Cryptography

Acknowledgment

- Based on slides by prof. Cristina Nita-Rotaru

Shift Cipher

- A substitution cipher
- The Key Space:
  - [1 .. 25]
- Encryption given a key K:
  - each letter in the plaintext P is replaced with the K'th letter following corresponding number (shift right)
- Decryption given K:
  - shift left

History: K = 3, Caesar's cipher

3

People have always wanted to encrypt communication such that only intended recipients have access to the content.

The shift cypher substitutes a character with the character with the character located *k* positions over in the alphabet.

Shift Cipher: An Example

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

P = CRYPTOGRAPHYISFUN

K = 11

C = NCJAVZRCLASJTDQFY

C → 2; 2+11 mod 26 = 13 → N

R → 17; 17+11 mod 26 = 2 → C

...

N → 13; 13+11 mod 26 = 24 → Y

4

Here a letter of the plain text message P was replaced with the letter 11 positions after the letter in the alphabet. Once the end of the alphabet is reached, one proceeds circularly and starts at A again.

Shift Cipher: Cryptanalysis

- Can an attacker find K?
  - YES: exhaustive search, key space is small (<= 26 possible keys).
- Once K is found, very easy to decrypt

5

Breaking such a code is trivial since all you need to guess is a number between 0 and 26 (i.e. the number of letters in the alphabet) and one can proceed by trial and error.

General Mono-alphabetical Substitution Cipher

- The key space: all permutations of $\Sigma = \{A, B, C, \ldots, Z\}$
- Encryption given a key (permutation) π:
  - each letter X in the plaintext P is replaced with $\pi(X)$
- Decryption given a key π:
  - each letter Y in the cipherext P is replaced with $\pi^{-1}(Y)$

Example:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
$\pi=$	B	A	D	C	Z	H	W	Y	G	O	Q	X	S	V	T	R	N	M	S	K	J	I	P	F	E	U

BECAUSE $\rightarrow$ AZDBJSZ

6

A more powerful cypher is to define a key as a permutation of the letters of the alphabet. Each letter is replaced with the corresponding letter as defined by the permutation.

Strength of the General Substitution Cipher

- Exhaustive search is infeasible
  - key space size is $26! \approx 4*10^{26}$
- Dominates the art of secret writing throughout the first millennium A.D.
- Thought to be unbreakable by many back then

7

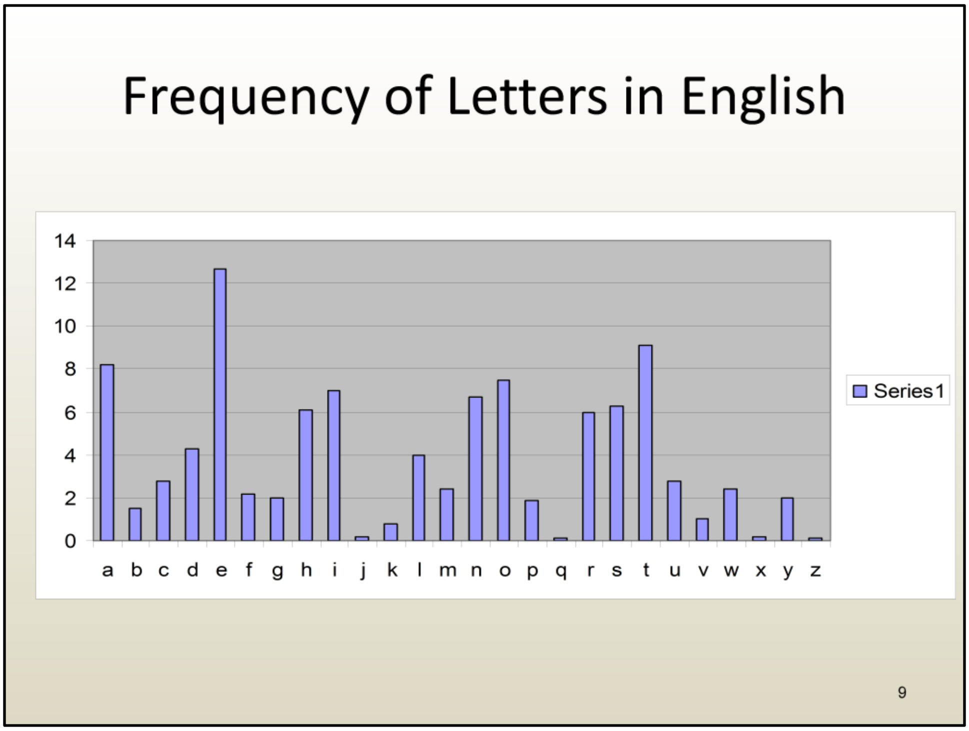
There are 26! permutations of the alphabet, so a brute force approach of trying all permutations doesn’t work.

Cryptanalysis of Substitution Ciphers: Frequency Analysis

- Basic ideas:
  - Each language has certain features: frequency of letters, or of groups of two or more letters.
  - Substitution ciphers preserve the language features.
  - Substitution ciphers are vulnerable to frequency analysis attacks.

8

However, one can break substitution cyphers based on the known frequency of letters in text in a given language. A letter is always substituted by the same letter, thus the frequency is preserved in the cypher text.



Based on this graph, one should replace the most frequent letter in the cypher text with E, second most frequent with T and so on.

Other Frequency Features of English

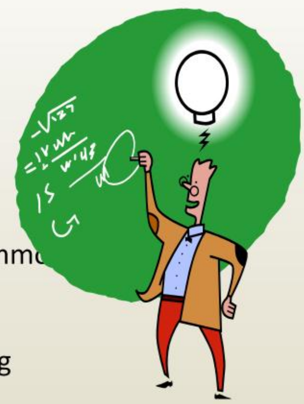
- Vowels, which constitute 40 % of plaintext, are often separated by consonants.
- Letter A is often found in the beginning of a word or second from last.
- Letter I is often third from the end of a word.
- Letter Q is followed only by U
- And more …

10

There are additional properties of the English language that come in handy when trying to break a substitution cypher.

Substitution Ciphers: Cryptanalysis

- The number of different ciphertext characters or combinations are counted to determine the frequency of usage.
- The cipher text is examined for patterns, repeated series, and comm combinations.
- Replace ciphertext characters with possible plaintext equivalents using known language characteristics.



Frequency Analysis History

- Discovered in Arabia
  - earliest known description of frequency analysis is in a book by the ninth-century scientist al-Kindi
- Rediscovered or introduced in Europe during the Renaissance
- Frequency analysis made substitution cipher insecure

Improve the Security of Substitution Cipher

- Using nulls
  - e.g., using numbers from 1 to 99 as the ciphertext alphabet, some numbers representing nothing are inserted randomly
- Deliberately misspell words
  - e.g., "Thys haz thi ifekkt off diztaughting thi ballans off frikwenseas"
- Homophonic substitution cipher
  - each letter is replaced by a variety of substitutes
- These make frequency analysis more difficult, but not impossible

13

Marginal improvement, substitution cyphers just aren't safe.

Summary

- Shift ciphers are easy to break using brute force attacks, they have small key space.

- Substitution ciphers preserve language features and are vulnerable to frequency analysis attacks.

14

So far we have talked about shift cyphers and substitution cyphers.

Towards the Polyalphabetic Substitution Ciphers

- Main weaknesses of monoalphabetic substitution ciphers
  - each letter in the ciphertext corresponds to only one letter in the plaintext letter
- Idea for a stronger cipher (1460's by Alberti)
  - use more than one cipher alphabet, and switch between them when encrypting different letters
- Developed into a practical cipher by Vigenère (published in 1586)

15

The Vigenère Cipher

Definition:

Given a key $K = (k_1, k_2, \ldots, k_m)$ and a plain text message $P = (p_1, p_2, \ldots, p_n)$

Encryption:

$c_i = (k_{i(\text{mod } m)} + p_i) \pmod{26}$, for i = 1 to n

Decryption:

$p_i = (e_i - k_{i(\text{mod } m)}) \pmod{26}$, for i = 1 to n

Example:

Plaintext P (n = 12):	C R Y P T O G R A P H Y
Key K (m = 4):	L U C K L U C K L U C K
Ciphertext C:	N L A Z E I I B L J J I

16

The key "luck" is repeated as many times needed to cover the entire plaintext P. This way a letter is replaced by different letters, which doesn't preserve letter frequency.

Security of Vigenere Cipher

- Vigenere masks the frequency with which a character appears in a language: one letter in the ciphertext corresponds to multiple letters in the plaintext. Makes the use of frequency analysis more difficult.
- Any message encrypted by a Vigenere cipher is a collection of as many shift ciphers as there are letters in the key.

17

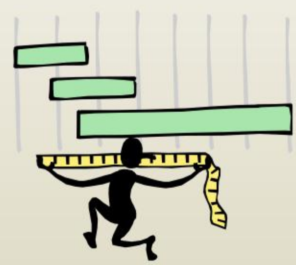
Vigenere Cipher: Cryptanalysis

- Find the length of the key.
- Divide the message into that many shift cipher encryptions.
- Use frequency analysis to solve the resulting shift ciphers.
  - how?

18

How to Find the Key Length?

- For Vigenere, as the length of the keyword increases, the letter frequency shows less English-like characteristics and becomes more random.
- One method to find the key length:
  - Kasisky test



19

Kasisky Test

- Two identical segments of plaintext, will be encrypted to the same ciphertext, if the they occur at a distance that is a multiple of m
  - m is the key length
- Algorithm:
  - Search for pairs of identical segments of length at least 3
  - Record distances between the two segments: Δ1, Δ2, …
  - m divides greatest common divisor of Δ1, Δ2, …

20

Example of the Kasisky Test

Key	K I N G K I N G K I N G K I N G K I N G K I N G
PT	t h e s u n a n d t h e m a n i n t h e m o o n
CT	D P R Y E V N T N **B U K** W I A O X **B U K** W W B T

8 characters =
4 x 2

21

Here the key length m is 4. There are three appearances of the substring “the” in the plaintext. Two of them map to the same part of the key and are thus transformed to the same substring “buk”. The Kasisky test finds the two substrings, the distance between them is 8, and thus the length of the key is 8 or 4 (2 is too short).

Summary

- Vigenère cipher is vulnerable: once the key length is found, a cryptanalyst can apply frequency analysis.

22

Cryptography today—RSA algorithm

- Invented in **1978** by Ron **R**ivest, Adi **S**hamir and Leonard **A**dleman
- Security relies on the difficulty of factoring large composite numbers
  - Numbers with 1,024 bits
- Essentially the same algorithm was discovered in 1973 by Clifford Cocks, who works for the British intelligence

23

Modern cryptography is based on the difficulty of factoring numbers n obtained by multiplying two large primes p and q, i.e. n = pq.