A SCHEDULING POLICY WITH MAXIMAL STABILITY REGION FOR RING NETWORKS WITH SPATIAL REUSE

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Abstract

A slotted ring that allows simultaneous transmissions of messages by different users is considered. Such a ring network is commonly called ring with *spatial reuse*. It can achieve significantly higher throughput than standard token rings but it also raises the issue of fairness since some nodes may be prevented from accessing the ring for long time intervals. Policies that operate in cycles and guarantee that a certain number (quota) of packets will be transmitted by every node in every cycle have been considered before to deal with the fairness issue. In this paper we address the problem of designing a policy that results in a stable system whenever the end-to-end arrival rates are within the stability region of the ring with spatial reuse (the stability region of the ring is defined as the set of end-to-end arrival rates for which there is a policy that makes the ring stable). We provide such a policy, which does not require knowledge of end-to-end arrival rates. The policy is an adaptive version of the quota policies and can be implemented with the same distributed mechanism. We use the Lyapunov test function technique together with methods from the theory of regenerative processes to derive our main results.

KEYWORDS: Ring Networks, Spatial Reuse, Stability Analysis, Scheduling, Lyapunov Function Method.

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1 Introduction

We consider a ring with spatial reuse, i.e., a ring in which multiple simultaneous transmissions are allowed as long as they take place over different links (cf. Cidon and Ofek [4], Falconer and Adams [7], Georgiadis *et al.* [8]). Time is divided in slots and each slot is equal to the smallest transmission unit, called packet. We assume zero propagation delay. A node can transmit a packet at the outgoing link at the same time that it receives another packet at the incoming link. A node receiving a packet with destination another node on the ring, may retransmit the packet in the outgoing link *in the same slot*, i.e., the ring has *cut-through* capabilities.

Falconer and Adams [7] and Cidon and Ofek [4] have proposed the following policy for the operation of the ring. Each node is assigned a number called "quota". The policy operates in cycles. A node is allowed to transmit during a cycle as long as the number of transmitted packets does not exceed its assigned quota. An analysis of the throughput characteristics of this policy when all nodes have nonempty queues is provided by Georgiadis *et al.* [8]. The quota policy ensures the fair access to the ring when the packet arrival rates to the nodes fluctuate and may even cause the system to operate in an unstable regime. Since the quotas are fixed, however, this policy does not have maximal stability region (Georgiadis *et al.* [9]). That is, there are end-to-end arrival rates for which the system becomes unstable under the quota policy, while it can be stabilized if other policies are employed.

In this paper we address the problem of designing a policy for the ring with spatial reuse, that has maximal stability region. This type of policies is useful in situations where it can be guaranteed by some higher level process, e.g. reservations, that the traffic streams at the various nodes are well behaved and do not saturate the system. We provide a policy with maximal stability region, which is an adaptive version of the quota policy. The policy is distributed and does not require that the nodes have knowledge of the end-to-end arrival rates or of the states of the other nodes on the ring. During the operation of the system, each node readjusts its quota based only on the size of its queue. We denote such a policy as Π. Specifically, the proposed policy operates in cycles. At the beginning of a cycle each node allocates itself "quota" equal to the number of packets at its buffer. During a cycle a node can transmit no more messages than the quota allocated to it. A cycle ends when *all* the quotas of all nodes are delivered to their destination. The proposed policy requires a distributed mechanism by which every node realizes that the quotas of all nodes have been delivered to their destination and thus a cycle ends. Such a mechanism is provided in the paper by Falconer and Adams [7].

Related work on the stability of the ring has been done by Bambos and Nquyen [2]. The system is studied for stationary arrival processes and a policy is proposed that equalizes the arrival and departure rates as long as the arrival rates belong to the stability region. We study the ring with i.i.d. arrivals, markov modulated arrivals, and arrivals with deterministic burstiness bounds. The policy proposed in this paper is considerably simpler to implement than the policy proposed by Bambos and Nquyen [2] and guarantees existence of the lth moment of the queue lengths when the $(l+1)$ moment of the number of packets arrived in a slot exists, while for deterministic arrivals it guarantees bounded backlogs.

Another policy that has attracted attention is the the so called "greedy" policy under which a nonempty node is instructed to transmit whenever it finds an available slot. Sufficient conditions for stability of the greedy policy are provided in Cruz [6] and Yaron and Sidi [12]. These conditions, however, are for special traffic destinations and are stronger than necessary. For arrivals with deterministic burstiness bounds and general destinations, it has been shown recently by Tassiulas and Georgiadis [11], that any work conserving policy, in particular the greedy policy, has maximal stability region. In a recent study, Coffman *et al.* [5] proved stability of the greedy policy under stochastic arrivals, but for symmetric rings and for specific destination distributions, and provided asymptotic results on the queue lengths as the number of nodes increases. However, the stability of the greedy policy for stochastic arrivals with general destinations remains still an open problem.

The paper is organized as follows. In the next section we present our main results and their consequences. In particular, we establish the stability region for the adaptive policy, and show that it is maximal. In Section 3 we present the proof of the main result. In Section 4 we show that the main results remain valid for models that involve correlated arrivals.

2 The system model the policy and the main result

The network consists of a number of nodes arranged in a circle. Each node i receives exogenous traffic with destination potentially any other node j of the ring. The packets are queued in the nodes until transmission. All the transmissions follow the unique direction of the ring. The packet transmission time is constant and the transmissions of all nodes are synchronized. The time is slotted. A packet transmitted by node i is transferred within one slot to its destination node j unless there is some node k preceding j in the transmission direction of the ring, which is transmitting another packet in the same slot. In the latter case, node k stores node's i packet for later transmission. Therefore, the parts of the ring traversed by different packets at the same slot, are non-overlapping. The decisions of whether a node will or will not transmit in each slot, as well as the service discipline among the different traffic streams in the node is determined by the transmission policy. The following policy is proposed here.

Transmission policy Π

The policy operates in cycles and is based on the idea of allocating quotas to the nodes. Let τ_k be the beginning of the kth cycle and set $\tau_1 = 1$. At time τ_k each node allocates itself "quota" $\nu_i(k) = Q_i(\tau_k)$, where $Q_i(t)$ is the queue size of node i at time t. Node i can transmit up to $\nu_i(k)$ packets during cycle k according to *any fixed nonidling* policy, i.e., the only restriction that is imposed on the transmissions is that the node transmits a packet in its outgoing link whenever either its queue is nonempty, or a message is received in the same slot in its incoming link with destination another node on the ring. Cycle k ends when *all* the quotas of all nodes are delivered to their destination.

Remarks:

1. The most important nonidling transmission policy for applications, is the policy where a node always gives nonpreemptive priority to the packets that arrive at the incoming link with destination another node. This way, only a single buffer capable of holding a maximum packet size message is needed to hold the traffic that arrives at the incoming link of a node. For details see the paper by Cidon and Ofek [4].

2. The proposed policy requires a distributed mechanism by which every node realizes

that the quotas of all nodes are delivered to their destinations. Such a mechanism, which can be easily adapted to the model considered in this paper, is provided in the paper by Falconer and Adams [7]. The implementation of this mechanism will increase the cycle length by two slots and does not alter the stability region of the policy. For simplicity in the exposition we do not consider the extra slots needed to implement this mechanism.

We introduce some notation before we proceed. Let M be the number of nodes and set $\mathcal{M} = \{1, \ldots, M\}$. The operations $i \oplus j$ and $i \ominus j$ denote respectively, addition and subtraction modulo M, with the convention that index 0 refers to node M. Furthermore, when i, j refer to node indices we denote $\sum_{k=i}^{j} x_k := x_i + x_{i\oplus 1} + \ldots + x_{j\ominus 1} + x_j$. We assume that the nodes are arranged on the ring according to their index so that the outgoing link to node i is the incoming link for node $i \oplus 1$. Node i may receive external traffic with destination any other node j in the system. Let $R_{ij}(t)$ be the number of packets that arrive at node i from the outside with destination node j at time t. If $i = j$, then it is assumed that the packet has to cross all the nodes on the ring until it is received by the originating node, i.

Throughout this section we adopt the following assumption.

(A) The vector process $\{\mathbf{R}(t)\}_{t=1}^{\infty}$, where $\mathbf{R}(t) = \{R_{ij}(t),\ i, j \in \mathcal{M}\}$, consists of i.i.d. vectors. We denote $R_{ij} := R_{ij}(1)$. Note that do not make any independence assumptions for the work arriving in various nodes at the same slot. To avoid technical difficulties we will also assume that $\Pr(R_{ij}(t) = 0,\ i, j \in \mathcal{M}) > 0$.

In section 4 we will see that the above assumption can be relaxed in certain ways without affecting significantly the validity of our results. In order to formulate our main results in a compact form, we need some additional notation. Let

$$\rho_{ij} := \lim_{n\to\infty} \frac{\sum_{t=1}^{n} R_{ij}(t)}{n} = ER_{ij}$$

be the (end-to-end) arrival rate of packets that arrive to node i with destination node j. We also define $\alpha_{im} := \sum_{j=m\oplus 1}^{i} \rho_{ij}$ and $r_m = \sum_{i\in\mathcal{M}} \alpha_{im}$. Note that α_{im} is the average number of packets per slot that are generated by node i and have to cross node m in order to reach their destination. Therefore, r_m is the average number of packets that cross node m during a slot. Finally, we set $r = \max\{r_m : m \in \mathcal{M}\}$.

Since at most one packet can be transmitted in a slot by node m, the condition $r_m \leq 1$, $m \in \mathcal{M}$, is necessary for stability. Therefore, the stability region of any policy is a subset of the region

$$\mathcal{R} = \{\boldsymbol{\rho} = (\rho_{ij})_{i,j=1}^{M} : r = \max_{1 \leq m \leq M} r_m \leq 1\}$$

In this paper we show that as long as the end-to-end arrival rates belong to

$$\mathcal{R}^o = \{\boldsymbol{\rho} : r = \max_{1 \leq m \leq M} r_m < 1\},$$

policy Π stabilizes the network in a strong sense. Specifically we show that (i) the queue length $Q_i(t)$ possesses a limiting distribution; (ii) its lth moment $EQ_i^l(t)$ as $t \to \infty$ exists provided that $ER_{ij}^{l+1} < \infty$; (iii) the queue length $Q_i(t)$ as $t \to \infty$ has an exponential tail (i.e., large backlogs are very unlikely), provided that the same is true for R_{ij}. We summarize our main results in the following theorem. Its proof is presented in the next section.

Theorem 1 (i) *Under policy* Π, *the process of queue lengths* $\{Q_i(t),\ i \in \mathcal{M}\}_{t=1}^{\infty}$ *converges in distribution to a random vector* $\{\widetilde{Q}_i,\ i \in \mathcal{M}\}$ *having a honest distribution, if* $r < 1$.

(ii) *If* $r < 1$ *and* $ER_{ij}^{l+1} < \infty$ *for some* $l \geq 1$ *and all* $i, j \in \mathcal{M}$, *then*

$$\lim_{t \to \infty} EQ_i^l(t) = E\widetilde{Q}_i^l < \infty \tag{1}$$

(iii) *If* $r < 1$ *and for* $\vartheta > 0$ *the moment generating function of* R_{ij} *exists, that is,* $E\exp(\vartheta R_{ij}) < \infty$ *for all* $i, j \in \mathcal{M}$, *then there exists* $\vartheta' > 0$ *such that*

$$E\exp(\vartheta' \widetilde{Q}_i) < \infty \tag{2}$$

for every $i \in \mathcal{M}$. ∎

Remark.

As a direct consequence of Theorem 1 and Chebyshev's Inequality, we can estimate the tail of the queue length distribution. In particular, under the hypothesis of Theorem 1(ii), the tail of the queue length distribution decays polynomially fast, that is, for some constant $C > 0$

$$\Pr\{\widetilde{Q}_i > k\} \leq \frac{C}{k^l} \,.$$

While, under the hypothesis of Theorem 1(iii), the queue length $\widetilde{Q}_i$ has an exponential tail, that is,

$$\Pr\{\widetilde{Q}_i > k\} \leq C e^{-\vartheta k}$$

for some $C > 0$, $\vartheta > 0$ and for all $k \geq 0$.

Another consequence of Theorem 1 is a coupling property that holds between the queue length process and a stationary version of it. The queue length process in our system is regenerative and from Theorem 1 it has a stationary distribution; hence a stationary version of it exists and let this be $\{Q_i^s(t), i \in \mathcal{M}\}_{t=-\infty}^{\infty}$. Let $\{Q_i(t), i \in \mathcal{M}\}_{t=0}^{\infty}$ be the queue length process when the system starts from an arbitrary state and the time instant $t = 0$ is the beginning of a scheduling cycle.

Corollary 1 *There is a process $\hat{\mathbf{Q}}(t)$ with the same probability law as $\mathbf{Q}(t)$ for which the following coupling relationship holds*

$$\lim_{t_0 \to \infty} P(\hat{\mathbf{Q}}(t) = \mathbf{Q}^s(t), \forall t \geq t_0) = 1.$$

Proof: Let $\{\tau_n^s\}_{n=1}^{\infty}$ be the regeneration points of the stationary process which are greater than zero. Let $\{\tau_n\}_{n=1}^{\infty}$ be the regeneration points of the system starting at time 0 and operating independently of the stationary process. Consider the random time defined as

$$\sigma = \min\{n : \tau_n = \tau_m^s, \text{ for some } m\}.$$

By slightly modifying the coupling theorem for continuous time renewal processes in Asmussen [1], pp.144, and using the fact that the discrete-time renewal processes $\{\tau_n^s\}_{n=1}^{\infty}$, $\{\tau_n\}_{n=1}^{\infty}$ are aperiodic, it follows that σ is finite almost surely. Let

$$\hat{\mathbf{Q}}(t) = \begin{cases} \mathbf{Q}(t), & t \leq \tau_\sigma \\ \mathbf{Q}^s(t), & t > \tau_\sigma. \end{cases}$$

It is easy to see that the process $\{\hat{Q}_i(t), i \in \mathcal{M}\}_{t=1}^{\infty}$ has the same probability law as $Q(t)$, while by construction and because of the fact that σ is almost surely finite

$$\lim_{t_0 \to \infty} P(\hat{\mathbf{Q}}(t) = \mathbf{Q}^s(t), \forall t \geq t_0) = 1.$$

Hence the corollary follows.■

3 Stability Analysis

Let $Q_{ij}(t)$ be the number of packets at node i with destination node j, including both external packets and packets received from the incoming link to node i. Clearly, $Q_i(t) = \sum_{j \in \mathcal{M}} Q_{ij}(t)$. Let also $\mathbf{Q}(t) := \{Q_{ij}(t) : \ i, j \in \mathcal{M}\}$. From the operation of the policy and assumption (A) we conclude that the process $\{\mathbf{Q}(\tau_n)\}_{n=1}^{\infty}$ is an embedded Markov chain. We first obtain some properties of the embedded Markov chain $\mathbf{Q}(\tau_n)$ and based on those we prove that the process $\mathbf{Q}(t)$ is stable in the sense defined in Theorem 1.

Analysis of the Embedded Markov Chain

We prove that under the condition of Theorem 1 the process $\{\mathbf{Q}(\tau_n)\}_{n=1}^{\infty}$ converges weakly to a honest random vector $\hat{\mathbf{Q}}$. Since by assumption $\Pr(R_{ij}(t) = 0, \ i, j \in \mathcal{M}) > 0$, it can be seen that the embedded Markov chain has only one irreducibility set and if restricted to this set, the chain is aperiodic. To prove ergodicity of the embedded Markov chain $\mathbf{Q}(\tau_n)$ we use the Lyapunov function method. Specifically, we use the following result of Meyn and Tweedie [10] , which we present in a form appropriate for the problem under consideration.

Theorem 2 *Suppose that $\{X_n\}_{n=1}^{\infty}$ is an aperiodic and irreducible Markov chain with countable state space S. Let $f(x)$ be a non-negative real function on the state space. If A is a finite set such that $f(x) \geq \epsilon > 0, \ x \in A^c$,*

$$E(f(X_2)|X_1 = x) < \infty, \quad x \in A \tag{3}$$

and for some $\delta > 0$,

$$E(f(X_2)|X_1 = x) < (1 - \delta) f(x), \quad x \in A^c, \tag{4}$$

then the Markov chain is ergodic and

$$Ef(\hat{X}) < \infty,$$

where $\hat{X}$ *has the steady state distribution of the Markov chain* $\{X_n\}_{n=1}^{\infty}$. ∎

In order to apply the above result, one must find a suitable function $f(\cdot)$ of $\mathbf{Q}(\tau_n)$ such that (3) and (4) hold. As we will see in the following, such a function is the maximum of the *node degrees*. The degree of node i at time t, $N_i(t)$, is defined as the total number of packets on the ring at time t that have to cross node i in order to reach their destination. Let $N(t) := \max\{N_i(t) : \ i \in \mathcal{M}\}$. The following result of Georgiadis *et al.* [8] is essential to our analysis.

Theorem 3 *For any of the policies* Π *defined above, the cycle length* $T_k := \tau_{k+1} - \tau_k$ *satisfies*

$$T_k = \max\{1, N(\tau_k)\} \tag{5}$$

for all $k \geq 1$.

Proof: We give here a more straightforward proof of this result than the one appeared in Georgiadis *et al.* [8]. It is based on the following observation: Provided that new arrivals after time τ_k are not included in the calculation of node degrees

$$N(t) - N(t+1) = 1 \quad \text{if} \quad N(t) > 0, \ \ \tau_k < t < \tau_{k+1} \tag{6}$$

Since the time at which the quota of all nodes are delivered to their destination is the first time after τ_k at which $N(t) = 0$, provided that new arrivals are not included in the calculation of node degrees, the theorem will follow directly from (6). Note that since there are no arrivals in the ring, (6) is equivalent to the following.

$$N_i(t) - N_i(t+1) = 1 \quad \text{if} \quad N_i(t) = N(t) > 0, \quad i = 1, .., M \tag{7}$$

If node i is nonempty at t, then (7) is obvious. Assume now that node i is empty at t and satisfies $N_i(t) = N(t)$. Let j be the first nonempty node "upstream" from node i, i.e., in the opposite direction of the ring transmission direction. The packet transmitted by j at time t has to cross node i, since otherwise the degree on node j will be strictly larger than the degree of node i which contradicts the fact that $N_i(t) = N(t)$. This implies that node i will transmit in its outgoing link at time t the packet sent by node j. Since no new arrivals are assumed, the degree of node i decreases by 1. ■

Clearly, the node degree $N(t)$ is a function of the queue length $\mathbf{Q}(\tau_n)$. To apply Theorem 2 we either define $f_l(\mathbf{Q}(\tau_n)) = N^l(\tau_n)$ or $f(\mathbf{Q}(\tau_n)) = \exp(\vartheta N(\tau_n))$ for some $\vartheta > 0$, and we then prove (cf. Corollary 2) that (4) of Theorem 2 holds for this choice of functions. For this, we need the following two lemmas which are consequences of Theorem 3. To preserve the clarity of the main line of the argument, the proofs of these lemmas are given in the appendix.

Lemma 1 *If for some $l \geq 1$ we have $ER_{ij}^l < \infty$ for all $i, j \in \mathcal{M}$, then*

$$\lim_{n\to\infty} E\left(\left(\frac{N(\tau_2)}{T_1}\right)^l \middle| T_1 = n\right) = \max\left\{r_m^l :\ m \in \mathcal{M}\right\}.$$

Lemma 2 *If $r < 1$ and $E\exp(\vartheta R_{ij}) < \infty$ for some $\vartheta > 0$ and all $i, j \in \mathcal{M}$, then there exists $\vartheta' > 0$ such that*

$$\lim_{n\to\infty} E\left(\left(\frac{\exp(\vartheta' N(\tau_2))}{\exp(\vartheta' T_1)}\right) \middle| T_1 = n\right) = 0.$$

¿From Lemmas 1 and 2 we easily conclude our next result that actually establishes (4) of Theorem 2.

Corollary 2 (i) *If $r < 1$ and $ER_{ij}^l < \infty$ for all $i, j \in \mathcal{M}$ and some $l \geq 1$, then there exist $\delta > 0$ and $B > 0$ such that*

$$E(N^l(\tau_2)|N(\tau_1) = n) \leq (1-\delta)n^l \qquad n \geq B\ .$$

(ii) *If $r < 1$ and $E\exp(\vartheta R_{ij}) < \infty$ for some $\vartheta > 0$ and all $i, j \in \mathcal{M}$, then there exists $\vartheta' > 0$ such that*

$$E\left(\exp(\vartheta' N(\tau_2))|N(\tau_1) = n\right) \leq (1-\delta)\exp(\vartheta' n) \qquad n \geq B\ .$$

Proof. Since $r_m \leq r < 1$, we have that $\max\{r_m^l : m \in \mathcal{M}\} \leq r < 1 - \delta$, $\delta > 0$. Using this observation, part (i) of the Corollary 2 follows directly from from (5) and Lemma 1. Part (ii) follows directly from (5) and Lemma 2. ■

Some further notation should be introduced in order to state the properties of the embedded Markov chain, to be used in the proof of Theorem 1. Let $\mathbf{Q}(\tau_1) = \mathbf{0}$, and we define two stopping times, namely: θ_k and $\mathcal{T}_k$. For the former we set $\theta_1 = 1$, and then

$$\theta_{k+1} := \inf\{n > \theta_k : \mathbf{Q}(\tau_n) = 0\} \ . \tag{8}$$

For the latter we set $\mathcal{T}_0 = 1$ and

$$\mathcal{T}_{k+1} = \min\{\tau_l : \ \tau_l > \mathcal{T}_k \ \text{ such that } \ \mathbf{Q}(\tau_l) = 0\} \ . \tag{9}$$

Note that $\mathcal{T}_k = \tau_{\theta_k}$. It will follow from Theorem 4 proved below that the times $\mathcal{T}_j$ are well defined for all k since the system will empty infinitely often almost surely. Let also $d_k = \theta_{k+1} - \theta_k$, and $D_k = \mathcal{T}_{k+1} - \mathcal{T}_k$. Clearly, d_k and $\mathcal{T}_k$ for $k = 1, \ldots,$ are i.i.d. Now, we are ready to prove some basic properties for the embedded Markov chain $\mathbf{Q}(\tau_n)$.

Theorem 4 (i) *If $r < 1$ then the Markov chain $\{\mathbf{Q}(\tau_n)\}_{n=1}^{\infty}$ is ergodic and*

$$E \sum_{n=1}^{\theta_2 - 1} N(\tau_n) < \infty \ . \tag{10}$$

(ii) *If $r < 1$ and for $l \geq 2$ we have $ER_{ij}^l < \infty$ for all $i, j, \in \mathcal{M}$, then*

$$E \sum_{n=1}^{\theta_2 - 1} N^l(\tau_n) < \infty \ , \tag{11}$$

(iii) *If $r < 1$ and for some $\vartheta > 0$ we have $E\exp(\vartheta R_{ij}) < \infty$ for all $i, j \in \mathcal{M}$, then there exists $\vartheta' > 0$ such that*

$$E \sum_{n=1}^{\theta_2 - 1} \exp(\vartheta' N(\tau_n)) < \infty \ . \tag{12}$$

Proof. Define $f_l(\mathbf{Q}(\tau_n)) = N^l(\tau_n)$, $\ l \geq 1$ and let $B \geq 1$. Clearly, the set $A := \{\mathbf{Q} : f_l(\mathbf{Q}) < B\}$ is finite. Also, if $\mathbf{Q}(\tau_1) \in A$, then since by (5) $T_1 = N(\tau_1) + 1 < B^{(1/l)+1}$, using arguments similar to those used in the proof of Lemma 1 it can be easily seen that

$$E\left(N^l(\tau_2) | \mathbf{Q}(\tau_1) = \mathbf{Q} \in A\right) < \infty,$$

provided that $ER_{ij}^l < \infty$. From the above discussion, Corollary 2 and Theorem 2 we conclude that $\{\mathbf{Q}(\tau_n)\}_{n=1}^{\infty}$ is ergodic, and provided that $ER_{ij}^l < \infty$, $i,j \in \mathcal{M}$ for some $l \geq 1$,

$$E\hat{N}^l < \infty,$$

where $\hat{N}^l = f_l(\hat{\mathbf{Q}})$, and $\hat{\mathbf{Q}}$ has the steady state distribution of $\{\mathbf{Q}(\tau_n)\}_{n=1}^{\infty}$. Now observe that the sequence $\{N(\tau_n)\}_{n=1}^{\infty}$ is regenerative with respect to the renewal sequence $\{\theta_n\}_{n=1}^{\infty}$. Since the ergodicity of $\{\mathbf{Q}(\tau_n)\}$ implies $Ed_1 < \infty$, from the regenerative theorem, Asmussen [1, Corollary 1.4] and the fact that $N(t)$ is non-negative, we have that

$$\frac{E\sum_{n=1}^{\theta_2-1} N^l(\tau_n)}{Ed_1} = E\hat{N}^l < \infty.$$

for every $l \geq 1$.

For part (iii), the proof is along the same lines with $f(\mathbf{Q}(\tau_n)) = \exp(\vartheta N(\tau_n))$. ■

Proof of Theorem 1

Assume that $\mathbf{Q}(1) = \mathbf{0}$. Consider the times $\mathcal{T}_k$, $k = 0, 1, ..$ defined in (9). The process $\{\mathbf{Q}(t)\}_{t=1}^{\infty}$ is regenerative with respect to the renewal process $\{\mathcal{T}_n\}_{n=1}^{\infty}$. From (5) and Theorem 4 we have

$$ED_1 = \sum_{n=1}^{\theta_2-1} T_n \leq 1 + E\sum_{n=1}^{\theta_2-1} N(\tau_n) < \infty.$$

Since the assumption $\Pr(R_{ij}(t) = 0,\ i,j \in \mathcal{M}) > 0$ implies that D_k is aperiodic, applying the regenerative theorem we conclude that $\{\mathbf{Q}(t)\}_{t=1}^{\infty}$ converges in distribution to a honest random variable $\widetilde{\mathbf{Q}}$, thus establishing part (i) of the theorem.

Let now $F(\cdot)$ be a nonnegative nondecreasing function (i.e., in our case either $F(x) = x^l$ or $F(x) = \exp(\vartheta x)$). Using the non-negativity of $Q_i(t)$, $N(t)$, problem 1.4, chapter 5 in Asmussen [1]), we conclude that

$$\lim_{t\to\infty} E\left[F(Q_i(t))\right] = E\left[F(\widetilde{Q}_i)\right] \leq E\left[F(\widetilde{N})\right] = \frac{E\sum_{t=1}^{\mathcal{T}_1-1} F(N(t))}{ED_1}. \tag{13}$$

Parts (ii) and (iii) of Theorem 1 will follow from (13) if we show that

$$E \sum_{t=1}^{\mathcal{T}_1 - 1} F(N(t)) < \infty,$$

under the specified conditions. Observe that we can write

$$\begin{aligned}
\sum_{t=1}^{\mathcal{T}_1 - 1} N^l(t) &\leq \sum_{k=1}^{\theta_2 - 1} T_k \left[N(\tau_k) + A(\tau_{k+1}) - A(\tau_k) \right]^l, \\
&\leq 2^{l-1} \sum_{k=1}^{\theta_2 - 1} T_k \left(N^l(\tau_k) + (A(\tau_{k+1}) - A(\tau_k))^l \right),
\end{aligned}$$

and, in view of (25),

$$\sum_{t=1}^{\mathcal{T}_1 - 1} \exp(\vartheta N(t)) \leq \sum_{k=1}^{\theta_2 - 1} T_k \left[\exp(2\vartheta N(t)) + \exp(2\vartheta(A(\tau_{k+1} - A(\tau_k))) \right] . \tag{14}$$

So, it suffices to show that

$$E \sum_{k=1}^{\theta_2 - 1} T_k F(N(\tau_k)) \ < \ \infty \tag{15}$$

and

$$E \sum_{k=1}^{\theta_2 - 1} T_k F(A(\tau_{k+1}) - A(\tau_k)) \ < \ \infty. \tag{16}$$

where either $F(x) = x^l$ or $F(x) = \exp(2\vartheta x)$.

Since by (5) $T_k = N(\tau_k)$ whenever $N(\tau_k) \geq 1$, we have

$$E \left(\sum_{k=1}^{\theta_2 - 1} T_k N^l(\tau_k) \right) \leq 1 + E \sum_{k=1}^{\theta_2 - 1} (N(\tau_k))^{l+1}, \tag{17}$$

and

$$E \left(\sum_{k=1}^{\theta_2 - 1} T_k \exp(2\vartheta N(\tau_k)) \right) \leq C(\vartheta) E \sum_{k=1}^{\theta_2 - 1} \exp(3\vartheta N(\tau_k)) \ , \tag{18}$$

where in the latter inequality we use the fact that $T_k \leq C(\vartheta) \exp(\vartheta T_k)$, for some constant $C(\vartheta)$. Based on (17) and (18) it is easy to see from Theorem 4 that (15) holds for both choices of the function $F(x)$) and appropriate choice of ϑ'.

We now concentrate on proving (16) for $F(x) = x^l$. Let $\mathcal{G}_k$ denote the sigma-field generated by $\mathbf{Q}(\tau_k)$, $k = 1, 2, \cdots$ and observe that θ_2 is a $\mathcal{G}_k$-stopping time. Using successively the facts that $\{\theta \geq k+1\} \in \mathcal{G}_k$, the process $\{\mathbf{Q}(\tau_k)\}_{k=1}^{\infty}$ is Markov and $T_k = \max(1, N(\tau_k))$ is $\mathcal{G}_k$-measurable, we get

$$\begin{aligned}
E \sum_{k=1}^{\theta_2 - 1} T_k (A(\tau_{k+1}) - A(\tau_k))^l &= \sum_{k=1}^{\infty} E\left[T_k (A(\tau_{k+1}) - A(\tau_k))^l 1_{\{\theta_2 - 1 \geq k\}}\right] \\
&= \sum_{k=1}^{\infty} E\left[E\left[T_k (A(\tau_{k+1}) - A(\tau_k))^l \mid \mathcal{G}_k\right] 1_{\{\theta_2 \geq k+1\}}\right] \\
&= \sum_{k=1}^{\infty} E\left[E\left[T_k (A(\tau_{k+1}) - A(\tau_k))^l \mid \mathbf{Q}(\tau_k)\right] 1_{\{\theta_2 \geq k+1\}}\right] \\
&= \sum_{k=1}^{\infty} E\left[E\left[(A(\tau_{k+1}) - A(\tau_k))^l \mid \mathbf{Q}(\tau_k)\right] T_k 1_{\{\theta_2 \geq k+1\}}\right] \quad (19)
\end{aligned}$$

Arguments similar to those used in the proof of Lemma 1, show that

$$(A(\tau_{k+1}) - A(\tau_k))^l \leq C_1 \sum_{i,j \in \mathcal{M}} \left(\sum_{t=\tau_k}^{\tau_{k+1}} R_{ij}(t) \right)^l,$$

where C_1 depends only on M and l. Therefore,

$$E\left[(A(\tau_{k+1}) - A(\tau_k))^l \;\middle|\; \mathbf{Q}(\tau_k)\right] \leq C_1 \sum_{i,j \in \mathcal{M}} E\left[\left(\sum_{t=\tau_k}^{\tau_{k+1}} R_{ij}(t)\right)^l \;\middle|\; \mathbf{Q}(\tau_k)\right].$$

Since by assumption (A.1) $R_{ij}(t),\ t = 1, 2, \cdots$ are i.i.d and $ER_{ij}^l < \infty$, using corollary 10.3.2 in Chow-Teicher [3], we conclude that for $l \geq 2$,

$$E\left[\left(\sum_{t=\tau_k}^{\tau_{k+1}} R_{ij}(t)\right)^l \;\middle|\; \mathbf{Q}(\tau_k)\right] \leq C_2 T_k^l,$$

where C_2 depends only on M, l and ER_{ij}^l. Clearly, the same inequality is true for $l = 1$. Using these estimates in (19), we finally have,

$$E \sum_{k=1}^{\theta_2 - 1} T_k [A(\tau_{k+1}) - A(\tau_k)]^l \leq CE\left(\sum_{k=1}^{\infty} T_k^{l+1} 1_{\{\theta_2 \geq k+1\}}\right)$$

$$
\begin{aligned}
&= CE\left(\sum_{k=1}^{\theta_2-1} T_k^{l+1}\right) \\
&\le CE\left(1+\sum_{k=1}^{\theta_2-1} (N(\tau_k))^{l+1}\right) \\
&< \infty,
\end{aligned}
\tag{20}
$$

where the last inequality follows from Theorem 4.

Now we focus on proving (16) for $F(x) = \exp(\vartheta x)$. We can use the same arguments as before together with (25) to obtain

$$
\begin{aligned}
E\sum_{k=1}^{\theta_2-1} T_k \exp(\vartheta(A(\tau_{k+1}) - A(\tau_k))) &\le C_1 E \sum_{k=1}^{\theta_2-1} \exp\left(\vartheta M \sum_{t=\tau_k}^{\tau_{k+1}} R_{ij}(t)\right) \\
&\le C_2 \sum_{k=1}^{\theta_2-1} E\exp(\vartheta' T_k) \\
&\le C_2 \sum_{k=1}^{\theta_2-1} E\exp(\vartheta' N(\tau_k)) + C_2\exp(\vartheta') \\
&< \infty.
\end{aligned}
$$

This completes the proof of the lemma, and also our main result Theorem 1. ■

4 Correlated arrival models

In the previous sections we assumed that packet arrivals are independent from slot to slot. In this section we show that the stability properties of the adaptive policy Π are maintained for other arrival models as well. Specifically we consider arrivals with bounded burstiness and Markov modulated arrivals.

In the arrival model with bounded burstiness it is assumed that for each arrival stream $\{R_{ij}(t)\}_{t=1}^{\infty}$ there are numbers ρ_{ij}, b_{ij} such that

$$
\sum_{t=t_1}^{t_2} R_{ij}(t) \le \rho_{ij}(t_2 - t_1) + b_{ij} \tag{21}
$$

If the vector $\{\rho_{ij}\}$ lie in region $\mathcal{R}^0$ then the system is stable under Π in the sense that the backlogs are uniformly bounded over time. To see this notice that by the definition of $N_m(t)$, relation (5) and inequality (21), we have that

$$\begin{aligned} N_m(\tau_{k+1}) &\leq r_m T_k + \sum_{i,j} b_{ij} \\ &\leq r_m(N(\tau_k)+1) + \sum_{i,j} b_{ij}, \end{aligned}$$

where the r_m's are defined in terms of the ρ_{ij}'s in the same manner as in the definition of $\mathcal{R}$ and T_k, τ_k are the same as in (5). Therefore,

$$N(\tau_{k+1}) \leq rN(\tau_k) + B + 1,$$

where $B = \sum_{i,j} b_{ij}$ and $r = \max\{r_m : \ m \in \mathcal{M}\} < 1$. We conclude that if the vector of ρ_{ij}'s lies in the region $\mathcal{R}$ then,

$$N(\tau_k) \leq \frac{B+1}{1-r} + r^k N(\tau_1),$$

Since $N_m(t) \leq N_m(\tau_k) + N_m(\tau_{k+1})$ whenever $\tau_k < t < \tau_{k+1}$, we can easily extend the previous bound for an arbitrary time t.

The proof of stability that we gave when the arrivals are i.i.d. goes through in the more general case where the arrivals are Markov modulated. Consider the following Markov modulated arrival model. There is a finite irreducible Markov chain $\{u(t)\}_{t=1}^{\infty}$ with state space $\mathcal{U}$ and a family of distributions $\{F_u : u \in \mathcal{U}\}$ such that the conditional distribution of $R_{ij}(t)$ given $u(t)$ is $F_{u(t)}$. Furthermore $R_{ij}(t)$ is independent of $\{R_{ij}(\tau) : \tau < t\}$ given $u(t)$. Assume finally that $\{u(t)\}_{t=1}^{\infty}$ is stationary therefore $\{R_{ij}(t)\}_{t=1}^{\infty}$ is stationary as well. With the above assumptions parts (i) and (ii) of Theorem 1 holds with minor modifications in the proofs. The only difference is that the queue length process at the beginnings of a cycle is not a Markov chain any more. However, the combination $(Q(\tau_n), u(\tau_n))$ of the queue length vector with the modulating chain constitutes a Markov chain and the proofs can be carried through based on this chain.

A Appendix

Proof of Lemma 1

According to the policy, the queue size at node i at time τ_2 consists of all the external packets that arrive in the interval $(1, \tau_2]$ to node i. From the definition of the degree of a node it follows that if $T_1 = n$,

$$N_m(\tau_2) = \sum_{t=1}^{n} \sum_{i \in \mathcal{M}} \sum_{j=m\oplus 1}^{i} R_{ij}(t). \tag{22}$$

Using the strong law of large numbers we conclude that

$$\lim_{n\to\infty} \frac{N_m(\tau_2)}{n} = \sum_{i \in \mathcal{M}} \sum_{j=m\oplus 1}^{i} r_{ij} = \sum_{i \in \mathcal{M}} \alpha_{im}.$$

Now let $F(\cdot)$ be a non-decreasing continuous function. In view of the above, we have for almost all sample paths,

$$\begin{aligned}
\lim_{n\to\infty} F(N(\tau_2)/n) &= \lim_{n\to\infty} F\left(\max\left\{N_m(\tau_2)/n :\ m \in \mathcal{M}\right\}\right) \\
&= \lim_{n\to\infty} \max\left\{F(N_m(\tau_2)/n) :\ m \in \mathcal{M}\right\} \\
&= \max\left\{\lim_{n\to\infty} F(N_m(\tau_2)/n) :\ m \in \mathcal{M}\right\} \\
&= \max\left\{F(r_m) :\ m \in \mathcal{M}\right\}.
\end{aligned} \tag{23}$$

The lemma will follow from (23) with $F(x) = x^l$, $l \geq 1$, if we show that the sequence $\{(N(\tau_2)/n)^l\}$ is uniformly integrable (u.i.).

Using the mean inequality

$$\left(\frac{\sum_{i=1}^{k} |a_i|}{k}\right)^l \leq \frac{\sum_{i=1}^{k} |a_i|^l}{k}, \quad l \geq 1,$$

we have

$$\begin{aligned}
N_m^l(\tau_2) &= \left(\sum_{i \in \mathcal{M}} \sum_{j=m\oplus 1}^{i} \left(\sum_{t=1}^{n} R_{ij}(t)\right)\right)^l \\
&\leq M^{2(l-1)} \sum_{i \in \mathcal{M}} \sum_{j=m\oplus 1}^{i} \left(\sum_{t=1}^{n} R_{ij}(t)\right)^l .
\end{aligned}$$

Therefore,

$$\begin{aligned}\left(\frac{N_m(\tau_2)}{n}\right)^l &\leq M^{2(l-1)} \sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} \left(\frac{\sum_{t=1}^{n} R_{ij}(t)}{n}\right)^l \\ &\leq M^{2(l-1)} \sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} \frac{\sum_{t=1}^{n} R_{ij}^l(t)}{n} \end{aligned} \tag{24}$$

Since by assumption $ER_{ij}^l(1) < \infty$ and the variables $\{R_{ij}(t)\}_{t=1}^{\infty}$ are i.i.d, it follows (see Chow and Teicher [3, exercise 4.2.7]) that the sequence $\{(\sum_{t=1}^{n} R_{ij}^l(t))/n\}_{n=1}^{\infty}$ is uniformly integrable. Therefore, the sequence

$$\sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} \left(\sum_{t=1}^{n} \frac{R_{ij}(t)}{n}\right)^l$$

is uniformly integrable since it is the sum of uniformly integrable sequences (see Chow-Teicher [3, page 94]). From (24) it follows that the sequence $\{(N_m(\tau_2)/n)^l\}_{n=1}^{\infty}$ is uniformly integrable and since $N(\tau_2) \leq \sum_{m\in\mathcal{M}} N_m(\tau_2)$, the same holds for the sequence $\{(N(\tau_2)/n\}_{n=1}^{\infty}$. Finally, from (23) and the uniform integrability of the sequence $\{(N(\tau_2)/n\}_{n=1}^{\infty}$ it follows that we can interchange limits and expectations, i.e.,

$$\lim_{n\to\infty} E\left(\frac{N_m(\tau_2)}{n}\right)^l = \max\{r_m^l : \quad m \in \mathcal{M}\}.$$

for all $l \geq 1$. ∎

Proof of Lemma 2

Observe first that if $X_i,\ i = 1, \ldots, K$ are random variables such that $E(\exp(\vartheta X_i)) < \infty,\ i = 1, \ldots, K$ for some ϑ, then

$$E\exp(\vartheta_1 \sum_{i=1}^{K} X_i) < \infty,$$

where $\vartheta_1 = \vartheta/K$. This follows by taking expectations in the following inequality that is a consequence of the convexity of the exponential function

$$\exp(\vartheta_1 \sum_{i=1}^{K} X_i) = \exp\left(\frac{\sum_{i=1}^{K} K\vartheta_1 X_i}{K}\right) \leq \frac{1}{K}\sum_{i=1}^{K} \exp(\vartheta_1 K X_i) \tag{25}$$

Applying the previous observation to the random variables

$$\tilde{R}_m(t) := \sum_{i\in\mathcal{M}} \sum_{j=m\oplus 1}^{i} R_{ij}(t), \quad m \in \mathcal{M},$$

we see that there is a $\vartheta_2 > 0$ such that

$$E\exp(\vartheta_2 \tilde{R}_m(t)) < \infty, \quad m \in \mathcal{M}.$$

Consider now the function $\Phi_m(\vartheta) = E\exp(\vartheta \tilde{R}_m(t) - \vartheta),\ 0 \le \vartheta \le \vartheta_2$. ¿From the previous discussion it can be seen that this function is well defined, continuous and differentiable in $[0, \vartheta_2]$. Since $\Phi_m(0) = 1$, $\Phi'_m(0) = E\tilde{R}_m(t) - 1 = r_m - 1 \le r - 1 < 0$, and $M < \infty$, it follows that there is a $\vartheta' > 0$ and a $\epsilon > 0$, such that for $m \in \mathcal{M}$, $\Phi_m(\vartheta') < 1 - \epsilon$, or equivalently,

$$\frac{E\exp(\vartheta' \tilde{R}_m(t))}{\exp \vartheta'} < (1-\epsilon)$$

¿From (22) we see that

$$N_m(\tau_2) = \sum_{t=1}^{n} \tilde{R}_m(t)$$

and since the random variables $R_m(t),\ t = 1, \ldots$ are i.i.d, we conclude that

$$\frac{E\exp(\vartheta' N_m(\tau_2))}{\exp(\vartheta' n)} = \left(\frac{E\exp(\vartheta' R_m(t))}{\exp(\vartheta')}\right)^n \le (1-\epsilon)^n. \tag{26}$$

Since

$$\begin{aligned} E\ \exp(\vartheta' N(\tau_2)) &= E\exp(\max\{\vartheta' N_m(\tau_2) :\ m \in \mathcal{M}\}) \\ &= E\max\{\exp(\theta' N_m(\tau_2)) :\ m \in \mathcal{M}\} \\ &\le \sum_{m\in\mathcal{M}} E\exp(\vartheta' N_m(\tau_2)), \end{aligned}$$

taking into account (26) we conclude

$$0 \le \lim_{n\to\infty} \frac{E\exp(\vartheta' N(\tau_2))}{\exp(\vartheta' n)} \le M \lim_{n\to\infty} (1-\epsilon)^n = 0$$

which completes the proof. ■

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