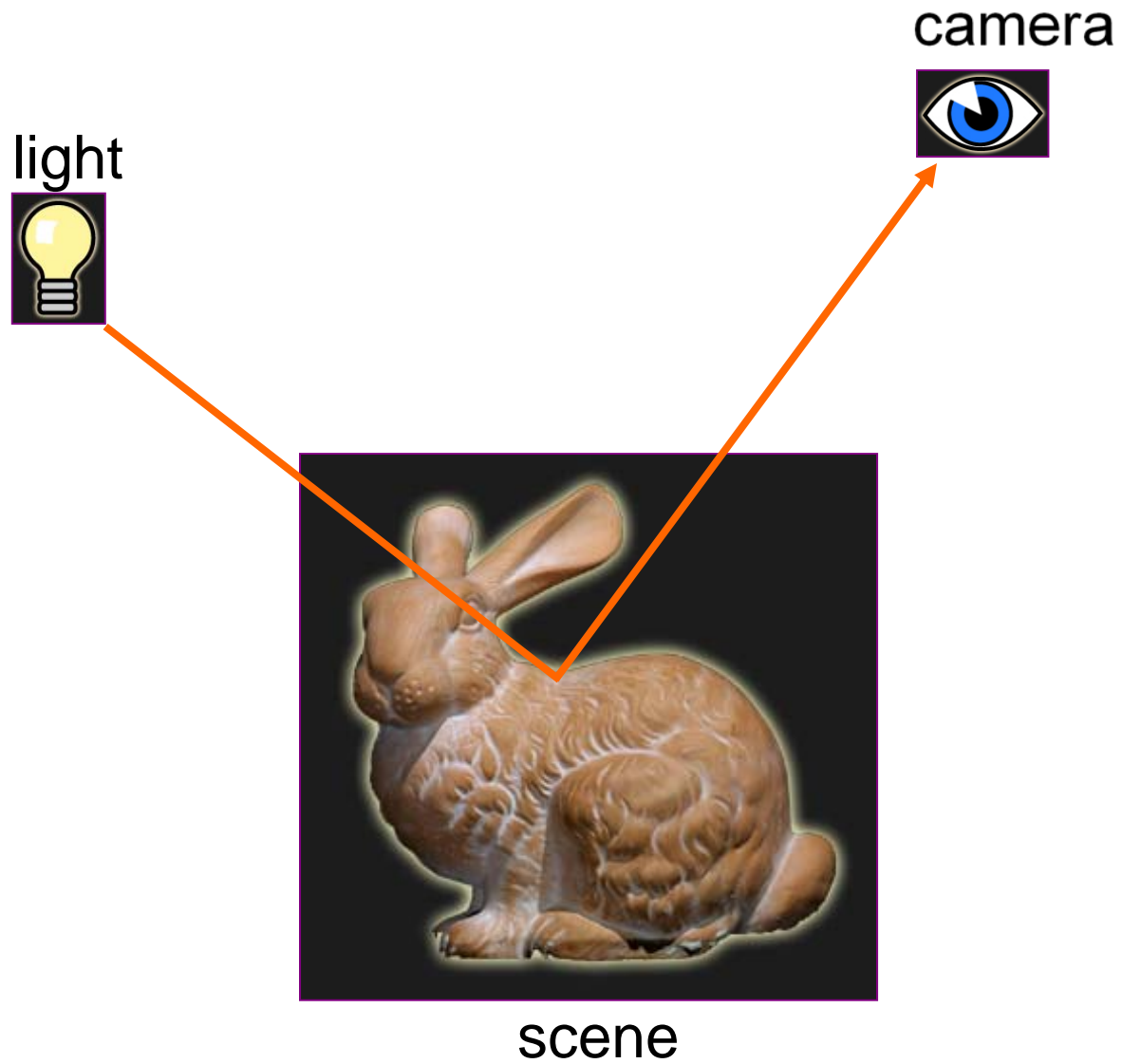


# Helmholtz Reciprocity





# Helmholtz Reciprocity



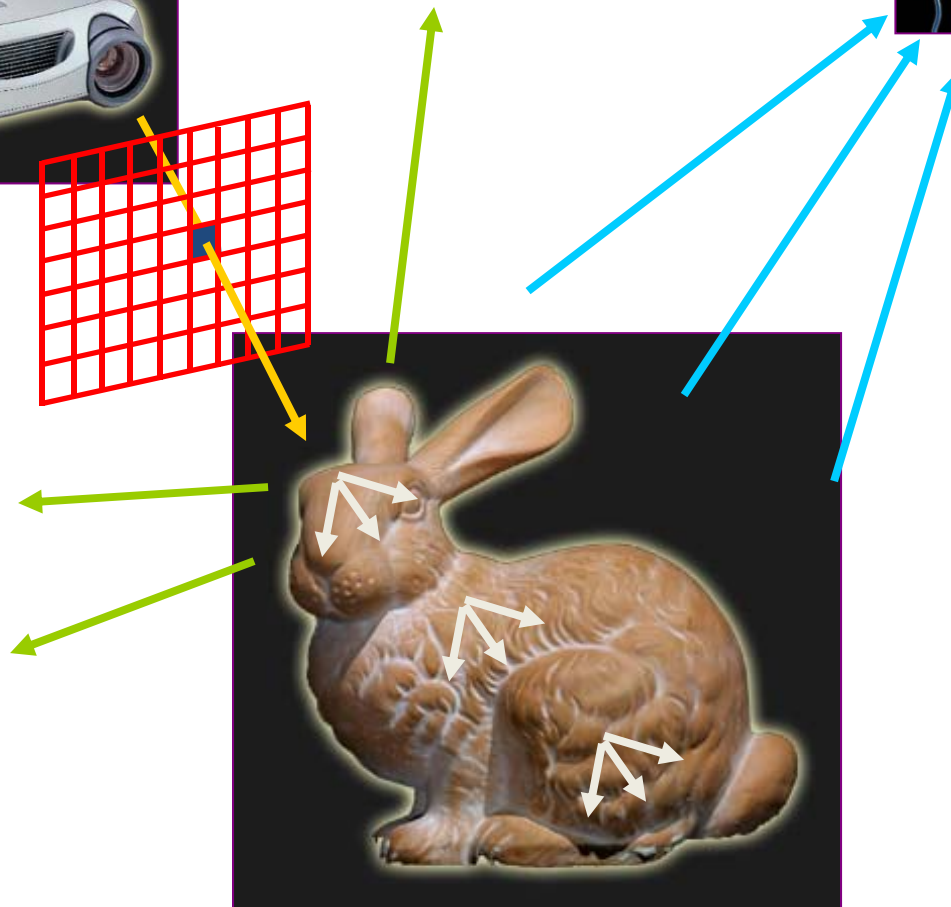
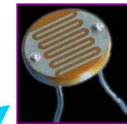
# Measuring transport along a set of paths



projector



photocell



scene

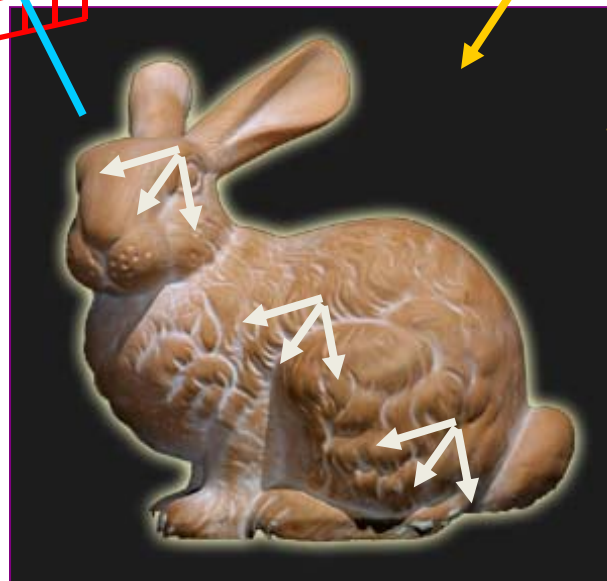
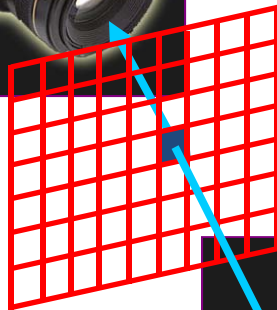
# Reversing the paths



camera



point light

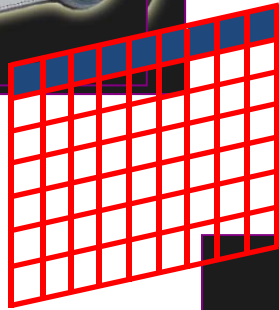


scene

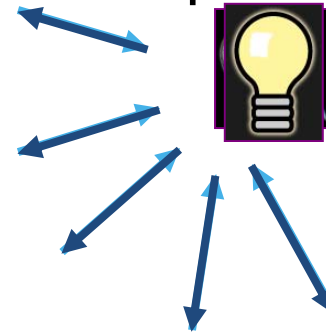
# Forming a dual photograph



“dual” camera  
projector



“dual” light  
protocol



scene



# Forming a dual photograph



“dual” camera

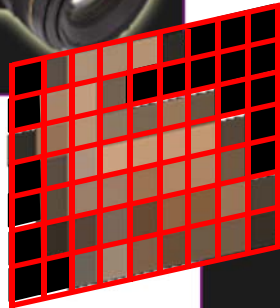
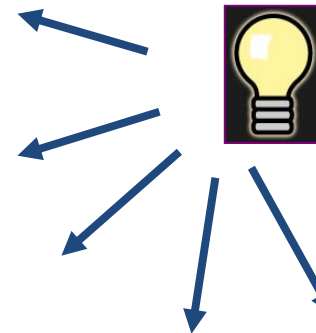


image of  
scene



scene

“dual” light



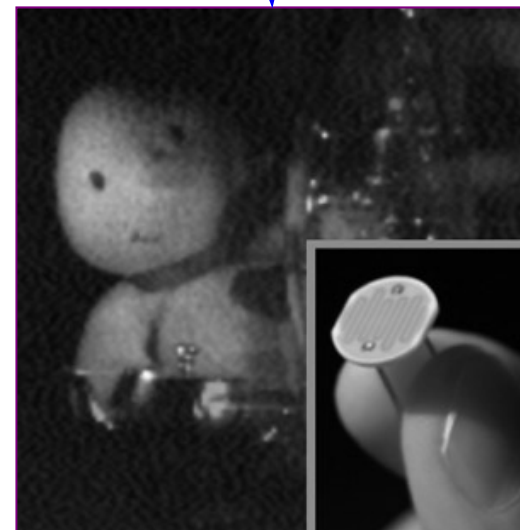


# Physical demonstration

- light replaced with projector
- camera replaced with photocell
- projector scanned across the scene



conventional photograph,  
with light coming from right



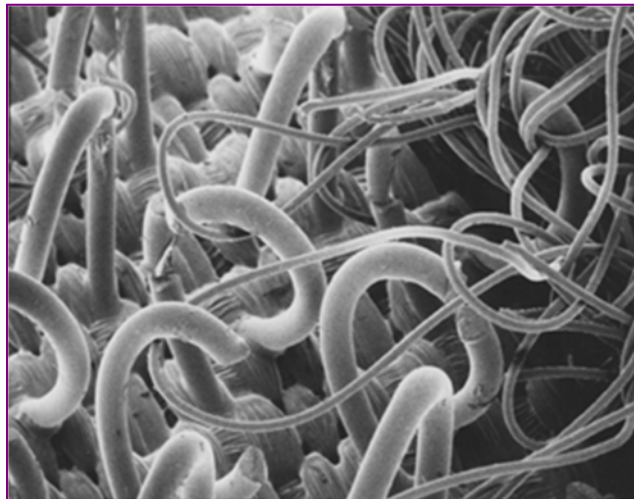
dual photograph,  
as seen from projector's position  
and as illuminated from photocell's position





# Related imaging methods

- time-of-flight scanner
  - if they return reflectance as well as range
  - but their light source and sensor are typically coaxial
- scanning electron microscope



Velcro® at 35x magnification,  
Museum of Science, Boston

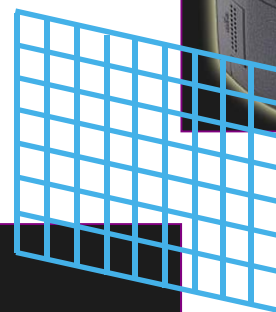
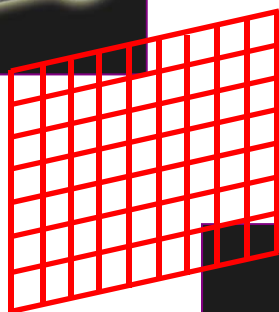
# The 4D transport matrix



projector

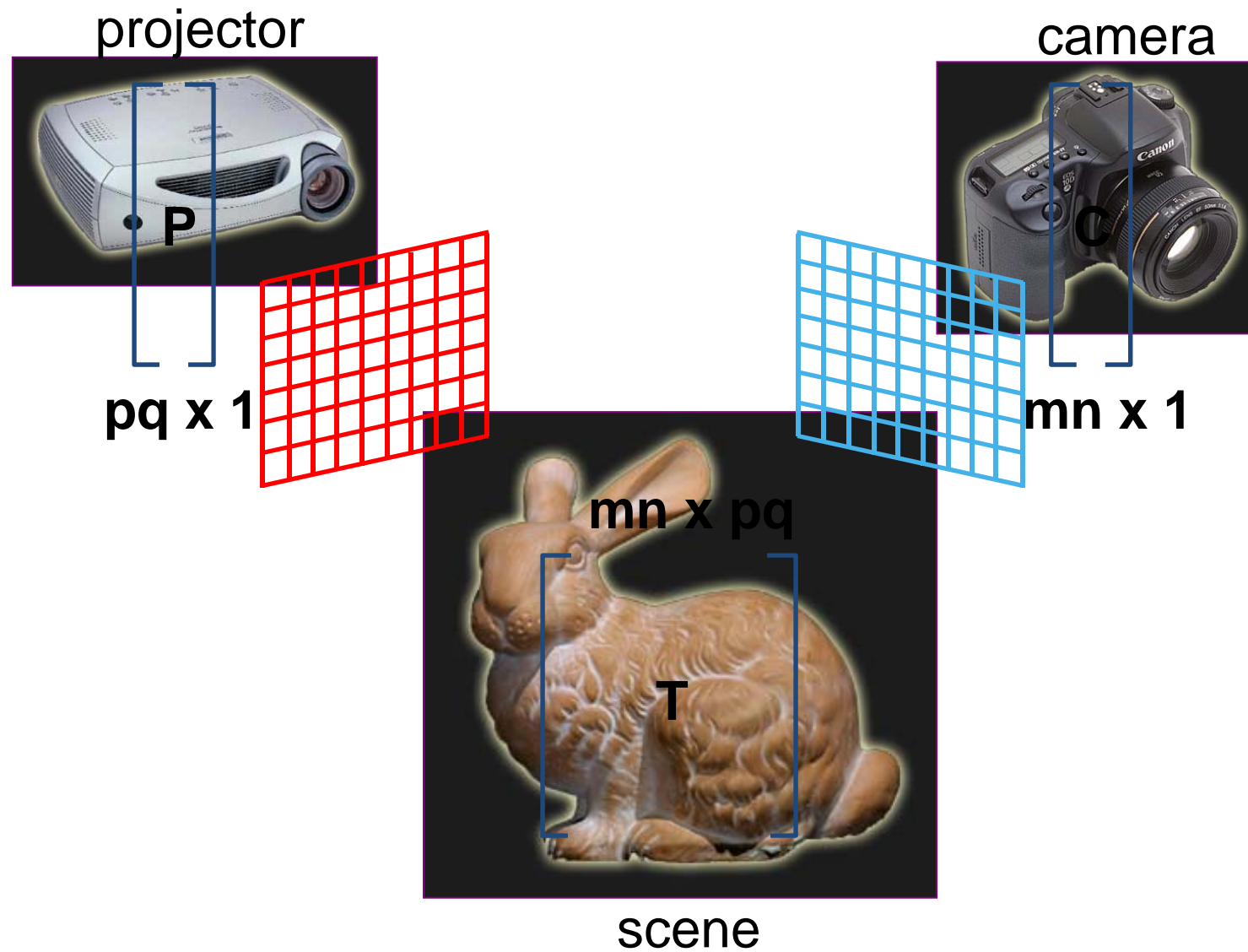


photocall



scene

# The 4D transport matrix



# The 4D transport matrix



$$\begin{matrix} \left[ \begin{array}{c} \mathbf{C} \end{array} \right] \\ mn \times 1 \end{matrix} = \begin{matrix} mn \times pq \\ \left[ \begin{array}{c} \mathbf{T} \end{array} \right] \end{matrix} \begin{matrix} \left[ \begin{array}{c} \mathbf{P} \end{array} \right] \\ pq \times 1 \end{matrix}$$

# The 4D transport matrix



$$\begin{array}{c} \left[ \begin{array}{c} C \end{array} \right] \\ mn \times 1 \end{array} = \begin{array}{c} mn \times pq \\ \left[ \begin{array}{c} \text{orange bar} \end{array} \right] \end{array} \begin{array}{c} T \\ \left[ \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right] \\ pq \times 1 \end{array}$$

# The 4D transport matrix



$$\begin{array}{c} \left[ \begin{array}{c} C \end{array} \right] \\ mn \times 1 \end{array} = \begin{array}{c} mn \times pq \\ \left[ \begin{array}{c} \text{yellow bar} \quad \text{orange bar} \end{array} \right] T \end{array} \begin{array}{c} \left[ \begin{array}{c} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right] \\ pq \times 1 \end{array}$$



# The 4D transport matrix



$$\begin{array}{c} \left[ \begin{array}{c} C \end{array} \right] \\ mn \times 1 \end{array} = \begin{array}{c} mn \times pq \\ \left[ \begin{array}{c} \text{3 vertical bars (2 yellow, 1 orange)} \\ T \end{array} \right] \end{array} \begin{array}{c} \left[ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{array} \right] \\ pq \times 1 \end{array}$$

# The 4D transport matrix



$$\begin{array}{c} \left[ \begin{array}{c} \mathbf{C} \end{array} \right] \\ mn \times 1 \end{array} = \begin{array}{c} mn \times pq \\ \left[ \begin{array}{c} \mathbf{T} \end{array} \right] \end{array} \begin{array}{c} \left[ \begin{array}{c} \mathbf{P} \end{array} \right] \\ pq \times 1 \end{array}$$

# The 4D transport matrix



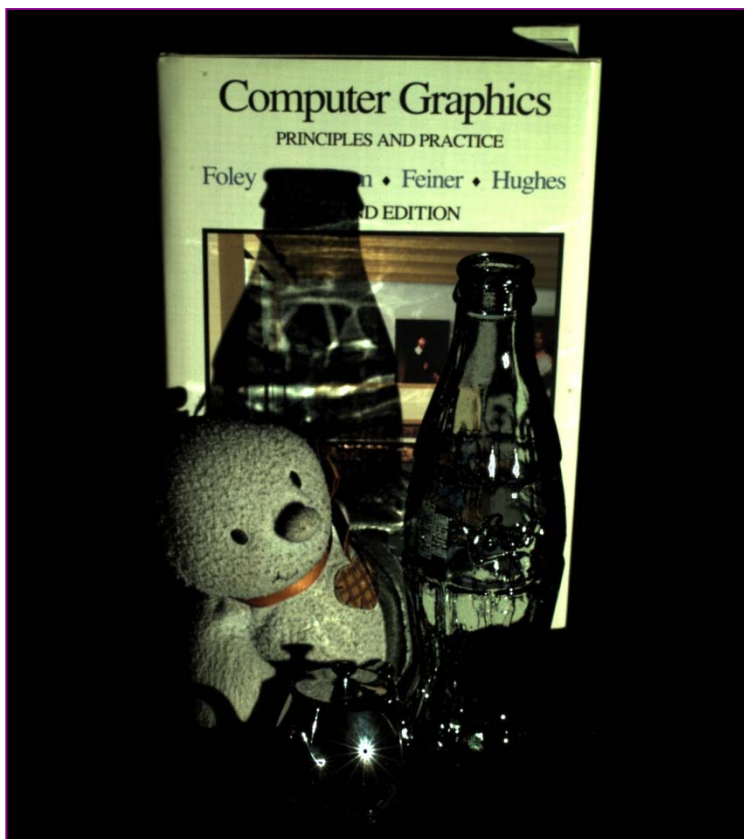
$$\begin{array}{c} \left[ \begin{array}{c} \mathbf{C} \end{array} \right] \\ mn \times 1 \end{array} = \begin{array}{c} mn \times pq \\ \left[ \begin{array}{c} \mathbf{T} \end{array} \right] \end{array} \begin{array}{c} \left[ \begin{array}{c} \mathbf{P} \end{array} \right] \\ pq \times 1 \end{array}$$

applying Helmholtz reciprocity...

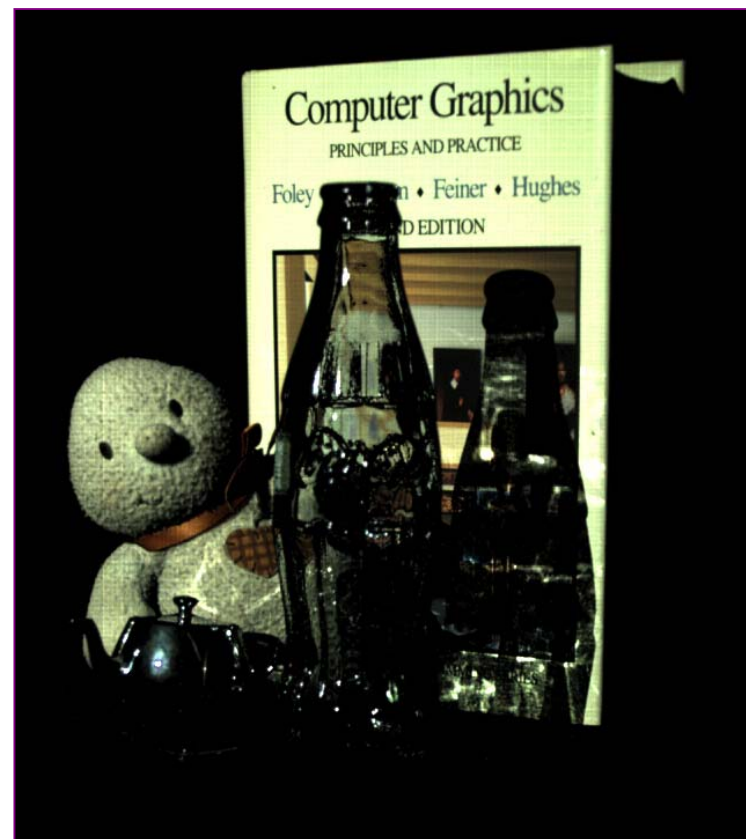
$$\begin{array}{c} \left[ \begin{array}{c} \mathbf{C}' \end{array} \right] \\ pq \times 1 \end{array} = \begin{array}{c} pq \times mn \\ \left[ \begin{array}{c} \mathbf{T}^T \end{array} \right] \end{array} \begin{array}{c} \left[ \begin{array}{c} \mathbf{P}' \end{array} \right] \\ mn \times 1 \end{array}$$



# Example



conventional photograph  
with light coming from right

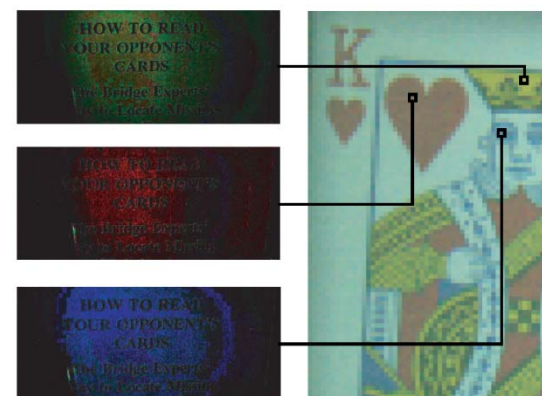
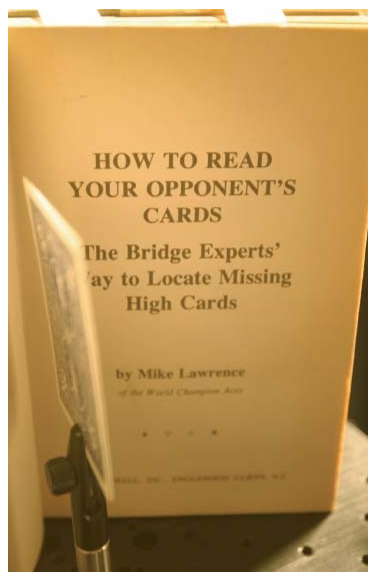
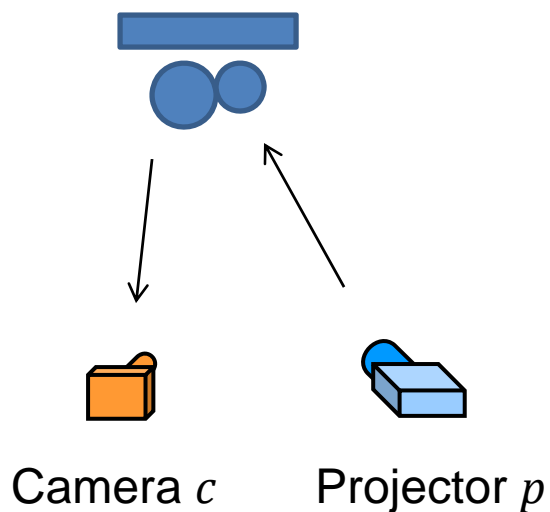


dual photograph  
as seen from projector's position



# Example

- Can encode light (or projector) to camera “transport” in a large matrix  $T$

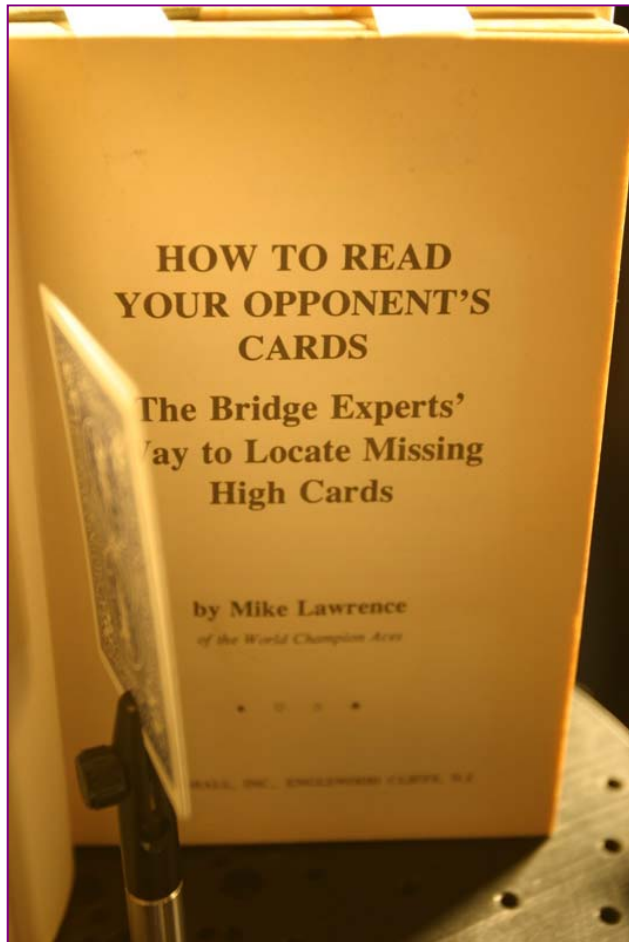


$$\begin{bmatrix} c \end{bmatrix} = \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} p \end{bmatrix}$$

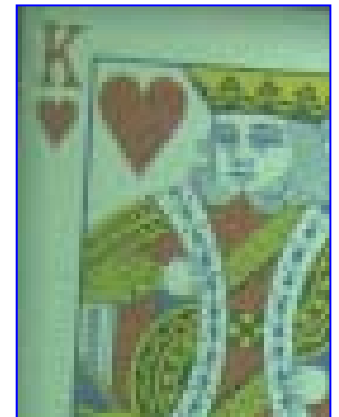
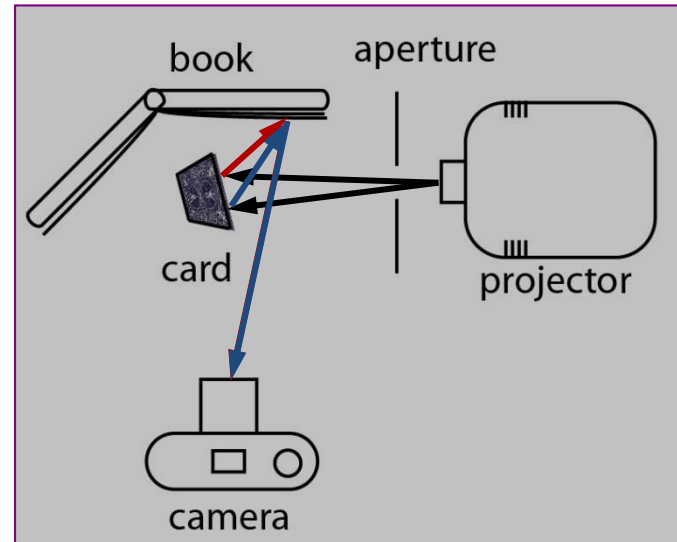
$$\begin{bmatrix} p \end{bmatrix} = \begin{bmatrix} T^t \end{bmatrix} \begin{bmatrix} c \end{bmatrix}$$

As seen from camera...      As seen from projector!!!

# Dual photography from diffuse reflections



the camera's view







# Properties of the transport matrix

- little inter-reflection  
→ sparse matrix
- many inter-reflections  
→ dense matrix
- convex object  
→ diagonal matrix
- concave object  
→ full matrix

Can we create a dual photograph entirely from diffuse reflections?



# Relighting



Paul Debevec's  
Light Stage 3

- subject captured under multiple lights
- one light at a time, so subject must hold still
- point lights are used, so can't relight with cast shadows



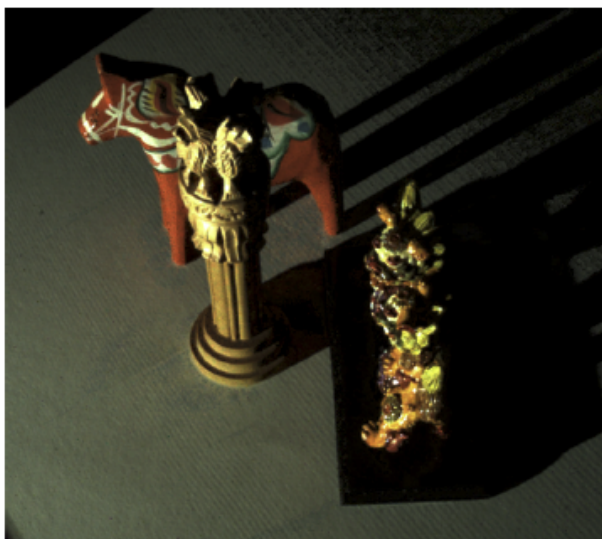
# Relighting



With Dual Photography...



# Relighting



With Dual  
Photography...





# Relighting



(a)



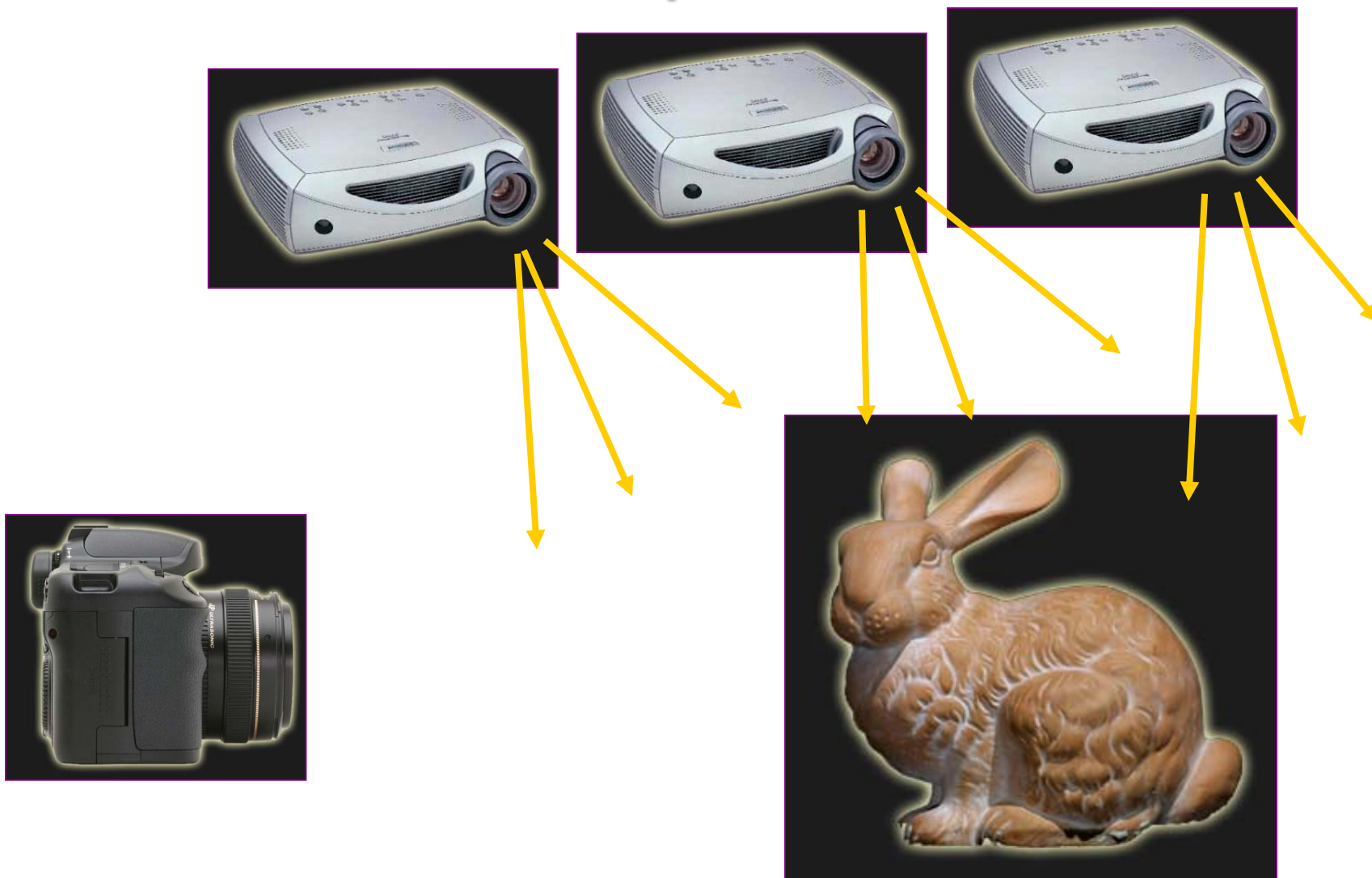
(b)



With Dual  
Photography...

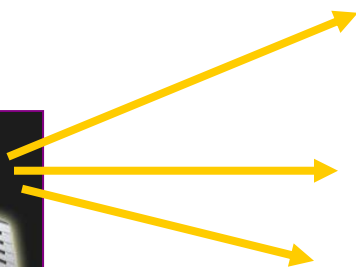


# The 6D transport matrix





# The 6D transport matrix





# The advantage of dual photography



- capture of a scene as illuminated by different lights cannot be parallelized
- capture of a scene as viewed by different cameras can be parallelized

# Measuring the 6D transport matrix



projector



camera array



scene



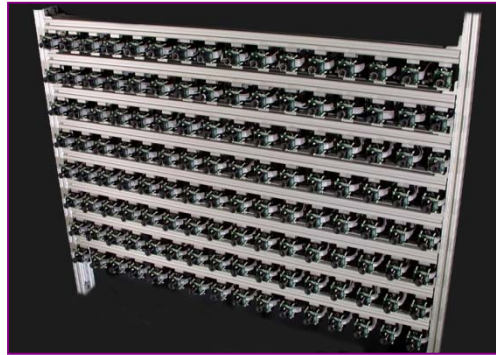
# Relighting with complex illumination



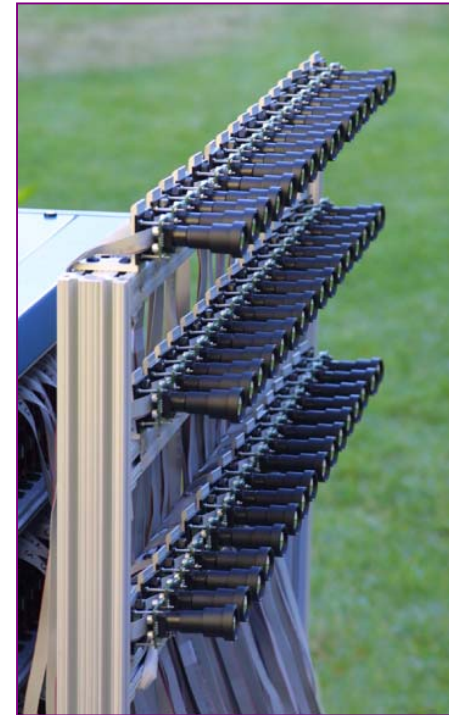
projector



camera array



scene



$$\begin{matrix} & & pq \times mn \times uv \\ \left[ \begin{matrix} C' \end{matrix} \right] & = & \left[ \begin{matrix} T^T \end{matrix} \right] \left[ \begin{matrix} P' \end{matrix} \right] \\ pq \times 1 & & mn \times uv \times 1 \end{matrix}$$

- step 1: measure 6D transport matrix  $T$
- step 2: capture a 4D light field
- step 3: relight scene using captured light field

# Running time

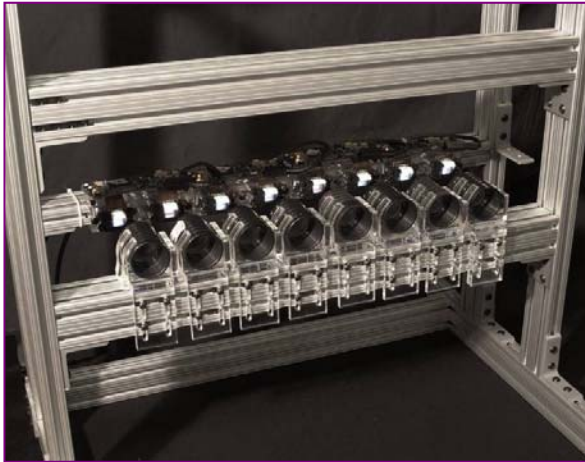


- the different rays within a projector can in fact be parallelized to some extent
- this parallelism can be discovered using a coarse-to-fine adaptive scan
- can measure a 6D transport matrix in 5 minutes

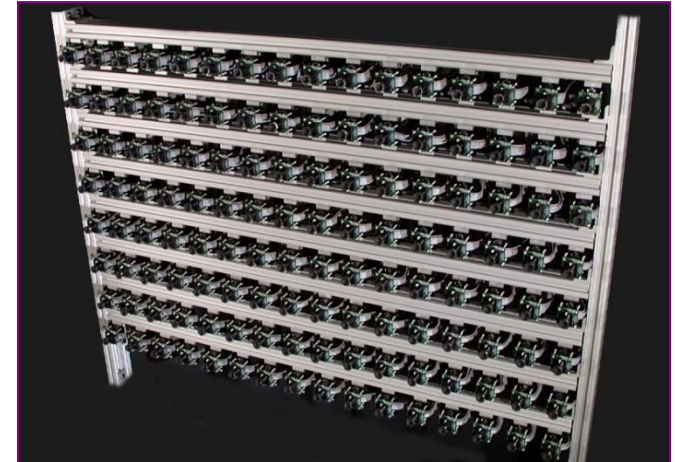
# Can we measure an 8D transport matrix?



projector array



camera array



scene