



# Collision Detection

CS434

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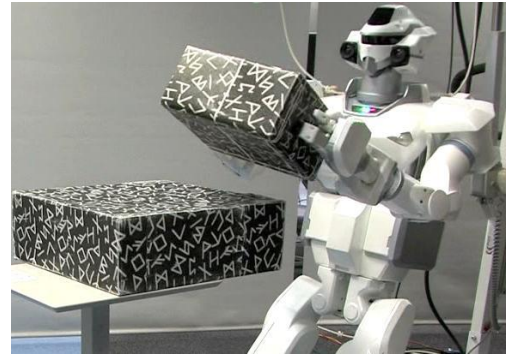


# Some Applications

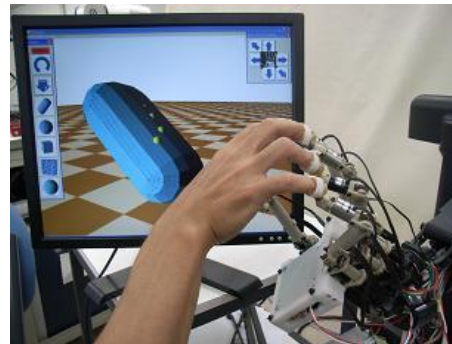
- Animations/games



- Robotic planning



- Haptics



# Some collision detection approaches...



- Minkowski Sum 2D collisions
  - <http://www.slideshare.net/crowdscontrol/minkowski-sum-on-2d-geometry>
- <http://gamma.cs.unc.edu/research/collision/>
- “Efficient Collision Detection for Animation and Robotics”, Lin’s PhD thesis, 1993
  - Slides based on those of Adit Koolwal and Benson Limketkai

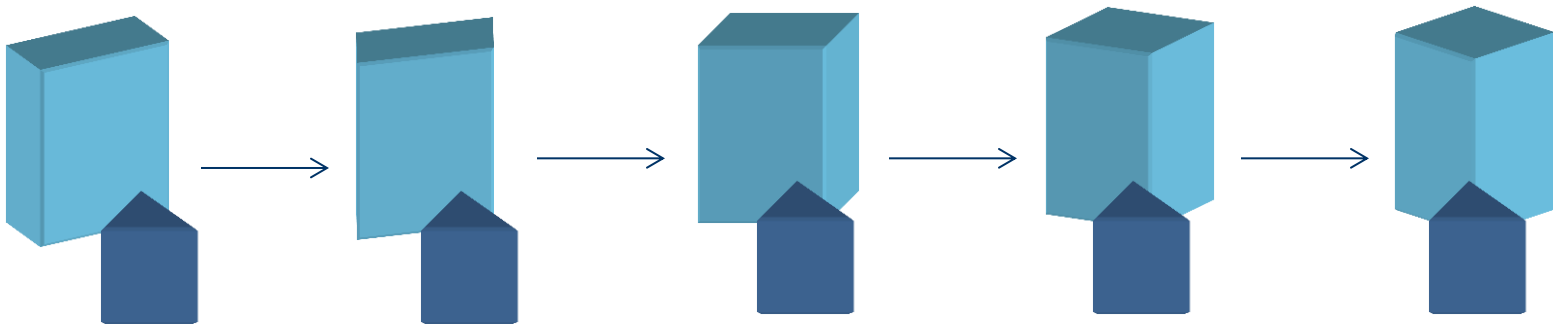
# an overview of the lin-canny algorithm



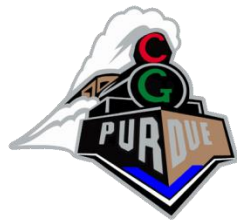
## The Lin-Canny algorithm:

After some preprocessing (discussed later), finds an initial pair of closest features between two polyhedra using an  $O(n^2)$  search.

At each timestep, checks if the current closest feature pair is still the closest. If not, it finds the new closest pair.

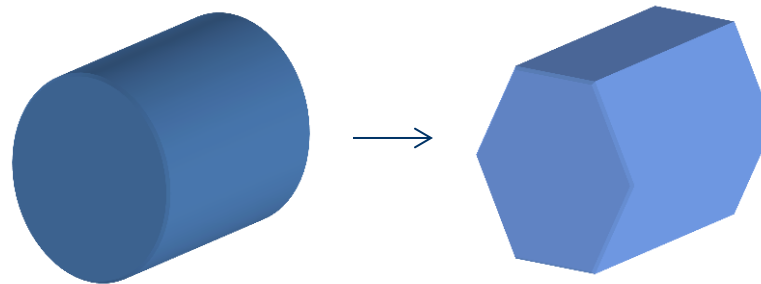


The Lin-Canny algorithm takes advantage of incremental motion because closest features change infrequently.

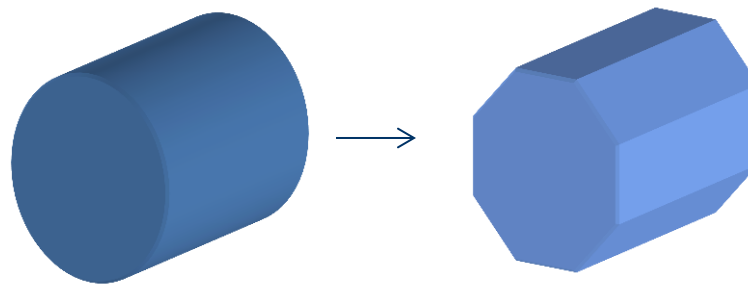


implementating lin-canny  
in 4 easy steps...

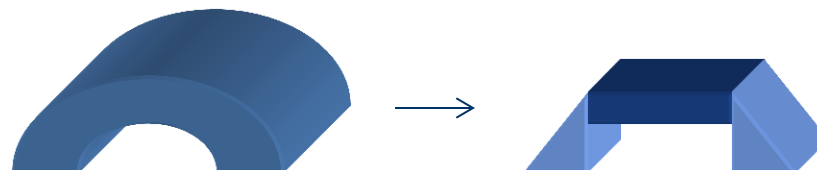
1. We first represent each object as a convex polyhedron, or a union of convex polyhedra.



We can improve the accuracy of the approximation by increasing the resolution of our representation.



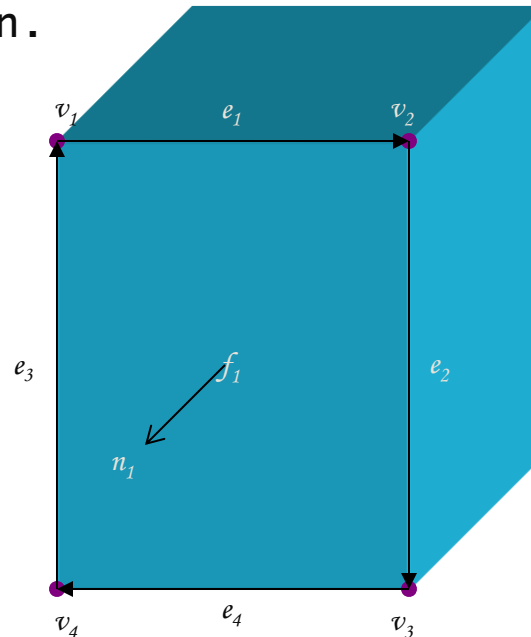
For non-convex objects we rely on subdivision into convex pieces, which can take up to quadratic time.





1. We first represent each object as a convex polyhedron or a union of convex polyhedra.
2. For each object we calculate the fields for each of its faces, edges, vertices, positions, and orientations.

A **FACE**  $f_i$  is parameterized by its outward normal and distance from the origin.



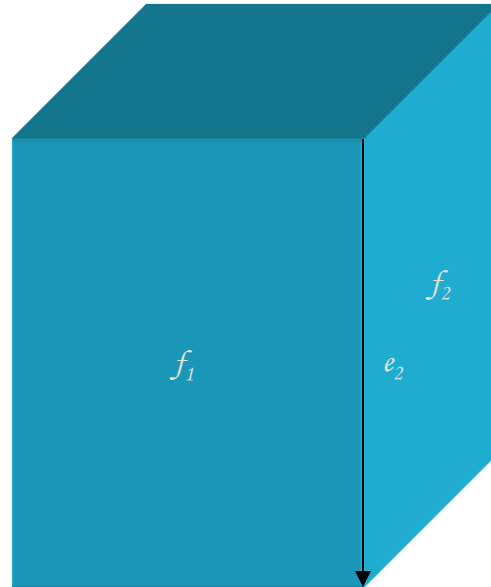
A **FACE** contains a list of **VERTICES**  $v_i$  which lie on its boundaries, and a list of **EDGES**  $e_i$  which bound the face.



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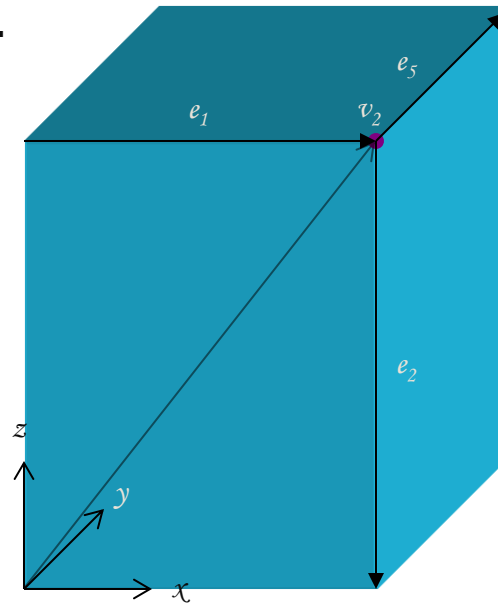




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A **FACE** contains a list of **VERTICES**  $v_i$  which lie on its boundaries, and a list of **EDGES**  $e_i$  which bound the face.

Each **VERTEX**  $v_i$  is described by its x,y,z coordinates and its **CO-BOUNDARY**, the set of edges that intersect it.

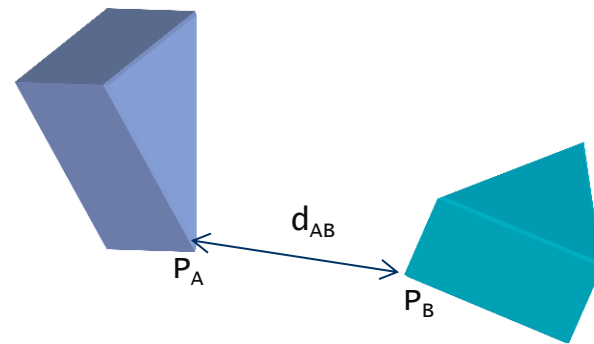


1. We first represent each object as a convex polyhedron or a union of convex polyhedra.
2. For each object we calculate the fields for each of its faces, edges, vertices, positions, and orientations.
3. For each pair of features between the two objects of interest, calculate the closest pair of points between those two features. Find the overall closest pair.

Define  $P_A$  to be the closest point of feature<sub>A</sub> to feature<sub>B</sub>

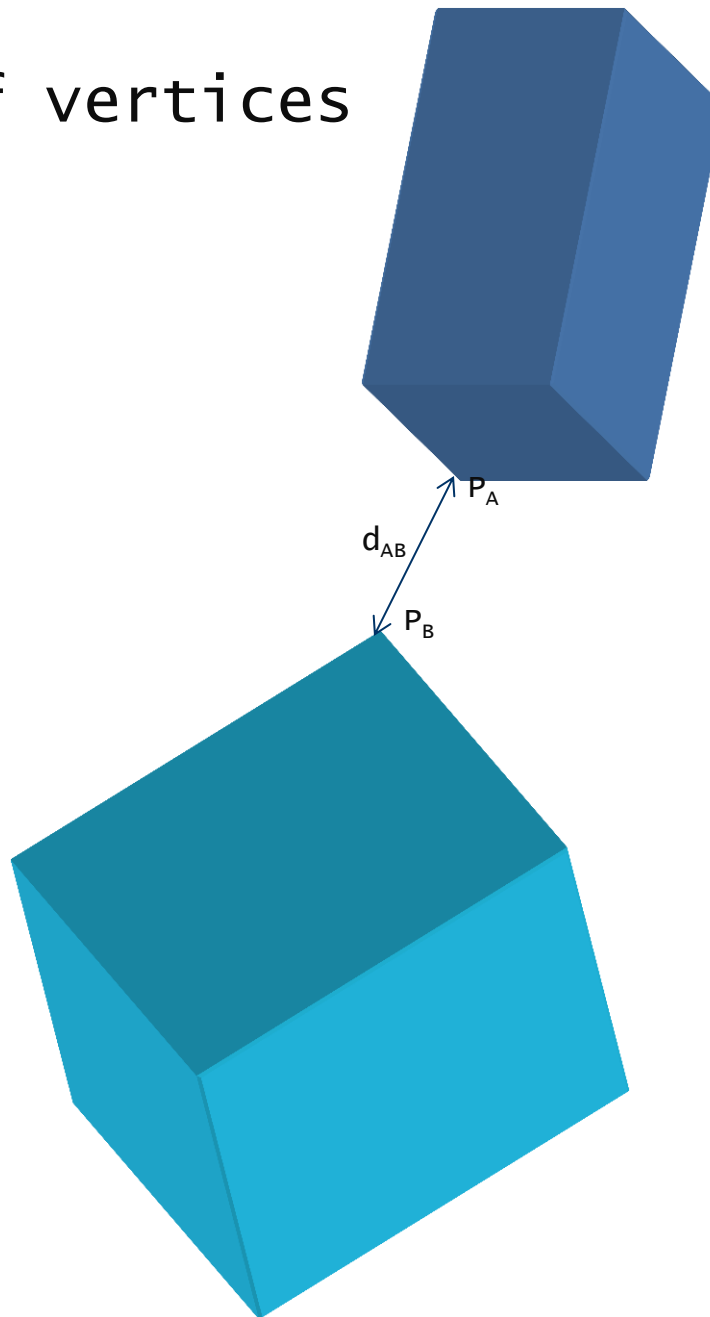
Define  $P_B$  to be the closest point of feature<sub>B</sub> to feature<sub>A</sub>

The distance  $d_{AB}$  is the Euclidean distance between  $P_A$  and  $P_B$

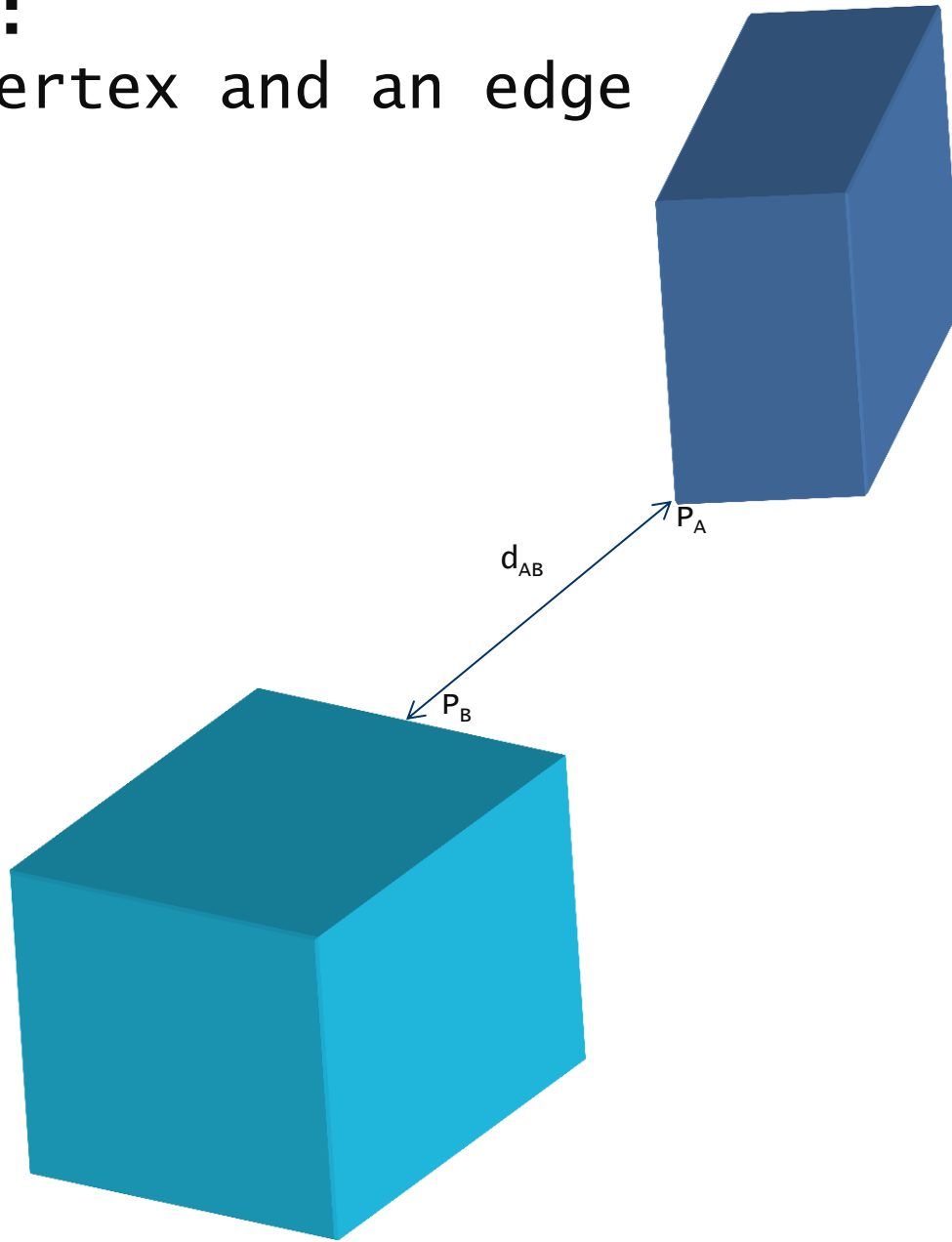


There are six different possible feature pairs that we will need to analyze...

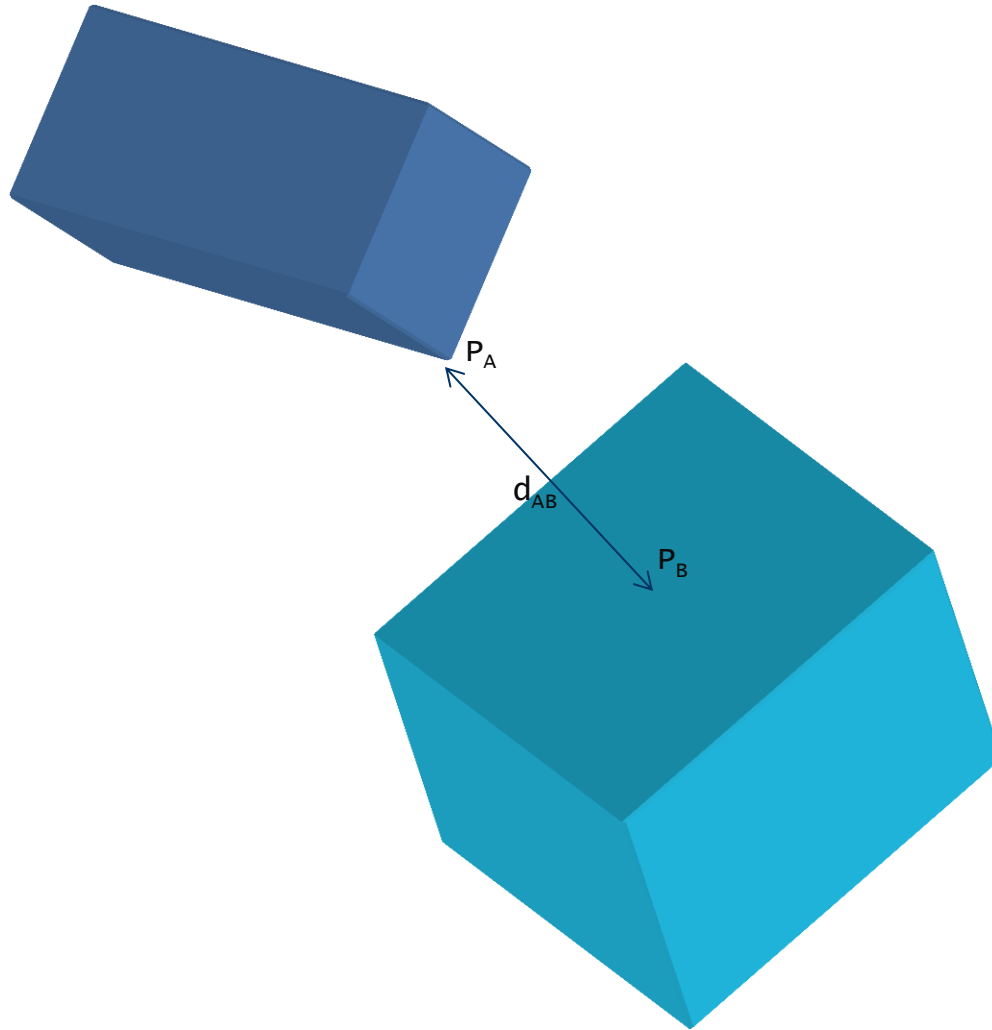
case one:  
a pair of vertices



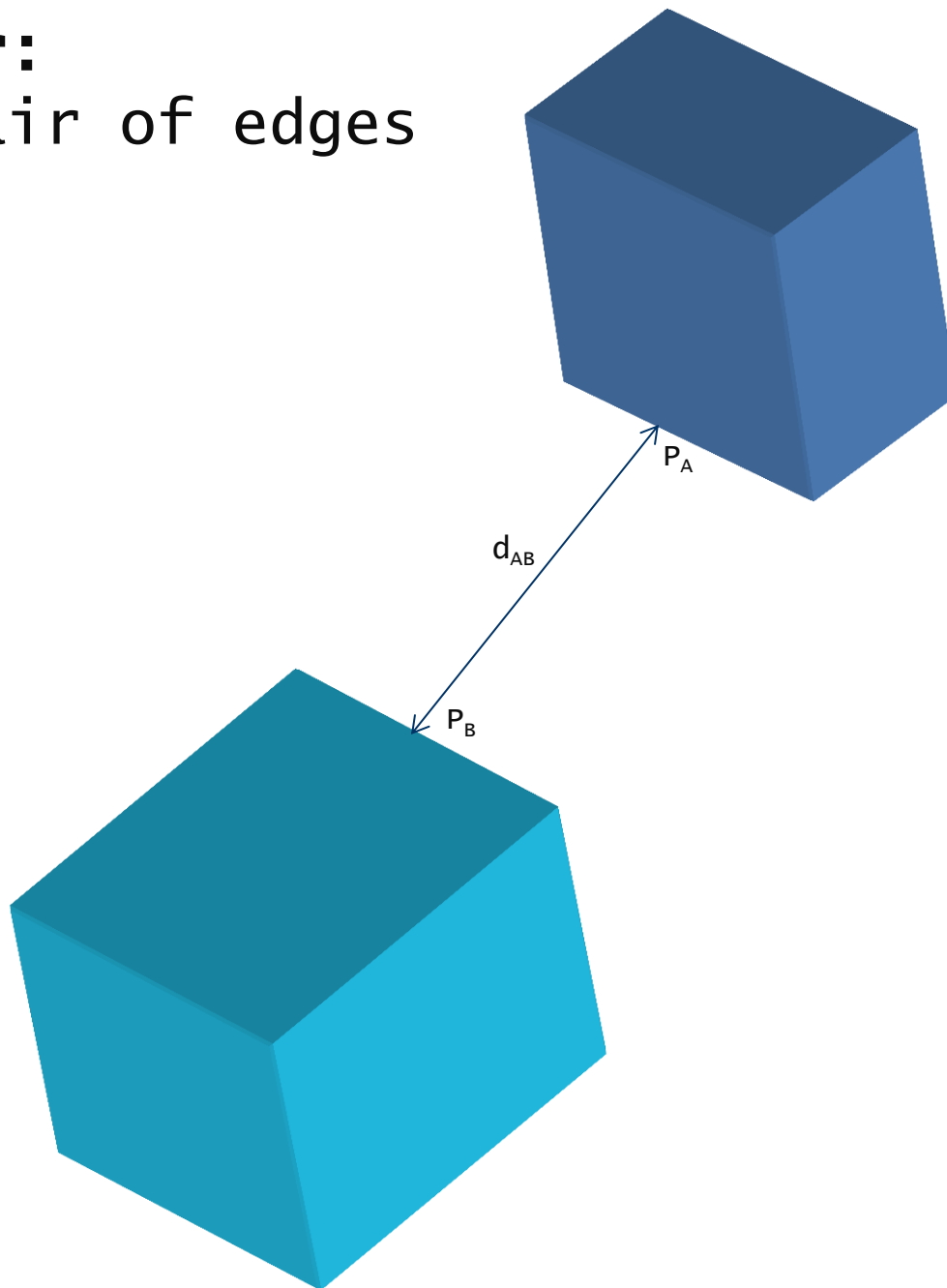
case two:  
a vertex and an edge



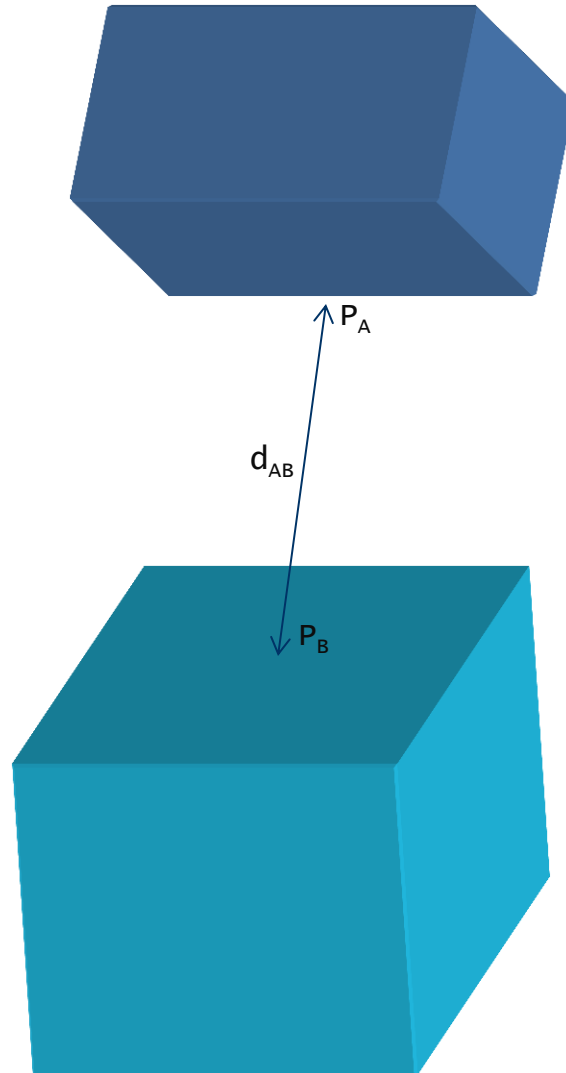
case three:  
a vertex and a face



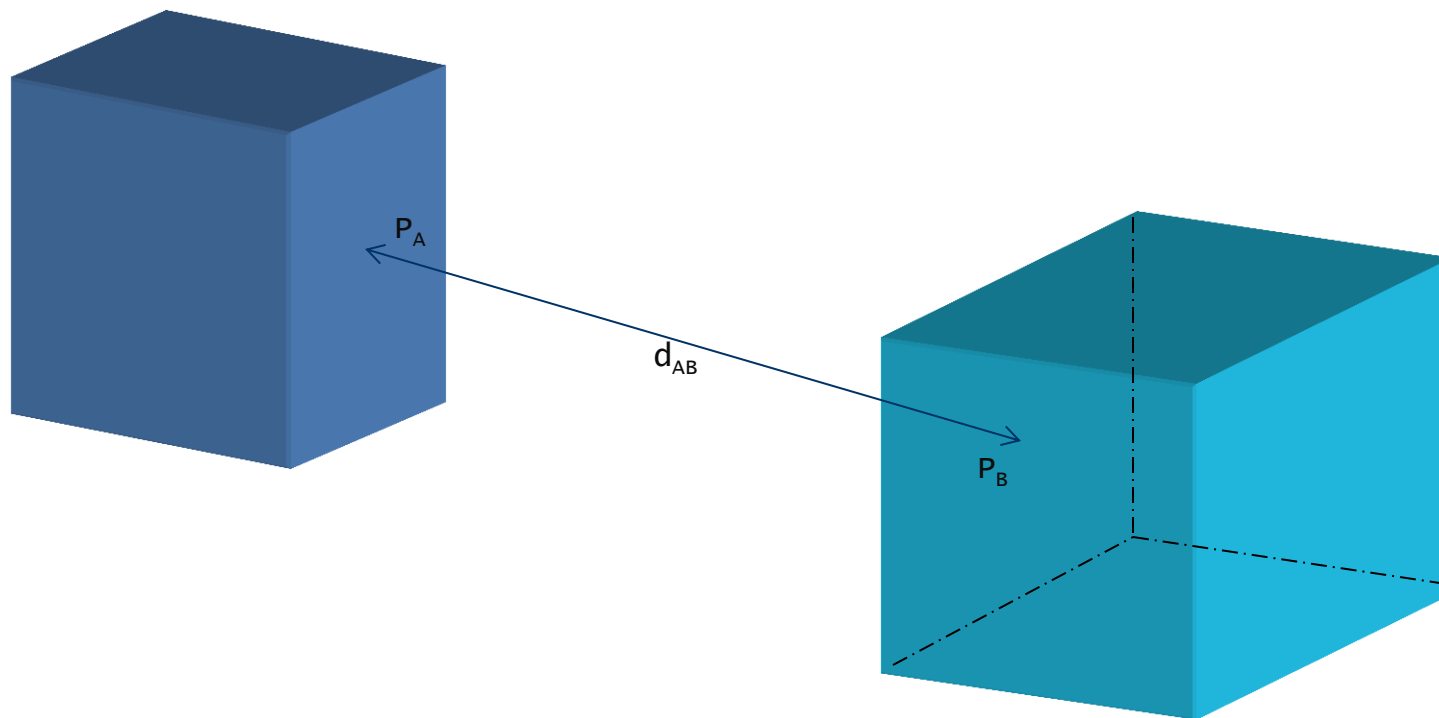
case four:  
a pair of edges



case five:  
an edge and a face



case six:  
a pair of faces (rare!)







1. We first represent each object as a convex polyhedron or a union of convex polyhedra.
2. For each object we calculate the fields for each of its faces, edges, vertices, positions, and orientations.
3. For each pair of features between the two objects of interest, calculate the closest pair of points between those two features. Find the overall closest pair.
4. Incrementally update the closest feature pair.

We utilize the following algorithm for updating:

1. Verify that  $P_A$  is the closest point of A to **feature<sub>B</sub>**, and that  $P_B$  is the closest point of B to **feature<sub>A</sub>**
2. If verification fails, choose a new feature pair and repeat step one
3. Eventually we will terminate on the closest feature pair



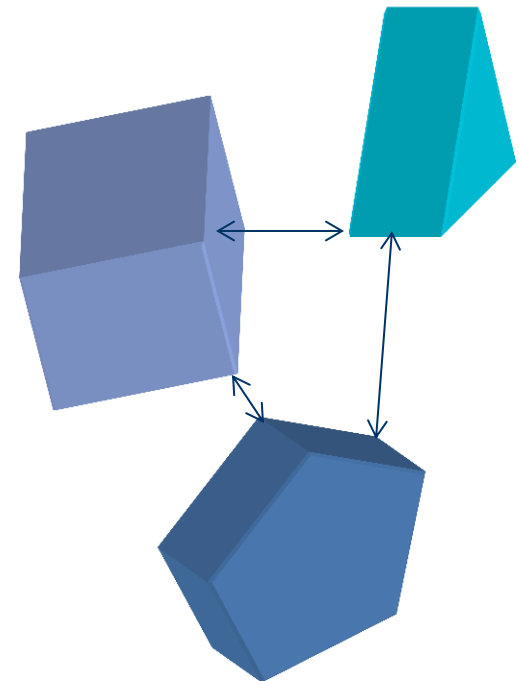
**question:** how can we tell if we've found a closest feature pair, or if we need to try a new pair?

**answer:** we have “applicability criteria” that each feature pair must satisfy in order to be the closest feature pair.

There are applicability criteria for the three of the feature pair combinations:

1. Point-Vertex (Vertex-Vertex)
2. Point-Edge (Vertex-Edge)
3. Point-Face (Vertex-Face)

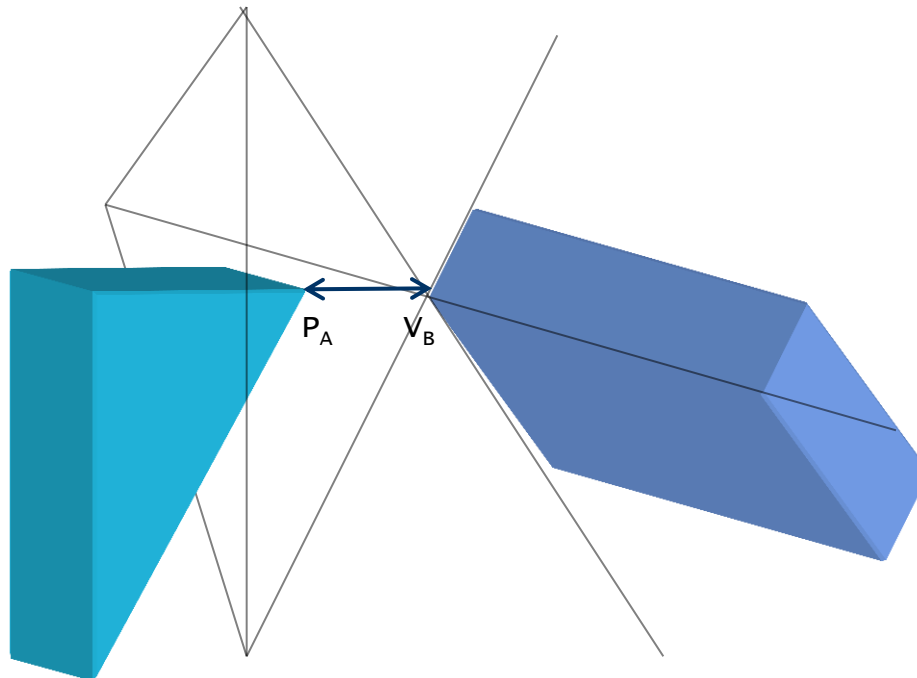
These criteria can be used in dealing with **Edge-Edge**, **Edge-Face**, and **Face-Face** feature pairs as well.



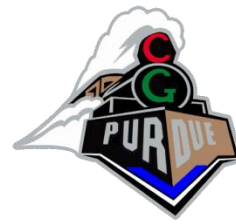
# point-vertex applicability criteria



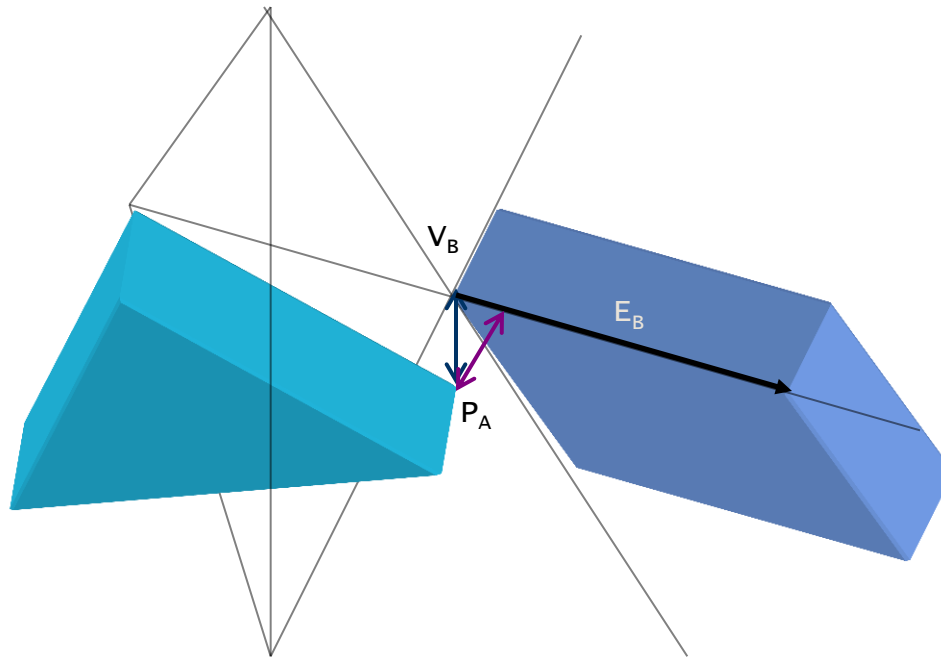
Point  $P_A$  and vertex  $V_B$  are the closest feature pair if  $P_A$  lies within the region bounded by the planes perpendicular to the edges touching  $V_B$ .



# point-vertex applicability criteria



When  $P_A$  lies outside the planar boundaries of  $V_B$  (described previously), then some edge  $E_B$  is closer to  $P_A$  than  $V_B$ .

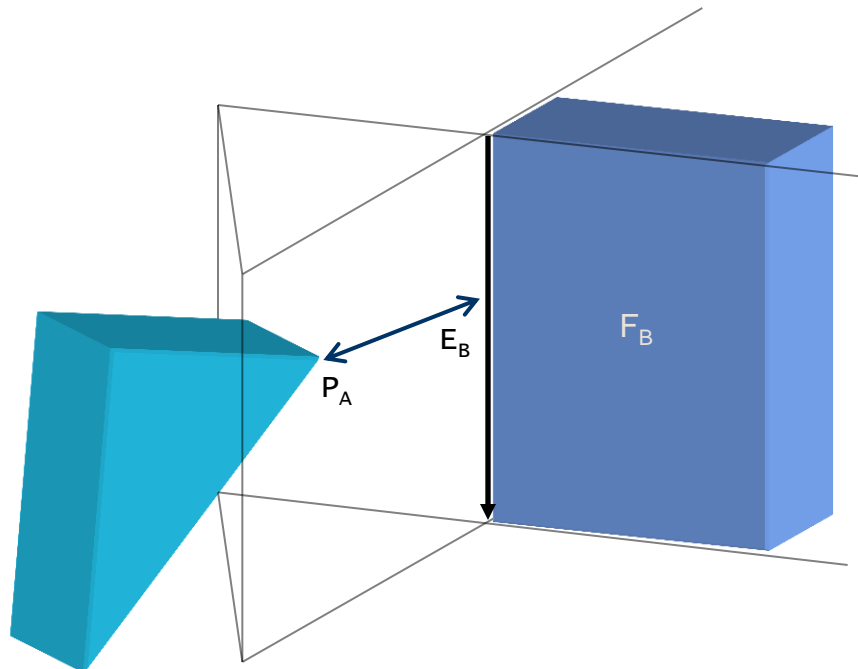


# point-edge applicability criteria

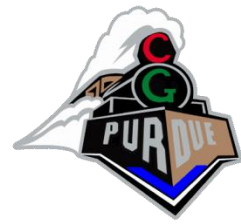


Point  $P_A$  and edge  $E_B$  are the closest feature pair if  $P_A$  lies within the region bounded by:

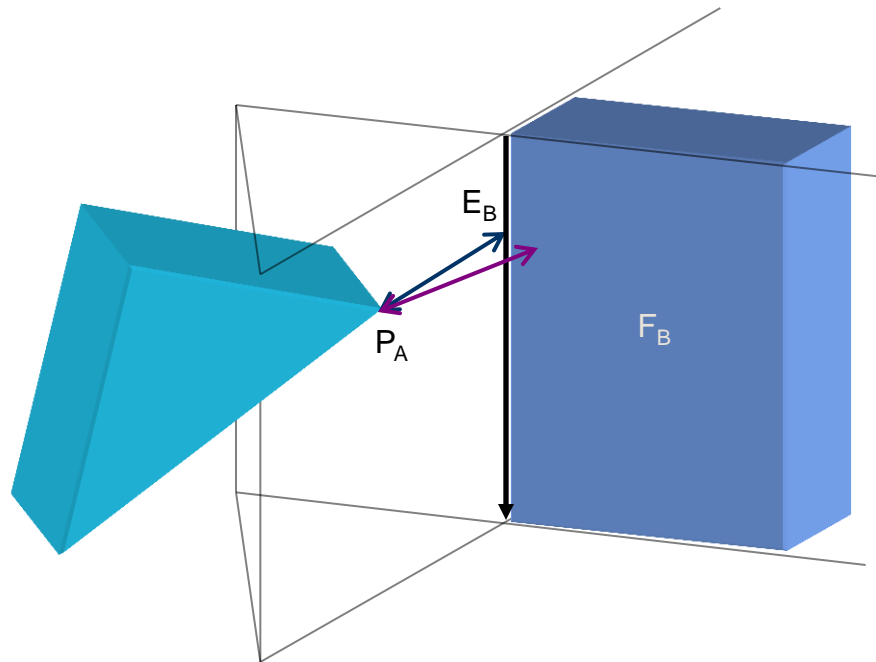
1. The two planes perpendicular to  $E_B$ , passing through its head and tail, *and*
2. The two planes perpendicular to the right and left faces of  $E_B$ .



# point-edge applicability criteria



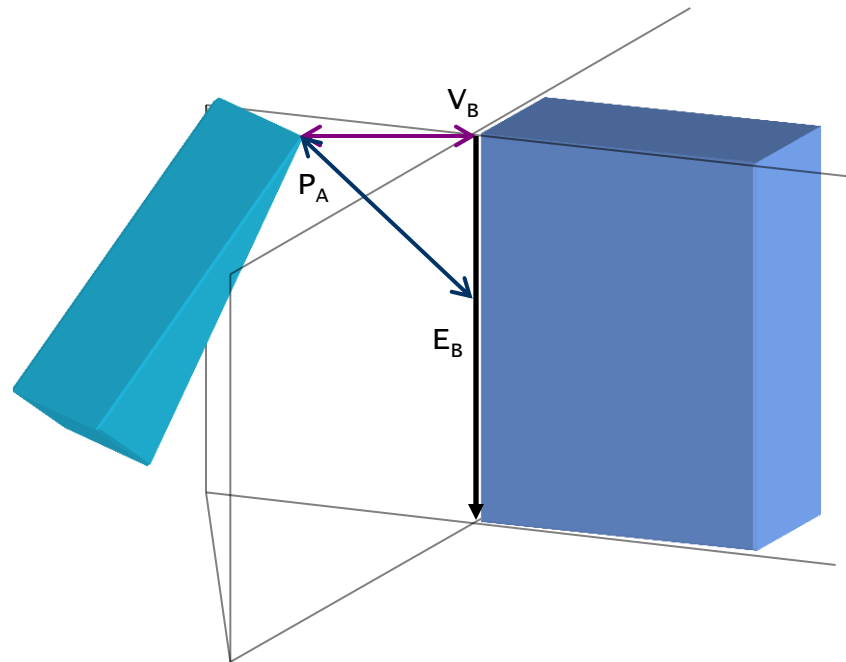
When  $P_A$  lies outside one of the two planes perpendicular to the right and left faces of  $E_B$ , then some face  $F_B$  is closer to  $P_A$  than  $E_B$ .



# point-edge applicability criteria



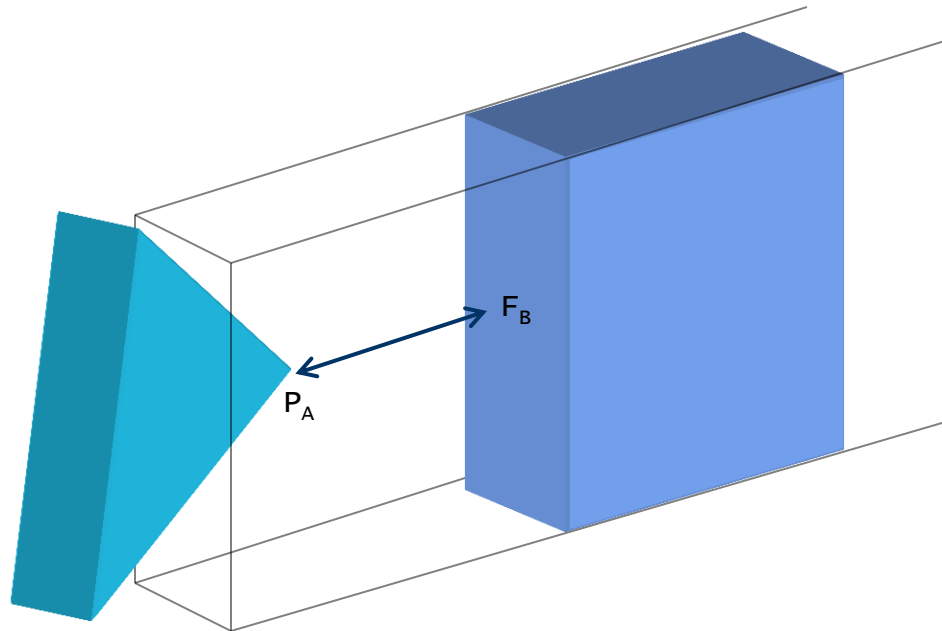
When  $P_A$  lies outside one of the two planes perpendicular to  $E_B$  and passing through either its head or its tail, then some vertex  $V_B$  is closer to  $P_A$  than  $E_B$ .



# point-face applicability criteria



Point  $P_A$  and face  $F_B$  are the closest feature pair if  $P_A$  lies within the region bounded by the planes that are both perpendicular to  $F_B$  and contain the boundaries of  $F_B$ .

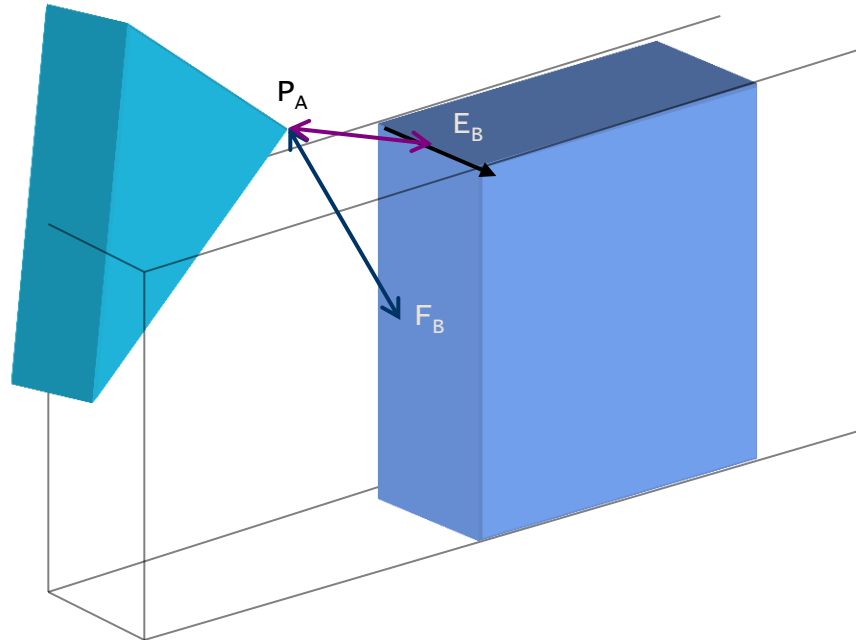




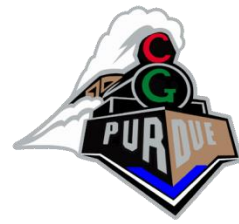
# point-face applicability criteria



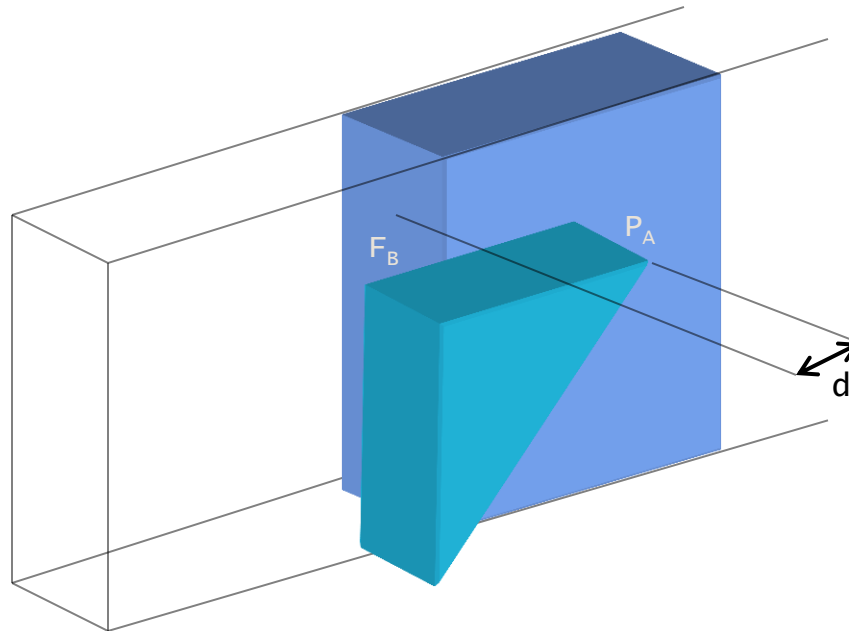
If  $P_A$  lies outside the planar boundaries of  $F_B$  (described previously) then some edge  $E_B$  is closer to  $P_A$  than  $F_B$ .

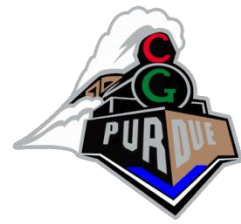


# point-face applicability criteria



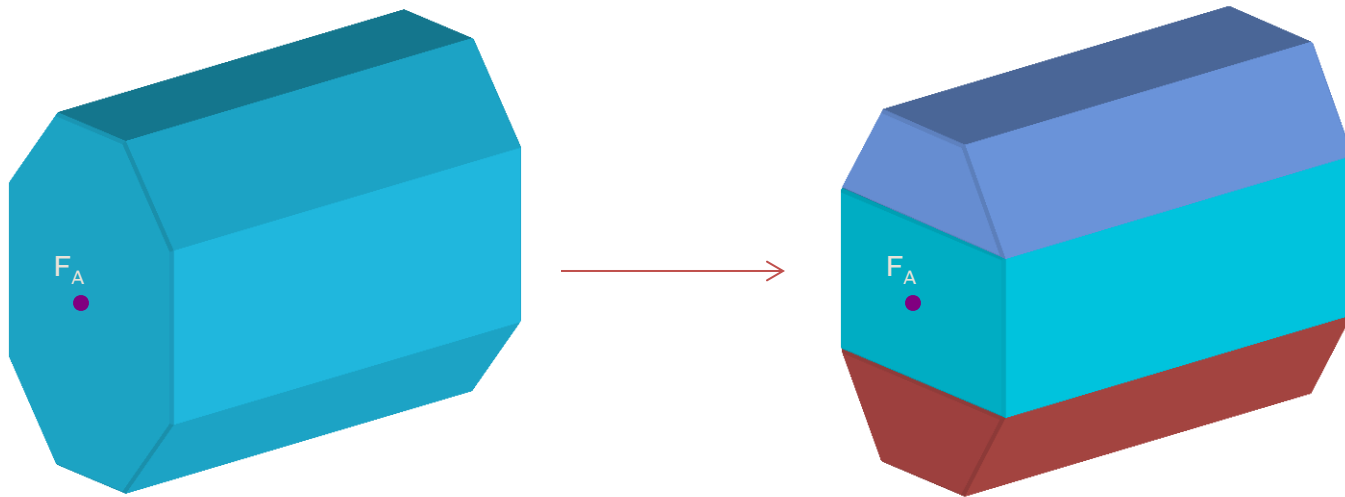
If  $P_A$  lies “below”  $F_B$  then some feature of B is closer to  $P_A$  than  $F_B$  (otherwise A would have collided into B). In this case, we must check every face of object B. This is NOT constant time. Note however that this can only happen during initialization; otherwise, the collision would have been detected.



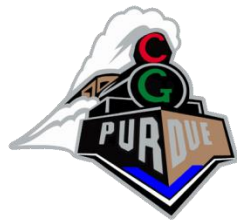


# preprocessing

Preprocessing steps ensure that objects have boundaries and co-boundaries of constant size (eg. limiting the number of edges going through each vertex and bounding each face to 4 or 5).



Constant sized boundaries allows constant time verification of a closest feature pair.



results and conclusions...



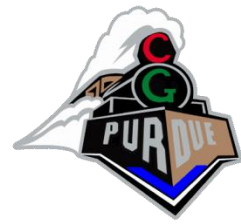
**lin-canny runs in quadratic time when initially finding a closest pair**

Finding the initial closest feature pair is in the worst case  $O(n^2)$  in number of features per object.

**lin-canny runs in constant time in an “incremental movement” framework**

Once an initial closest feature pair is found, it takes on average constant time to keep track of that closest pair.

# conclusions



Lin-Canny is a simple and efficient algorithm that is guaranteed to find the closest feature and point pair.

Results have applications in collision detection and motion planning.

Lin-Canny can be extended to handle non-convex objects without any significant increase in running time.