Proceedings of DETC'99
1999 ASME Design Engineering Technical Conferences
September 12-15, 1999, Las Vegas, Nevada, USA

DETC99/DAC-8703

COMPUTER-ASSISTED KINEMATIC TOLERANCE ANALYSIS OF A GEAR SELECTOR MECHANISM WITH THE CONFIGURATION SPACE METHOD

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ABSTRACT

This paper presents a case study in computer-assisted tolerancing with the configuration space method. We analyze part of a gear selector mechanism in an automotive automatic transmission. The model contains three complex parts with 99 functional parameters. The analysis, which takes less than a minute on a workstation, indicates that the critical kinematic variation occurs in third gear and identifies the parameters that cause the variation. The analysis program handles general planar systems of curved parts with contact changes, including open and closed kinematic chains. It computes the worst-case variation in the system kinematic function and detects potential failure modes due to unexpected qualitative variations in function, such as jamming. It constructs configuration space models of the kinematic variations of the pairs in the system then derives the system variation by composing the pair models.

INTRODUCTION

Improving design quality and reducing product development time are key factors in making companies competitive in the world market. Tolerance analysis plays a central role in both tasks. In current practice, tolerance analysis is an imperfect, laborious, and time consuming activity. Keeping the analysis effort affordable limits the aspects of a design that receive attention to those that are presumed critical. In mechanical systems, these are typically safety items, important clearances, and places where part interferences are expected. Even so, unproven assumptions and simplifications are often made to speed up the analysis. As a result, tolerance problems impose significant risks and uncertainties in design despite significant analysis efforts.

The generic shortcomings of tolerance analysis are acute in automotive power-train design. The orders to purchase the transfer lines, machines, and tools are placed when the tolerance analysis begins. When the tolerance analysis reveals a problem, modifications to the design often require very expensive changes to the manufacturing process. Engineers have three options to resolve the problem: conduct a statistical analysis to determine if the probability of trouble is acceptable; decrease the part tolerances, which increases costs; or accept higher warranty costs and customer dissatisfaction. During production, downstream consequences of yet-to-be discovered tolerance problems can stop production. Many tolerance problems reflect incorrect simplifying assumptions where engineers ignored an important feature or where the geometric complexity of the parts, their motions, and their interactions produced unexpected effects. Tolerance analysis of the mechanism kinematics is one of the most demanding and time consuming tasks.

This paper presents a case study in computer-assisted kinematic tolerancing with the configuration space method. We determine the effects of part variation on kinematic function (part motions and contacts). This is the most important form of functional tolerance analysis because kinematic function largely determines system function. Kinematic tolerance analysis consists of tolerance specification, variation modeling, and sensitivity analysis steps. Tolerance specifications define the allowable variation in the shapes of the parts of a system. The most common are parametric and geometric tolerance specifications (Requicha, 1993; Voelcker, 1993). Parametric tolerances are best suited to kinematic tolerancing because they are much simpler than geometric tolerances, yet capture the relevant part variations. Variation modeling derives the functional relationship between the tolerance parameters and the system kinematic function. Sensitivity analysis uses the model to estimate the worst-case or average variation of a kinematic quantity for given part variations. Worst-case results are generally preferred in functional tolerancing due to the high cost of delivering defective products.

Creating a variation model is the limiting factor in kinematic tolerance analysis. The analyst has to formulate and solve large systems of algebraic equations to obtain the relationship between the tolerance parameters and the kinematic function. The analysis grows much harder when we consider systems with contact changes. Contact changes occur in the nominal function of higher pairs, such as gears, cams, clutches, and ratchets. Part variation produces contact changes in systems whose nominal designs prescribe permanent contacts. The analysis has to determine which contacts occur at each stage of the work cycle, to derive the resulting kinematic functions, and to identify qualitative kinematic variations due to contact changes, such as play, undercutting, interference, and jamming. Once the variation model is obtained, sensitivity analysis can be performed by linearization, statistical analysis, or Monte Carlo simulation (Chase and Parkinson, 1991).

We have developed a worst-case kinematic tolerance analysis program for planar mechanical systems with parametric part tolerances (Sacks and Joskowicz, 1998; Joskowicz and Sacks, 1999). The program constructs a variation model for the system, derives worst-case bounds on the variation, and helps designers find failure modes. The variation model is a generalization of our configuration space representation of the part contacts in the nominal system (Sacks and Joskowicz, 95). The algorithm handles general planar systems of curved parts with contact changes, including open and closed kinematic chains. It analyzes systems with 50 to 100 parameters in under a minute, which permits interactive tolerancing of detailed functional models.

This paper presents the preliminary results of applying our methodology to the gear selector assembly of an automatic transmission (Figure 1). Transmission design is an excellent test case for our program because it involves complex mechanisms with nominal and unintended contact changes that can fail due to tolerances. We analyzed part of the gear selector assembly with known tolerance problems. We begin by describing the gear selector assembly and the tolerancing problems that arise. We then explain the configuration space representation of kinematic function, its extension to toleranced parts, and our kinematic tolerance analysis algorithm. We describe our results and conclude by discussing current and future work.

PREVIOUS WORK

Previous work on kinematic tolerance analysis of mechanical systems falls into three increasingly general categories: static (small displacement) analysis, kinematic (large displacement) analysis of fixed contact systems, and kinematic analysis of systems with contact changes. Static analysis of fixed contacts, also referred to as tolerance chain or stack-up analysis, is the most common. It consists of identifying a critical dimensional parameter (a gap, clearance, or play), building a tolerance chain based on part configurations and contacts, and determining the parameter variability range using vectors, torsors, or matrix transforms (Clemént et al, 1997; Whitney et al, 1994). Recent research explores static analysis with contact changes (Inui and Miura, 1995; Ballot and Bourdet, 1997). Configurations where unexpected failures occur can easily be missed because the user needs to identify the relevant system configurations (Schultheiss and Hinze). Kinematic analysis of fixed contact mechanical systems, such as linkages, has been thoroughly studied in mechanical engineering (Erdmann, 1993). It consists of defining kinematic relations between parts and studying their kinematic variation (Chase et al, 1997). Most commercial CAT systems include this capability for planar and spatial mechanism (Solomons et al, 1997). These methods are impractical for systems with many contact changes, such as automotive transmissions, and can miss failure modes due to unforeseen contact changes. Our method overcomes these limitations by automating variational contact model derivation and analysis for general planar systems.

CASE STUDY: THE GEAR SELECTOR MECHANISM

Figure 1(a) shows part of the gear selector mechanism of a Ford automatic transmission. The mechanism is shown from the side (View A), the front (View B) and the top (View C). It consists of a cam/piston assembly (left on view A, parts 1 to 4) and a gear shifting assembly (right on view A and view B, parts 6 to 10) connected by a rod (middle of views A and C, part 5). For the purposes of this paper, we focus on the cam/piston assembly.

Figure 1(b) shows the four main parts of the cam/piston assembly: the rooster cam, the pin, the piston, and the valve body. (The parts are labeled 1–4 in part a.) The cam rotates around an axis at its center. Its angular position is controlled by the shift stick (not shown). Its side pin is mounted on the piston's left end and causes it to slide back and forth inside the valve body, which is fixed. The different piston positions open and close the conducts on the valve. The pin, which is spring-loaded, temporarily locks the rooster cam in one of seven settings labeled 1, 2, 3, D, N, R, and P.

Each of the cam settings determines a nominal opening of the valves. The angular position of the cam is determined by the pin that pushes the lower cam profile. Variations in the pin, piston, and cam shapes and positions affect the piston displacement and thus the valve opening. The task of kinematic tolerance

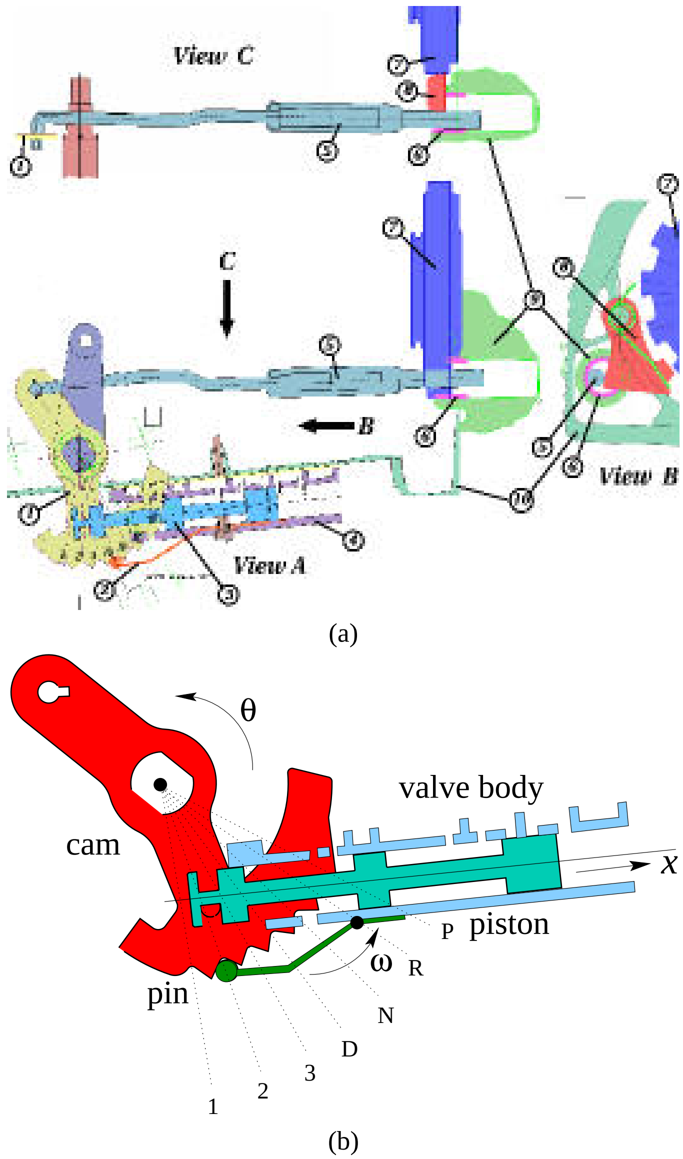

Figure 1. (A) CAD DRAWING OF GEAR SELECTOR MECHANISM; (B) CAM/PIN/PISTON ASSEMBLY.

analysis is to determine the maximum variation of the piston displacement for each cam setting. It is also important to determine which feature variations contribute the most to the piston variation: the cam axial position, its profile, the pin radius, or others. The many part features, complex kinematic relations, and contact changes make manual analysis impractical.

Kinematic tolerancing also plays an important role in certifying the safety of the gear selector design. Like the braking and steering systems, the automatic transmission gear selector mechanism is a safety item. The settings of the gear selector activate mechanical and electrical functions, such as locking the transmission output shaft in P, engaging the selected gear in settings 1, 2, 3, D, and R, and allowing the engine to start in P and N. Some countries require a tolerance analysis of critical gear selector functions as part of the vehicle certification. Kinematic tolerance analysis helps verify that the mechanism will always work. In the gear selection mechanism, the transition from one gear to another is of special interest because a mismatch of electrical, mechanical, or hydraulic functions is most likely there. The tolerance analysis has to demonstrate that the mechanism is safe despite the inevitable transition misalignment due to part variation.

CONFIGURATION SPACE

We model the nominal kinematic function of a mechanism within the configuration space representation of rigid part interaction (Latombe, 1991; Lozano-Pérez, 1983). We construct a configuration space for each pair of interacting parts in the mechanical system. The configuration space is a manifold with one coordinate per part degree of freedom. For this paper, we will assume that parts are planar and that each has a single degree of freedom: translation along a fixed axis or rotation around a fixed point. This yields a two-dimensional configuration space which can be readily computed and visualized (Sacks and Joskowicz, 1995). The same principle applies to general planar pairs, which are analyzed elsewhere (Sacks, 1998).

We illustrate these concepts on the gear selector assembly. The interacting pairs are the cam/pin and the cam/piston. The coordinates of the cam/pin configuration space are the orientation angles θ of the cam and ω of the pin. The coordinates of the cam/piston configuration space are θ and the offset x of the piston along its motion axis.

Configuration space partitions into three disjoint sets that characterize part interaction: blocked space where the parts overlap, free space where they do not touch, and contact space where they touch without overlap. Blocked space represents unrealizable configurations, free space represents independent part motions, and contact space represents motion constraints due to part contacts. The spaces have useful topological properties. Free and blocked space are open sets whose common boundary is contact space. Contact space is a closed set comprised of curves that represent contacts between pairs of part features. As the parts move, their configurations trace a curve through free and contact space. Contact changes occur when the configuration reaches contact curve endpoints.

Figure 2 shows the configuration spaces of the cam/piston and cam/pin pairs. Free space is white, blocked space is grey, and contact space is black. The solid circles mark the part configurations shown in Figure 1b. The cam/piston free space is a narrow diagonal channel bounded by contact curves. The top and bottom curves represent contacts between the cam pin and the left and right vertical segments of the pin slot. The space

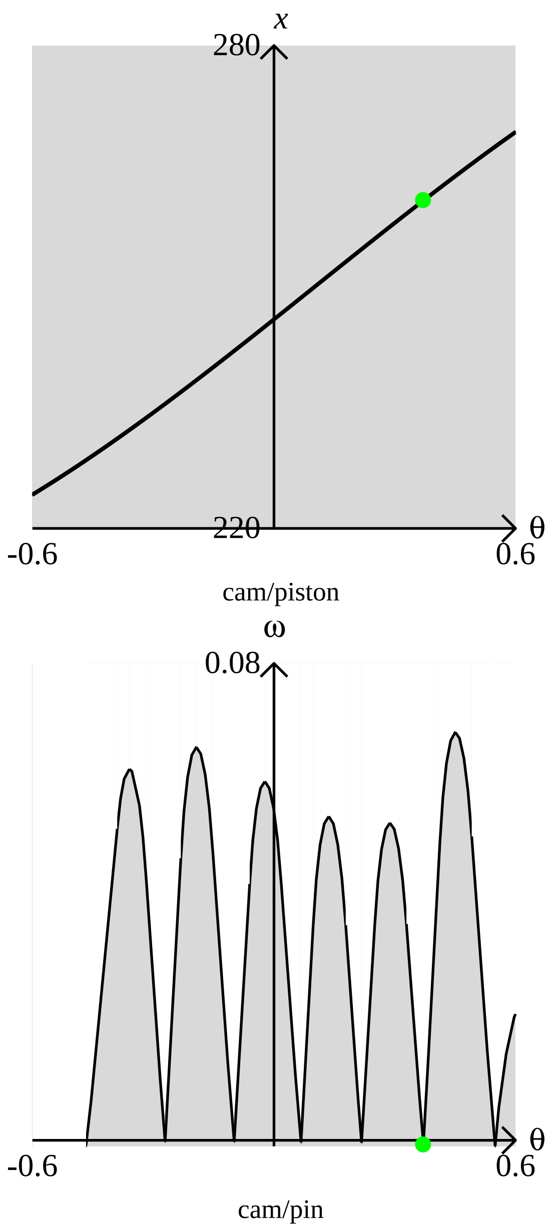

Figure 2. DETAIL OF THE GEAR SELECTOR CONFIGURATION SPACES. DISPLACEMENT x IS IN MILLIMETERS, ANGLES θ AND ω ARE IN RADIANS.

between the contact curves is the nominal functional play. The kinematic relation between the two is nearly linear. The cam/pin contact space consists of seven "valleys" and seven "hills". The valleys represent the seven cam settings where the pin is positioned between two cam teeth. The peaks represent transitions between settings where the pin is in contact with the tips of the six cam teeth.

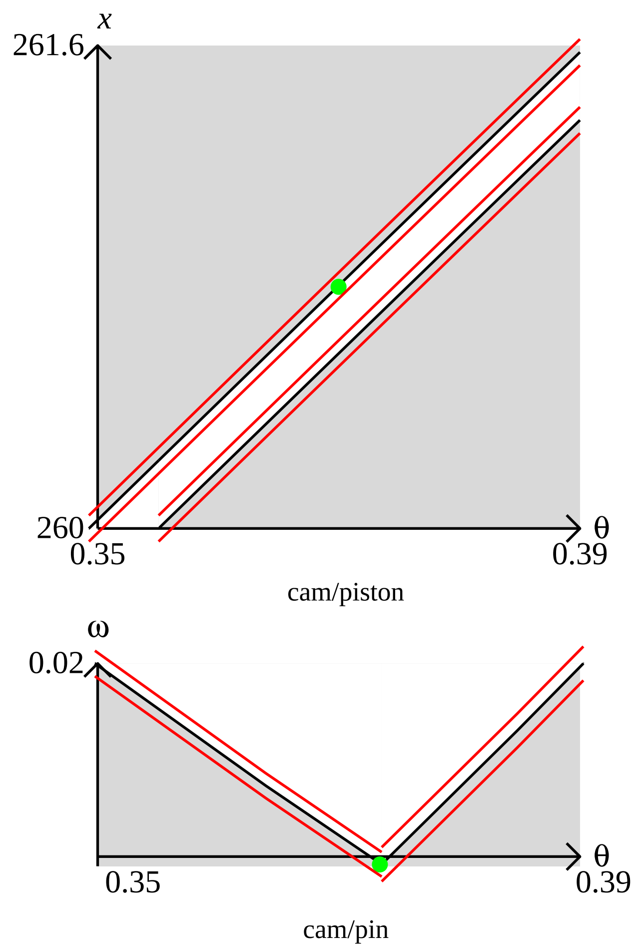

Figure 3. DETAILS OF THE PAIRWISE CONTACT ZONES AROUND THE 3RD GEAR POSITION. THE CENTER CURVE IS THE NOMINAL CONTACT CURVE, WHILE THE LOWER AND UPPER CURVES SHOW THE WORST-CASE KINEMATIC VARIATION.

KINEMATIC TOLERANCE ANALYSIS

We model kinematic variation by generalizing the configuration space representation to toleranced parts. The contact curves are parameterized by the part features in contact, which depend on the tolerance parameters. As the parameters vary around their nominal values, the curves vary in a band around the nominal contact space, which we call the contact zone. Figure 3 shows details of the cam/piston and cam/pin contact zones. The contact zone defines the kinematic variation in each contact configuration: every pair that satisfies the part tolerances generates a contact space that lies in the contact zone. Kinematic variations do not occur in free configurations because the parts do not interact.

Each contact curve generates a region in the contact zone that represents the kinematic variation in the corresponding part contact. The region boundaries encode the worst-case kinematic variation over the allowable parameter variations. They are smooth functions of the tolerance parameters and of the mechanism configuration in each region. They are typically discontinuous at region boundaries because the contact curves depend on different parameters and are unrelated. For example, the solid circle marks a discontinuity in the cam/pin zone where the pin touches two adjacent cam teeth. The variation at transition points is the maximum over the neighboring region endpoints. The contact zone also captures qualitative changes in kinematics, such as jamming, under-cutting, and interference (Sacks and Joskowicz, 1997; Sacks and Joskowicz, 1998).

We compute the contact zone from the parametric model of the pair. The inputs are the part models, the nominal values and allowable ranges of the parameters, and an error bound. The outputs are closed-form expressions for the contact zone. We require, as do other sensitivity analysis methods, that the part shapes and configurations depend smoothly on the parameters. Examples of non-smooth dependencies are parameters with integer values, such as a gear with n teeth, and models with singularities, such as a circular arc with radius $r = 0$.

Each contact curve is analyzed independently. The curve has the form $y = f(x, \mathbf{p})$ with x and y the degrees of freedom and $\mathbf{p}$ the vector of tolerance parameters. We compute the sensitivity vector $\partial y/\partial \mathbf{p}$ by numerical differentiation. Suppose parameter p_i has nominal value $\overline{p}_i$ and variation $\pm\delta p_i$. We approximate the worst-case kinematic variation as

$$\sum_i \left| \frac{\partial f}{\partial p_i} \right| \delta p_i \tag{1}$$

using the standard tolerancing approximation that it is linear in the parameter variations. The ith term represents the maximal linear variation in y induced by p_i variations.

The contact zone model generalizes from pairs to systems. The contact space is a semi-algebraic set in configuration space: a collection of points, curves, surfaces, and higher dimensional components. As the tolerance parameters vary around their nominal values, the components vary in a band around the nominal contact space, which is a higher-dimensional analog of the contact zone of a pair. We avoid general algebraic methods, performing kinematic tolerance analysis on individual operating modes.

System operating modes are defined by driving forces and initial conditions. We can perform the analysis for any number of modes, but cannot analyze the sensitivity to the continuously infinite space of all possible modes. Given the forces and initial conditions, the laws of physics determine the time evolution of the system. We can compute a nominal sequence of states by simulation or by physical measurement. This yields a nominal path in the system configuration space. We perform kinematic tolerance analysis by computing the kinematic variation at sampled configurations along the nominal path.

We compute the system kinematic variation at a sample configuration by combining the variations of its pairs according to the rules of calculus. For example, we compute the variation in the piston offset at the displayed configuration from the contact equations of the cam/piston and cam/pin pairs: $x = f(\theta, \mathbf{p})$ and $\theta = g(\omega, \mathbf{p})$. The chain rule yields

$$\frac{\partial x}{\partial p_i} = \frac{\partial f}{\partial \theta}\frac{\partial \theta}{\partial p_i} + \frac{\partial f}{\partial p_i} = \frac{\partial f}{\partial \theta}\frac{\partial g}{\partial p_i} + \frac{\partial f}{\partial p_i} \tag{2}$$

with the derivatives evaluated at the nominal parameter values. The worst-case variation is

$$\sum_i \left| \frac{\partial x}{\partial p_i} \right| \delta p_i \tag{3}$$

as before. The general algorithm is analogous to the example. The system is encoded as a contact graph whose nodes represent parts and whose links represent contact relations. The graph is traversed depth-first starting from driving parts and the chain rule is invoked to compute the kinematic variation of the driven parts.

TOLERANCE ANALYSIS OF THE GEAR SELECTOR MECHANISM

We performed kinematic tolerance analysis on the gear selector mechanism. We obtained the nominal boundary representation model of the gear selector cam subassembly from Ford. We constructed a parametric model of the subassembly by adding variation parameters to the functional features of the parts. For the cam, we toleranced the line segments that form the tooth sides, the small arc segments that form the tooth tips, the arc segments that connect the teeth, and the circular pin that engages the piston. For the piston, we toleranced the two vertical segments that are in contact with the cam pin. For the pin, we toleranced the single, circular feature. Line segments were toleranced by varying the coordinates of the two endpoints; arc segments were toleranced by varying the radius and the center coordinates. To account for uncertainties in the position of the rotation axes, we also toleranced the centers of rotation of the cam and the pin. Since we chose the piston as the reference part of the assembly, there was no need to tolerance the orientation of its translation axis. The model has 86 tolerance parameters for the cam, 8 for the piston, 5 for the pin, and 99 overall. We assigned every parameter an independent tolerance of ± 0.1 mm. Constructing the parametric models and inputting them to the program took one person two hours.

To determine the kinematic variations, we computed the contact zones of the cam/piston and cam/pin pairs, as shown in Figure 2. The computation took 20 seconds, using a Lisp program running on an Indigo 2 Workstation. In the cam/piston zone, the piston offset has a worst-case variation of between 0.41mm and 0.45 mm over the functional range of cam angles. In the cam/pin zone, the pin orientation has a worst-case variation of between 0.013 radians and 0.018 radians.

We examined the sensitivity to the individual tolerance parameters at the configuration $\theta = 0.371$ radians, $\omega = -0.0008$ radians, $x = 260.8$ mm where the gear selector is in third gear (Figure 1). The worst-case variation of x is 0.9 mm—roughly half from each pair. The main factors in the cam/piston variation are the cam center horizontal position (25%), vertical position (25%), tooth base x position (25%), and the x coordinates of the piston vertical segments (10%). The cam/pin variation is evenly distributed among the parameters of the touching features and the part centers of rotation.

CONCLUSIONS

We have presented a case study of kinematic tolerance analysis in transmission design using the contact zone model of kinematic variation. The study involves a three-part assembly with 99 tolerance parameters. The assembly is not amenable to tolerance analysis by hand or with prior software because of the complex kinematic function (many contacts and contact changes) and to the large number of parameters. Our program performs a complete analysis in under a minute and produces useful information for the Ford transmission designers. It supports fast, complete kinematic tolerance analysis of general planar mechanical systems with tens of parts and hundreds of tolerance parameters.

We are extending the algorithm to variational part models. The contact zone computation algorithm assumes that the part features are given functions of the tolerance parameters. This assumption is appropriate when the nominal geometry is specified explicitly, as is the case of manual design and in traditional, non-parametric CAD tools, such as Autocad. But a different approach is needed for variational CAD tools, such as ProEngineer, Catia, of Ford's custom tolerance analysis system, ADAPT. In these systems, the nominal geometry is specified implicitly as the solution set of constraint equations. The user specifies equations $\mathbf{h}(\mathbf{p},\mathbf{q}) = 0$ with $\mathbf{p}$ the part parameters and $\mathbf{q}$ the model parameters. For example, $\mathbf{p}$ could include the center and radius of a circular arc on the part boundary, while $\mathbf{q}$ could include the distance between the circle center and the origin. The system solves the equations for $\mathbf{p}$, which defines the nominal geometry, in terms of the nominal values of $\mathbf{q}$. We seek the variation in kinematic function due to a variation in $\mathbf{q}$, which induces a variation in $\mathbf{p}$. To handle this situation, the program must manipulate the variational equations, the contact equations, and the nominal solutions to compute the kinematic variation.

Our ultimate goal is to develop an automated configuration space-based program to support the design engineer with kinematic tolerance analysis and synthesis in all the steps of design. It will help engineers discard design concepts which are not robust, detect undesired part interactions and interferences, and optimize functional tolerances.

ACKNOWLEDGMENT

Sacks is supported by NSF grants CCR-9617600 and CCR-9505745 and by the Purdue Center for Computational Image Analysis and Scientific Visualization. Joskowicz is supported by a grant from the Authority for Research and Development, The Hebrew University of Jerusalem, Israel. Sacks and Joskowicz are supported by a Ford University Research Grant, the Ford ADAPT200 project, and by grant 98/536 from the Israeli Academy of Science.

REFERENCES

Eric Ballot and Pierre Bourdet. A computation method for the consequences of geometric errors in mechanisms. In *Proc. of the 5th CIRP Int. Seminar on Computer-Aided Tolerancing*, Toronto, 1997.

Kenneth Chase and Allan Parkinson. A survey of research in the application of tolerance analysis to the design of mechanical assemblies. *Research in Engineering Design*, 3(1):23–37, 1991.

Kenneth Chase, Spencer Magleby, and Charles Glancy. A comprehensive system for computer-aided tolerance analysis of 2d and 3d mechanical assemblies. In *Proc. of the 5th CIRP Int. Seminar on Computer-Aided Tolerancing*, Toronto, 1997.

André Clemént, Alain Rivière, Phillipe Serré, and Catherine Valade. The ttrs: 13 constraints for dimensioning and tolerancing. In *Proc. of the 5th CIRP Int. Seminar on Computer-Aided Tolerancing*, Toronto, 1997.

Arthur Erdman, *Modern Kinematics: developments in the last forty years*. John Wiley and Sons, 1993.

Matsamoto Inui and Masashiro Miura. Configuration space based analysis of position uncertainties of parts in an assembly. In *Proc. of the 4th CIRP Int. Seminar on Computer-Aided Tolerancing*, 1995.

Leo Joskowicz and Elisha Sacks. The configuration space method for kinematic tolerance analysis. In *Global consistency of tolerances, Proc. of the 6th CIRP Int. Seminar on Computer-Aided Tolerancing*, F. van Houten and H. Kals eds., Kluwer 1999.

Jean-Claude Latombe. *Robot Motion Planning*. Kluwer Academic Publishers, 1991.

Tomás Lozano-Pérez. Spatial planning: A configuration space approach. In *IEEE Transactions on Computers*, volume C-32, pages 108–120. IEEE Press, 1983.

Aristides A. G. Requicha. Mathematical definition of tolerance specifications. *Manufacturing Review*, 6(4):269–274, 1993.

Elisha Sacks. Practical sliced configuration spaces for curved planar pairs. *International Journal of Robotics Research*, 17(11), 1998.

Elisha Sacks and Leo Joskowicz. Computational kinematic analysis of higher pairs with multiple contacts. *Journal of Mechanical Design*, 117:269–277, 1995.

Elisha Sacks and Leo Joskowicz. Parametric kinematic tolerance analysis of planar mechanisms. *Computer-Aided Design*, 29(5):333–342, 1997.

Elisha Sacks and Leo Joskowicz. Parametric kinematic tolerance analysis of general planar systems. *Computer-Aided Design*, 30(9):707–714, 1998.

Otto Solomons, Fred van Houten, and Hubert Kals. Current status of CAT systems. In *Proc. of the 5th CIRP Int. Seminar on Computer-Aided Tolerancing*, Toronto, 1997.

Ralf Schultheiss and Uwe Hinze. Detect the unexpected - how to find and avoid unexpected tolerance problems in mechanisms. In *Global consistency of tolerances, Proc. of the 6th CIRP Int. Seminar on Computer-Aided Tolerancing*, F. van Houten and H. Kals eds., Kluwer 1999.

Herbert Voelcker. A current perspective on tolerancing and metrology. *Manufacturing Review*, 6(4):258–268, 1993.

Daniel Whitney, Olivier Gilbert, and Marek Jastrzebski. Representation of geometric variations using matrix transforms for statistical tolerance analysis. *Research in Engineering Design*, 6(4):191–210, 1994.