Three or four information time paradoxes revisited

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The paradox of the stalled watch

- This watch is stalled (swiss made but exhausted battery)

  - It desperately indicates 5:33':30"

This watch is absolutely accurate

- Twice a day!

  – It is useless I cannot tell when…

This watch is absolutely accurate

- But I can tell you when…

– Twice a day at 5:33':30''!

I can tell you when

- Two meanings: one paradox
  - 1. Information about time
  - 2. Time of information delivery

Paradox of the execution day

- On Friday afternoon
  - The judge sentences a man to be executed the next week

  - « in a day you wouldn't be able to predict the day before »
  - The man says « I will never be executed »

Paradox of the execution day

- The man:
  - I cannot be executed next Friday
    - Would be last day,
    - could predict on Thursday evening
  - Cannot on Thursday
    - Would be last day (since Friday out)
    - could predict on Wednesday evening
  - … Cannot on Saturday
    - Would be never

Paradox of the execution day

- Execution day Recursion:

sat	sun	mon	tue	wed	thu	fri
predicted	predicted	predicted	predicted	predicted	predicted	predicted

Paradox of the execution day

- The man will transmit every evening
    - « execution day is tomorrow »

- The man is like the stalled watch
    - Transmit always the same information
    - Will be right one day
    - In fact no information is transmitted (entropy zero)

Paradox of the 2,009 unfaithful wives of Baghdad

- Disclaimer: this paradox has nothing to do with or against
  - Women…
  - Faith…
  - Iraq…

Paradox of the 2,009 unfaithful wives of Baghdad

- Long time ago, the Caliph of baghdad issued the following order

  - *Not all our women are faithful wives.*

  - *I order that any betrayed husband kills his wife the morning after he discovers his unfortune*

Paradox of the 2,009 unfaithful wives of Baghdad

- At this time, there were 2,009 unfaithful wives in Baghdad.

  - Nothing happen during the 2,008 first mornings
  - The 2,009th morning all the 2,009 unfaithful wives have been killed

Paradox of the 2,009 unfaithful wives of Baghdad

- All the 2,009 unfaithful women killed on the 2,009th morning. How possible?
- Hypothesis:
  - Every husband knows the status of all other husbands in Baghdad
  - He does'nt know his own status. Nobody will tell him.
  - Information is very fast in Baghdad
  - Baghdadis are strict obedient to Caliph order
  - They are also very good mathematicians.
  - No polygamy

Resolution of the Baghdad paradox

- Assume only one unfaithful wife in Baghdad
  - All non betrayed husband sees one unfaithful women in Baghdad
  - The betrayed husband sees none

    -- But the Caliph says that there is at least one

    -- Thus the unfaithful wife is his own wife

    -- He kills her the next morning

Resolution of the Baghdad paradox

- Let assume *n* unfaithful wives
  - Let D_n be the number of days to wait before their killing
  $$D_n = n$$

    - A betrayed husband sees only *n*-1.
    - Since $D_{n-1} = n - 1$, if his wife was faithful, then he should read about n-1 murders in the n-1 th day newspaper
    - Since not: his wife is unfaithful and he kills her the next morning: $$D_n = D_{n-1} + 1$$

Baghdad paradox and information theory

- There is a communication code:
  - Every husband that sees *n* unfaithful wives
    - Observes *n* silent days
    - Kills his wife the *n*+1 th morning
    - The murder appears in newspaper.

Variation on Baghdad paradox

- New Caliph statement:
  - *More than 2,000 wives are unfaithful.*
  - *I order that any betrayed husband kills his wife the morning after he discovered his unfortune*

- The 9th morning, all the 2,009 unfaithful wives are killed

Variation on Baghdad paradox

- If only 2,001 unfaithful wives, then
  - the betrayed husbands see only 2,000

But the Caliph said there are more

Thus his wife is the 2,001st unfaithful wife

He kills her the morning after

$$D_{2,000+n} = n$$
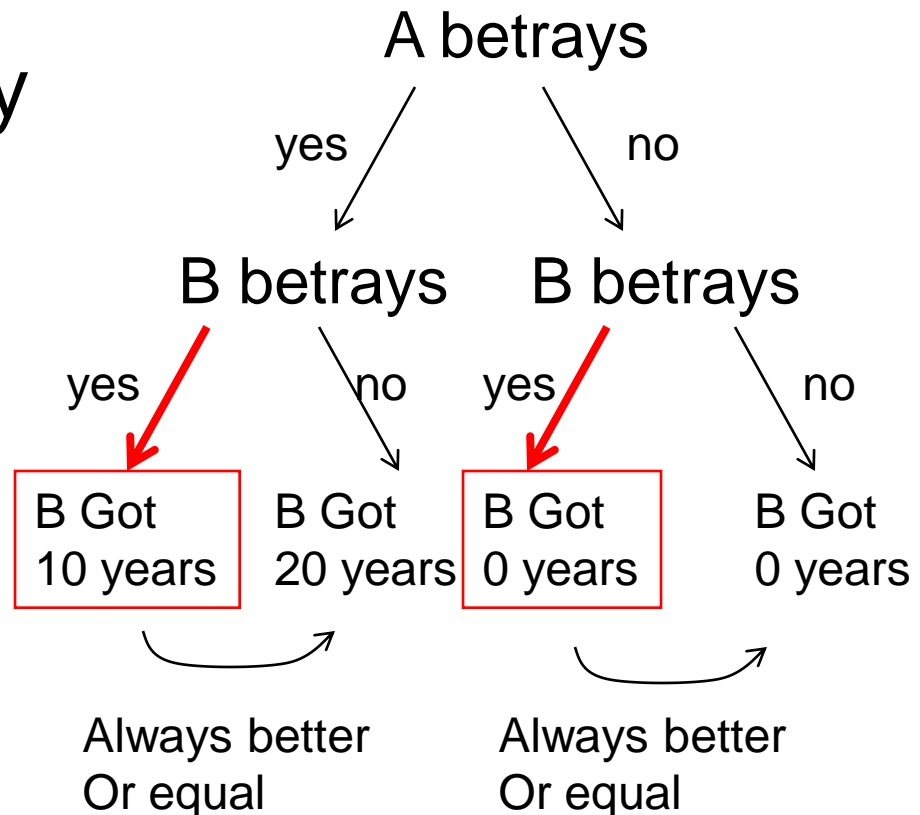
Variation on Baghdad paradox

- But there is no information in the statement:

  - *More than 2,000 wives in Baghdad are unfaithful.*

  - Betrayed husbands see 2,008, unbetrayed see 2,009, both more than 2,000.
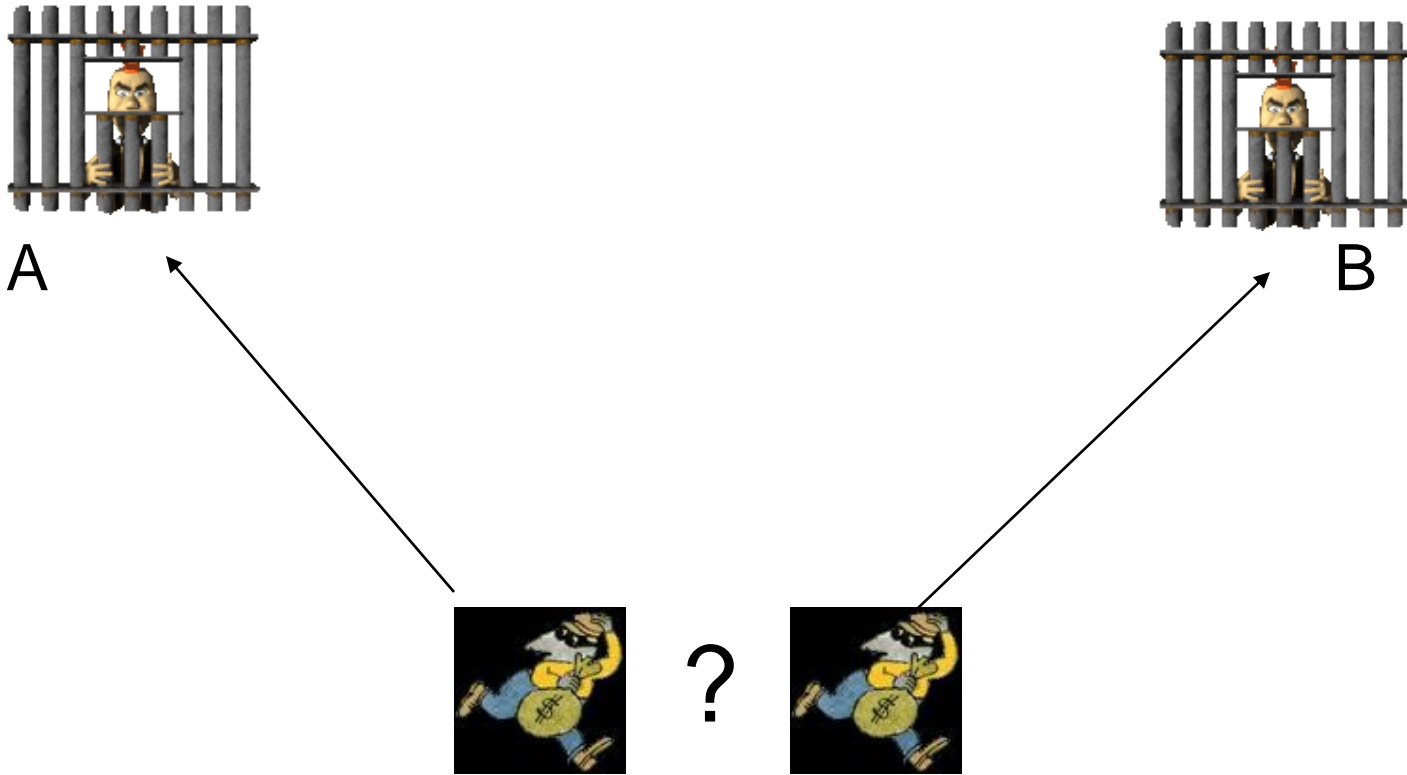 - How can it make a difference?

Prisonner paradox

- Most famous counterexample of Nash equilibrium
  - One robbery;
  - Two suspects, A and B, interrogated in separate prisons, cannot communicate
  - 20 years in jail to share
    - If A and B both betray each other, then 10 years each
    - If only is betrayed by the other:, then got full 20 years
    - If none betrays, zero year.

Prisonner paradox

- Collective objective would be no betrayal
- But both betray
  - B's reasoning

A betrays

yes / no

B betrays   B betrays

yes / no yes / no

| B Got 10 years | B Got 20 years | B Got 0 years | B Got 0 years |

Always better Or equal   Always better Or equal

Prisonner paradox

A   B

?

State(A)=State(B)=betraying if robbery
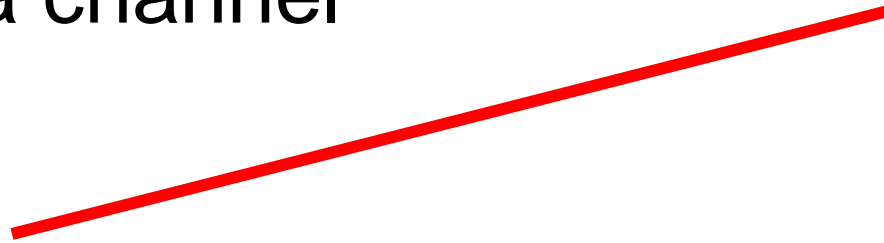State(A)=State(B)= not betraying otherwise

Prisonner paradox

- I(A,B)=H(robbery):
  - If P(robbery)=1/2, then I(A,B)>0
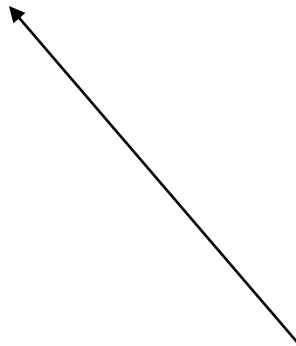 - But there is no channel between A and B

Prisonner paradox
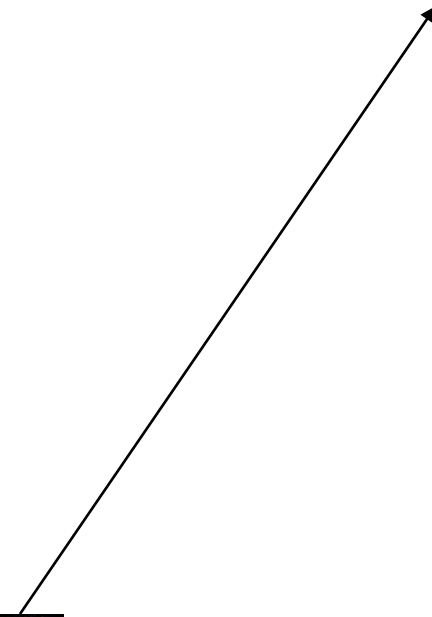
- Assume a channel

A

B

?

Prisonner paradox

- If State(A) can be communicated to B before B decides
  - Collective wealth prevails
  - Always State(A)=State(B)=non betraying
  - I(A,B)=0 whatever P(robbery)
    - No information shared!