Vectors

Outline and Reading

- The Vector ADT (§5.1.1)
- Array-based implementation (§5.1.2)

The Vector ADT

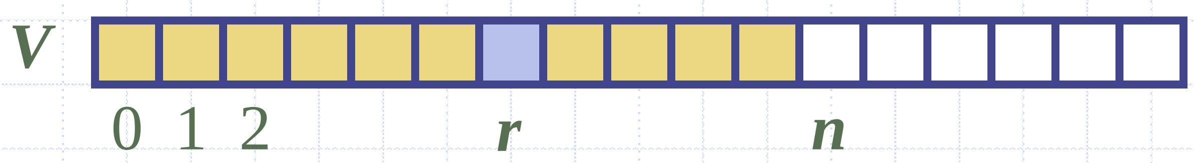
- The Vector ADT stores objects to which it provides direct access
- An element can be accessed, inserted or removed by specifying its rank (number of elements preceding it)
- An exception is thrown if an incorrect rank is specified (e.g., a negative rank)
- Main vector operations:
  - elemAtRank(int r): returns the element at rank r without removing it
  - replaceAtRank(int r, Object o): replace the element at rank r with o
  - insertAtRank(int r, Object o): insert a new element o to have rank r
  - removeAtRank(int r): removes the element at rank r
- Additional operations size() and isEmpty()

Applications of Vectors

- Direct applications
  - Sorted collection of objects (elementary database)
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

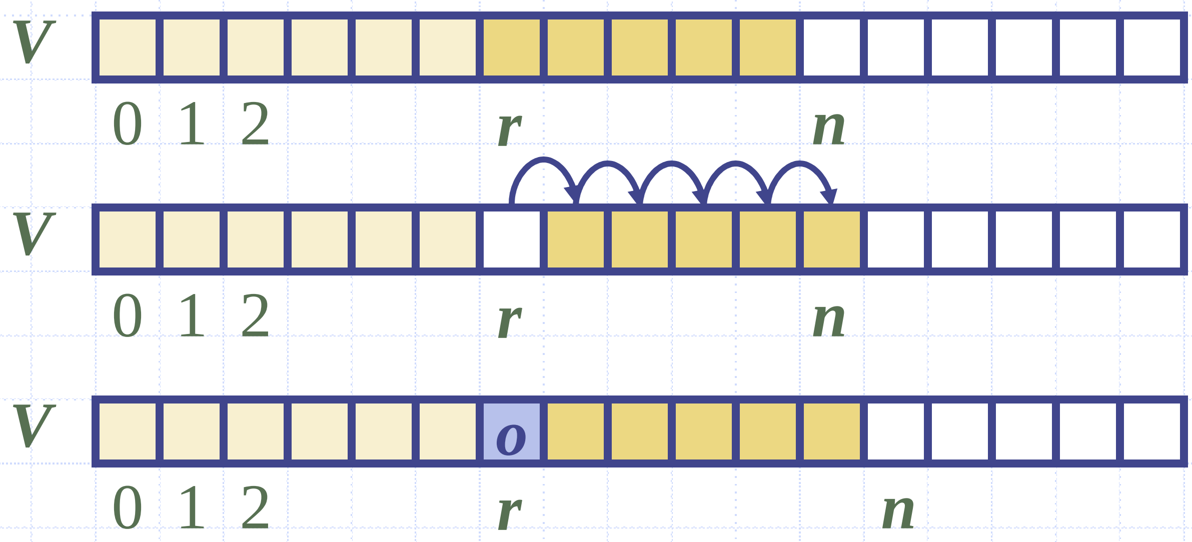
Array-based Vector

- Use an array V of size N
- A variable n keeps track of the size of the vector (number of elements stored)
- Operation ***elemAtRank***(r) is implemented in $O(1)$ time by returning $V[r]$



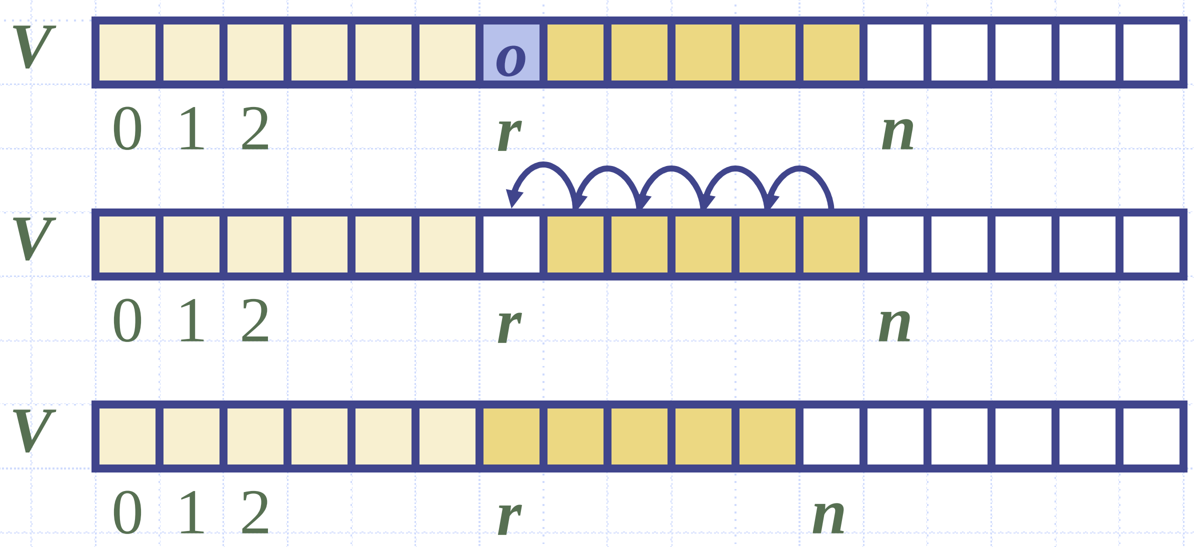
Insertion

- In operation ***insertAtRank***(r, o), we need to make room for the new element by shifting forward the $n - r$ elements $V[r], \ldots, V[n-1]$
- In the worst case ($r = 0$), this takes $O(n)$ time



Deletion

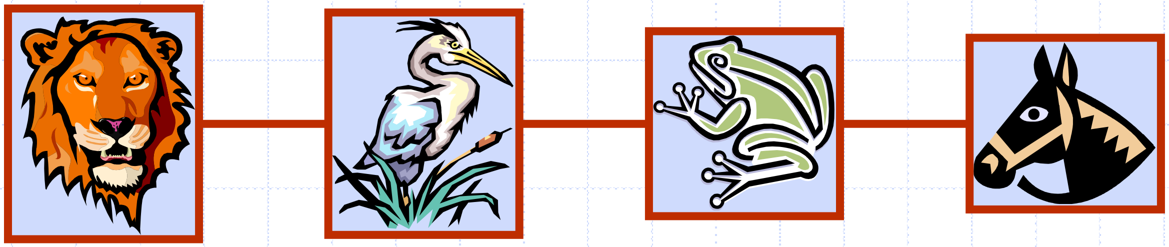
- In operation ***removeAtRank***(r), we need to fill the hole left by the removed element by shifting backward the $n - r - 1$ elements $V[r + 1], \ldots, V[n - 1]$
- In the worst case ($r = 0$), this takes $O(n)$ time



Performance

- In the array based implementation of a Vector
  - The space used by the data structure is $O(n)$
  - ***size***, ***isEmpty***, ***elemAtRank*** and ***replaceAtRank*** run in $O(1)$ time
  - ***insertAtRank*** and ***removeAtRank*** run in $O(n)$ time
- In an ***insertAtRank*** operation, when the array is full, instead of throwing an exception, we can replace the array with a larger one

Lists and Sequences

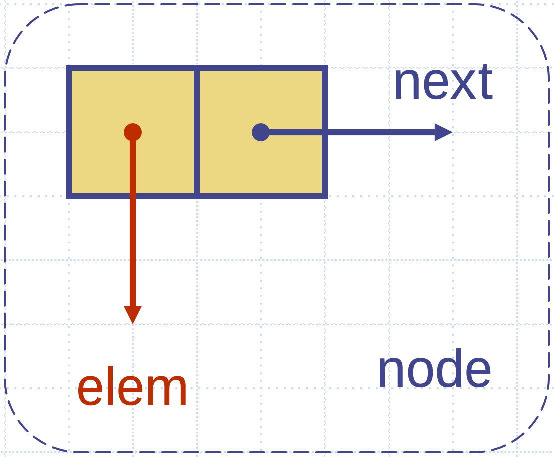


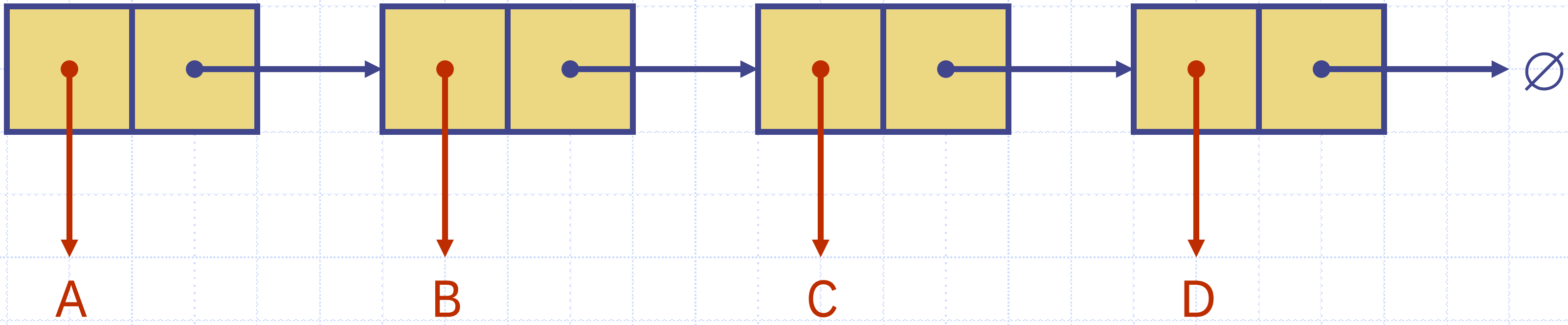
Outline and Reading

- Singly linked list
- Position ADT and List ADT (§5.2.1)
- Doubly linked list (§ 5.2.3)
- Sequence ADT (§5.3.1)
- Implementations of the sequence ADT (§5.3.3)

Singly Linked List

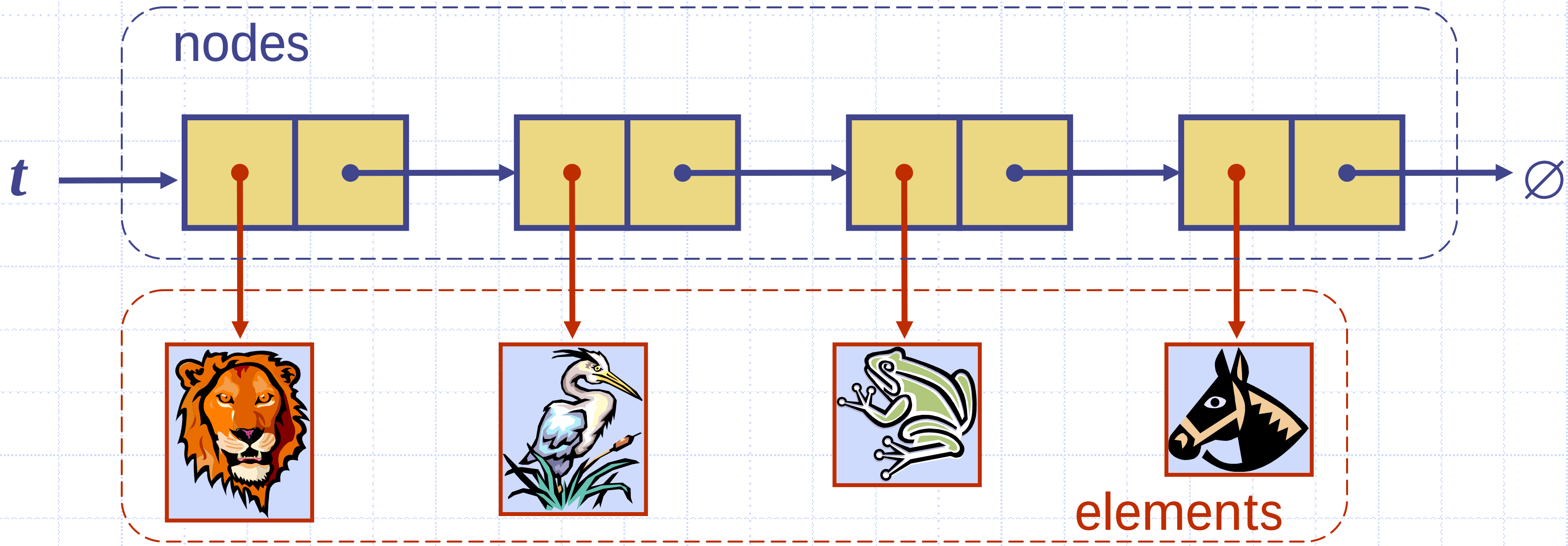
- A singly linked list is a concrete data structure consisting of a sequence of nodes
- Each node stores
  - element
  - link to the next node





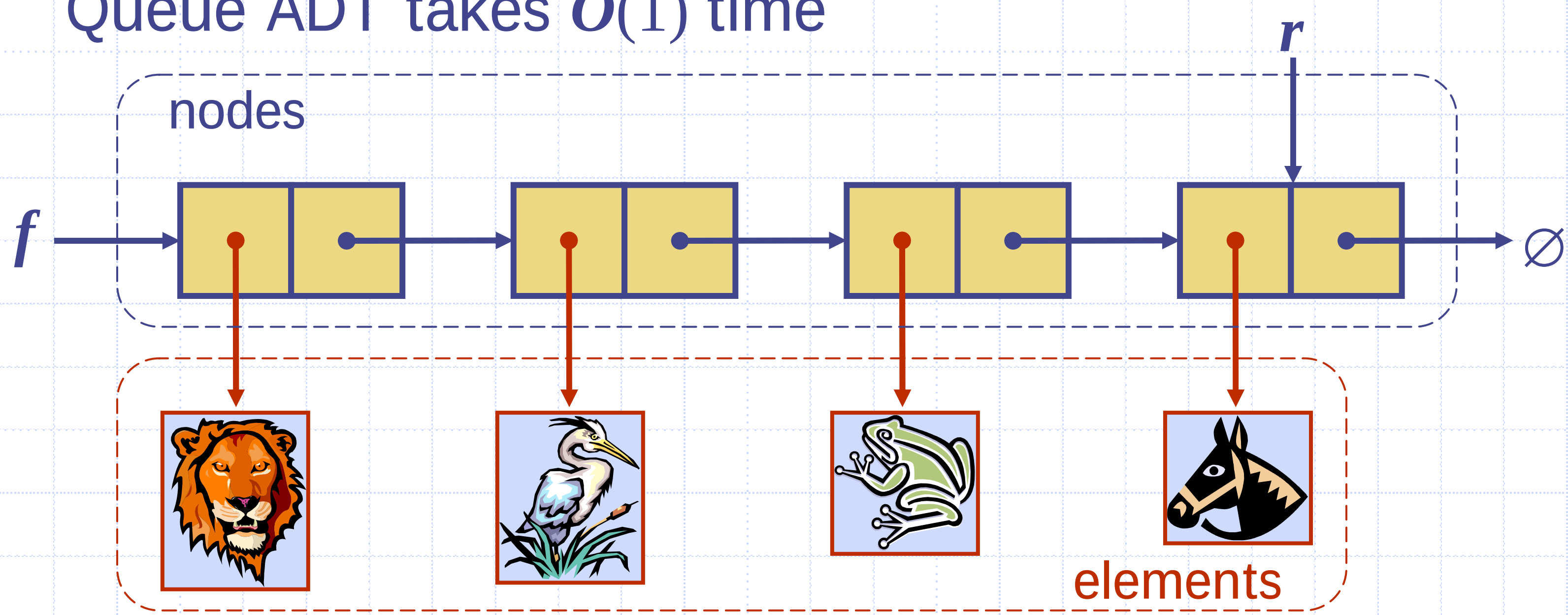
Stack with a Singly Linked List

- We can implement a stack with a singly linked list
- The top element is stored at the first node of the list
- The space used is $O(n)$ and each operation of the Stack ADT takes $O(1)$ time



Queue with a Singly Linked List

- We can implement a queue with a singly linked list
  - The front element is stored at the first node
  - The rear element is stored at the last node
- The space used is $O(n)$ and each operation of the Queue ADT takes $O(1)$ time



Position ADT

- The Position ADT models the notion of place within a data structure where a single object is stored
- A special **null** position refers to no object.
- Positions provide a unified view of diverse ways of storing data, such as
  - a cell of an array
  - a node of a linked list
- Member functions:
  - Object& element(): returns the element stored at this position
  - bool isNull(): returns true if this is a null position

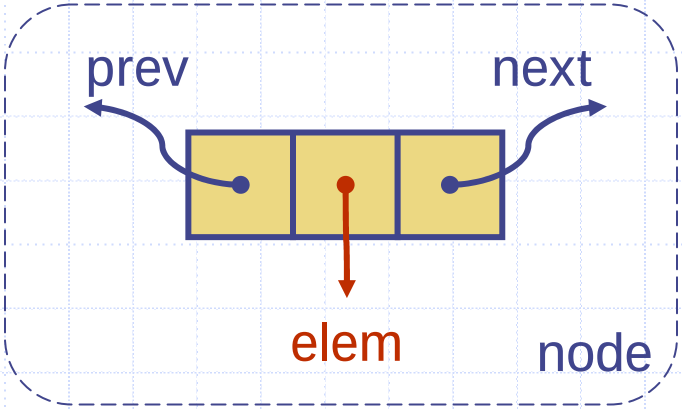
List ADT

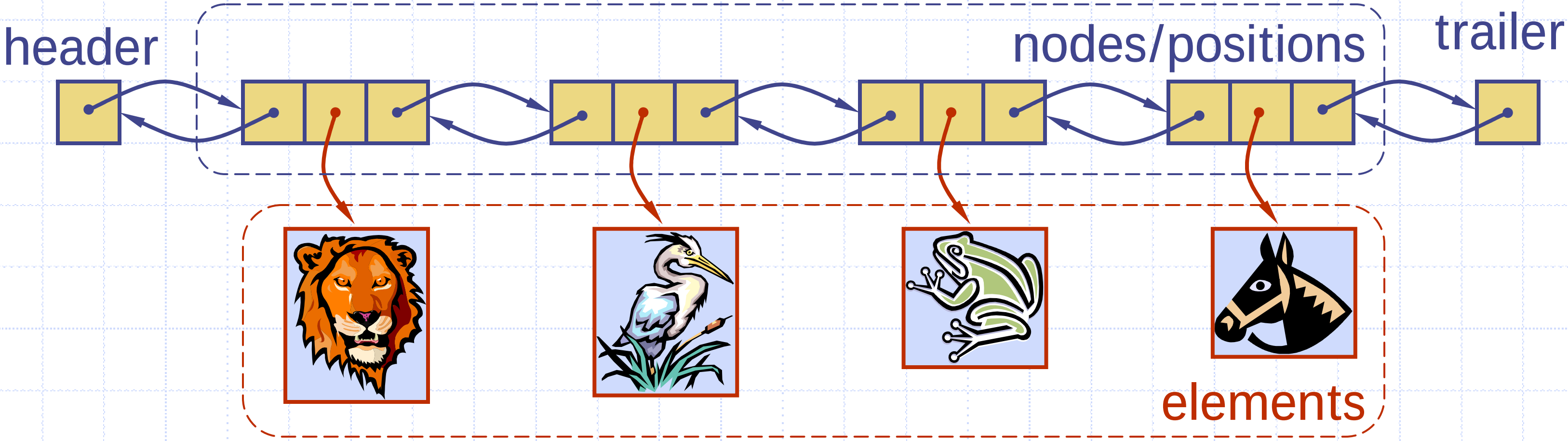
- The List ADT models a sequence of positions storing arbitrary objects
- It establishes a before/after relation between positions
- Generic methods:
  - size(), isEmpty()
- Query methods:
  - isFirst(p), isLast(p)

- Accessor methods:
  - first(), last()
  - before(p), after(p)
- Update methods:
  - replaceElement(p, o), swapElements(p, q)
  - insertBefore(p, o), insertAfter(p, o),
  - insertFirst(o), insertLast(o)
  - remove(p)

Doubly Linked List

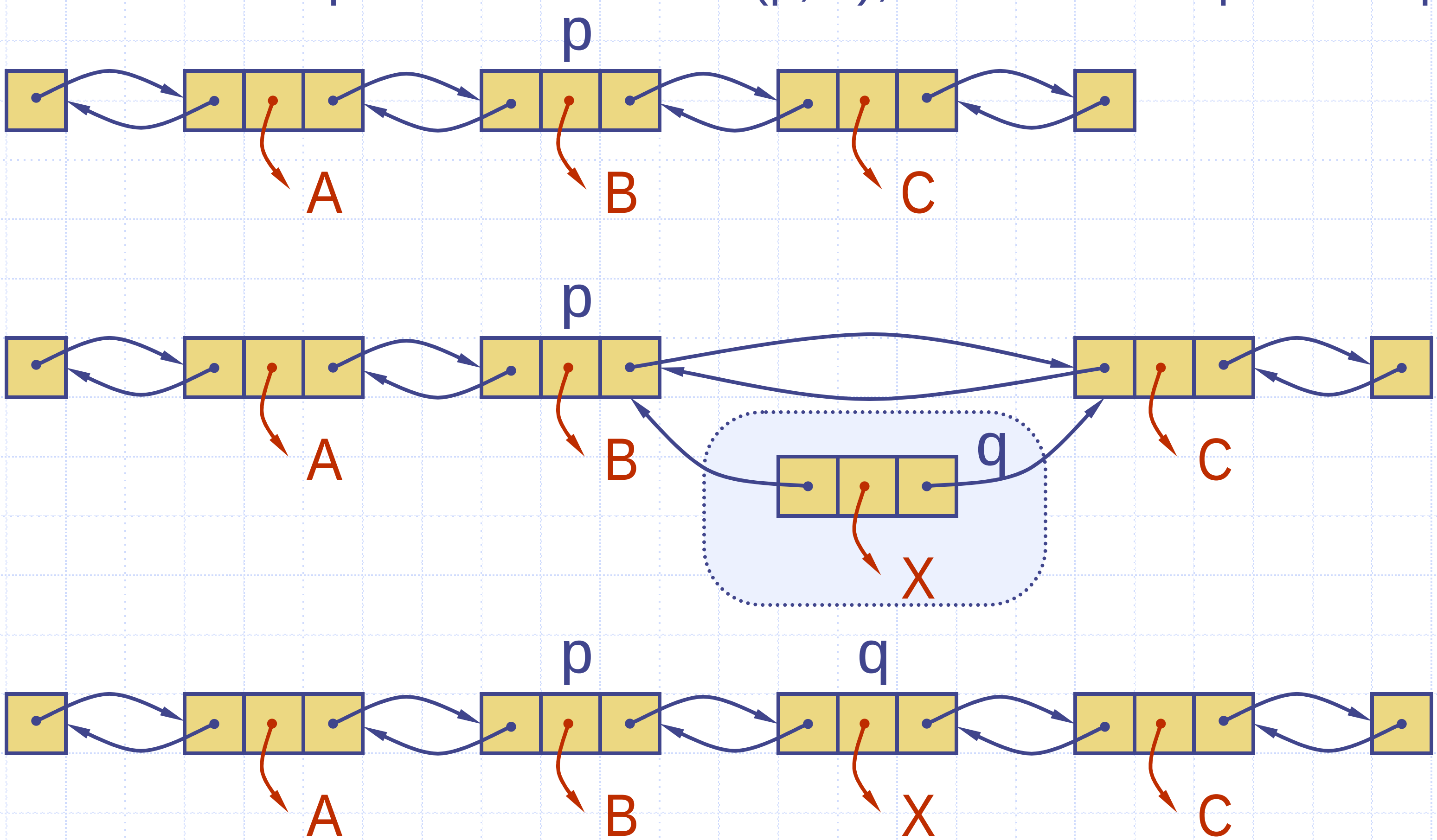
- A doubly linked list provides a natural implementation of the List ADT
- Nodes implement Position and store:
  - element
  - link to the previous node
  - link to the next node
- Special trailer and header nodes





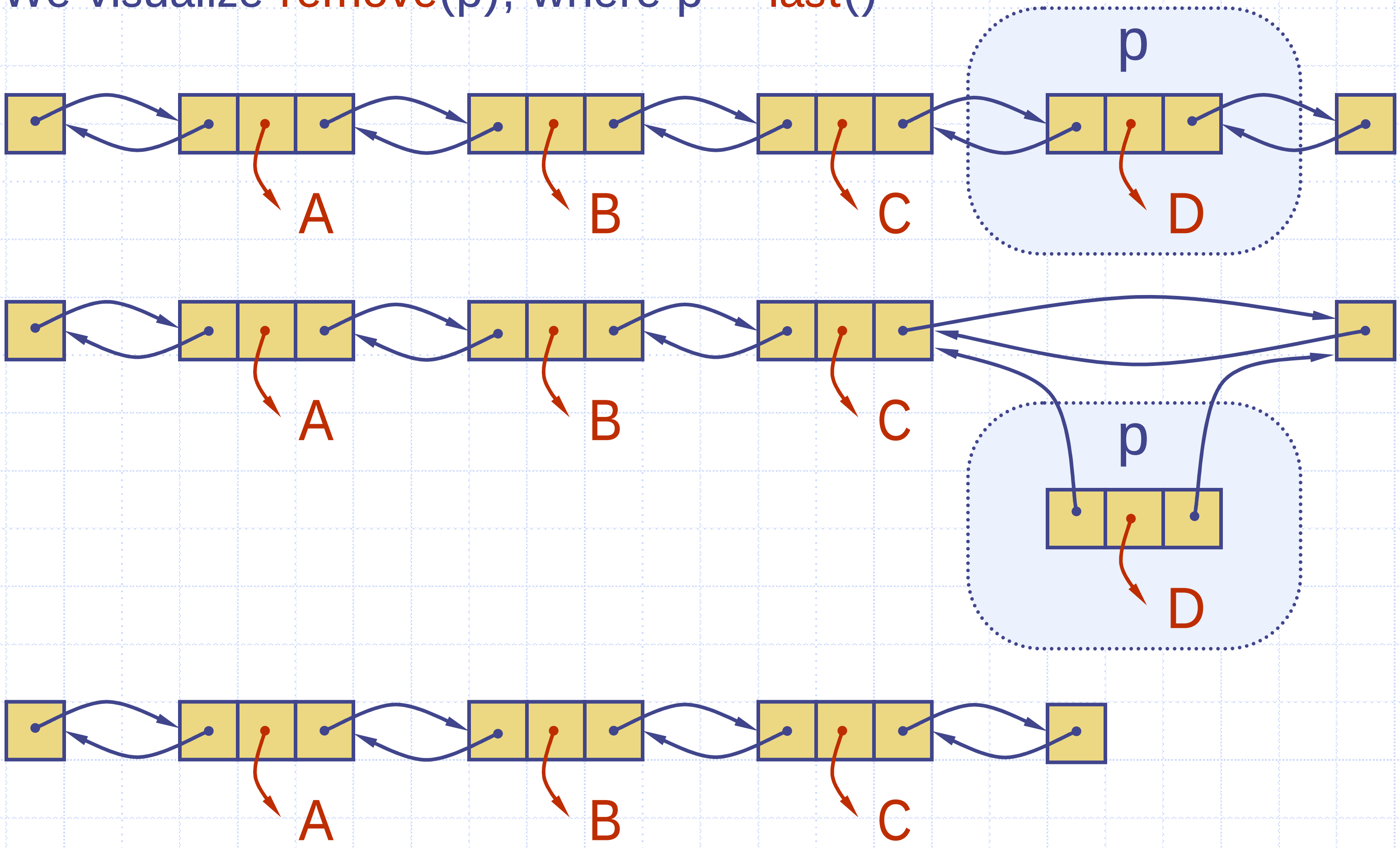
Insertion

- We visualize operation insertAfter(p, X), which returns position q



Deletion

- We visualize remove(p), where p = last()



Performance

- In the implementation of the List ADT by means of a doubly linked list
  - The space used by a list with n elements is $O(n)$
  - The space used by each position of the list is $O(1)$
  - All the operations of the List ADT run in $O(1)$ time
  - Operation element() of the Position ADT runs in $O(1)$ time

Sequence ADT

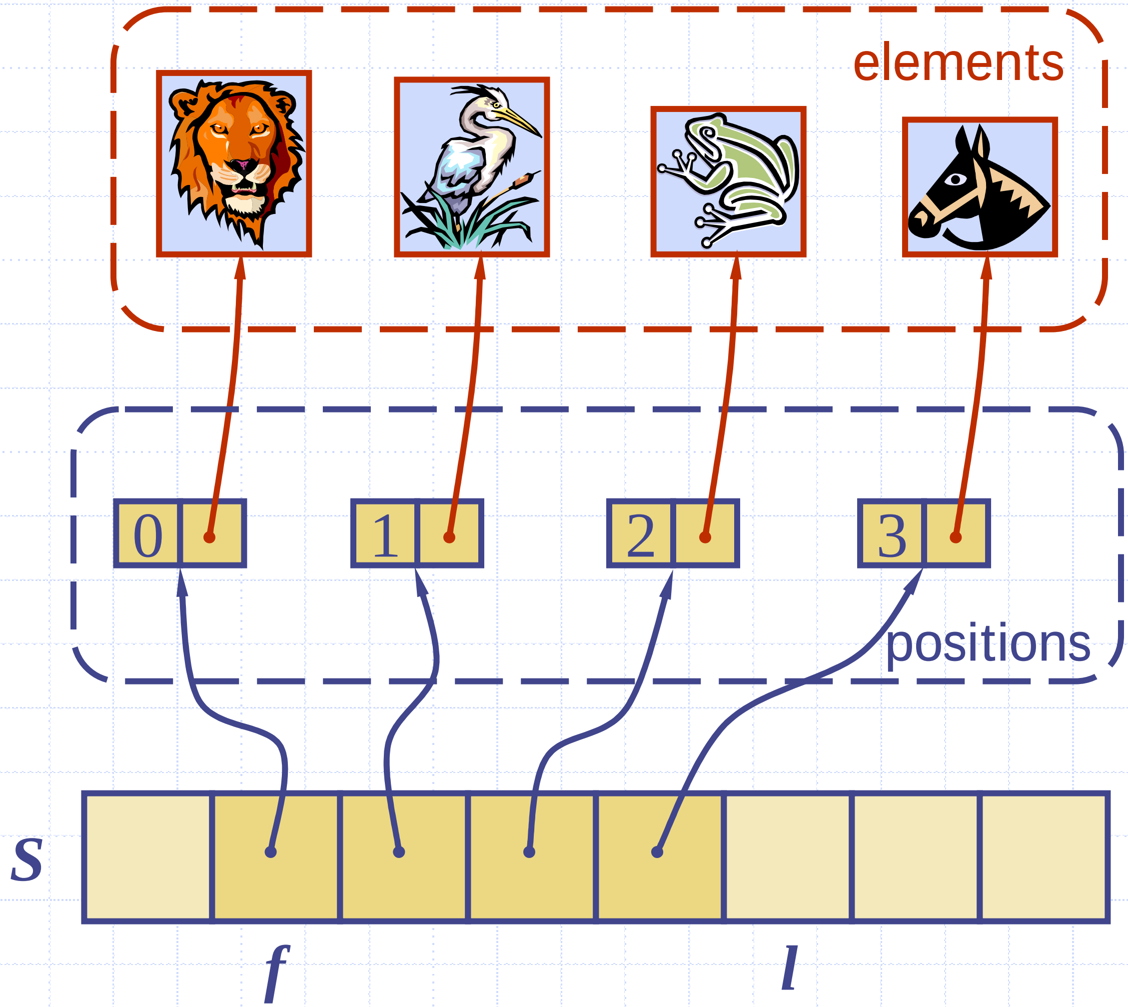
- The Sequence ADT is the union of the Vector and List ADTs
- Elements accessed by
  - Rank, or
  - Position
- Generic methods:
  - size(), isEmpty()
- Vector-based methods:
  - elemAtRank(r), replaceAtRank(r, o), insertAtRank(r, o), removeAtRank(r)
- List-based methods:
  - first(), last(), before(p), after(p), replaceElement(p, o), swapElements(p, q), insertBefore(p, o), insertAfter(p, o), insertFirst(o), insertLast(o), remove(p)
- Bridge methods:
  - atRank(r), rankOf(p)

Applications of Sequences

- The Sequence ADT is a basic, general-purpose, data structure for storing an ordered collection of elements
- Direct applications:
  - Generic replacement for stack, queue, vector, or list
  - small database (e.g., address book)
- Indirect applications:
  - Building block of more complex data structures

Array-based Implementation

- We use a circular array storing positions
- A position object stores:
  - Element
  - Rank
- Indices *f* and *l* keep track of first and last positions



Sequence Implementations

Operation	Array	List
size, isEmpty	1	1
atRank, rankOf, elemAtRank	1	n
first, last, before, after	1	1
replaceElement, swapElements	1	1
replaceAtRank	1	n
insertAtRank, removeAtRank	n	n
insertFirst, insertLast	1	1
insertAfter, insertBefore	n	1
remove	n	1