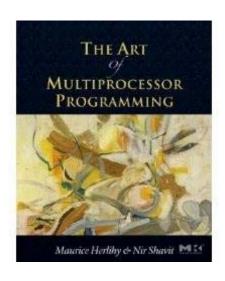
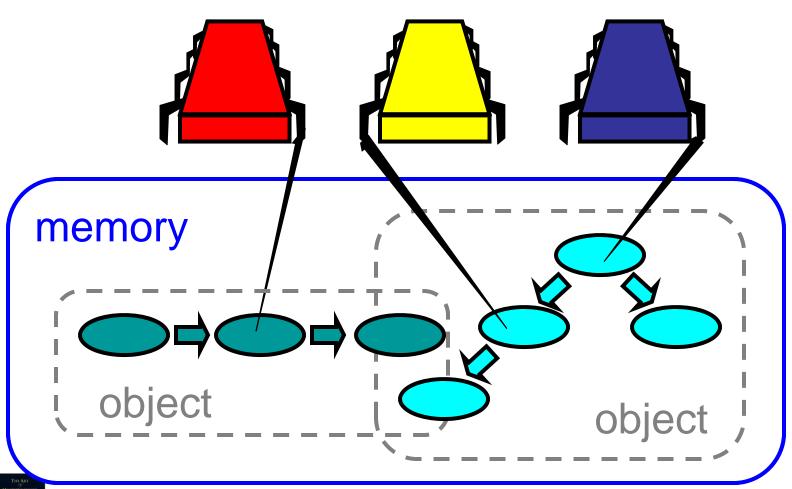
Concurrent Objects



Companion slides for
The Art of Multiprocessor Programming
by Maurice Herlihy & Nir Shavit

Concurrent Computation



Objectivism

- What is a concurrent object?
 - How do we describe one?
 - How do we implement one?
 - How do we tell if we're right?

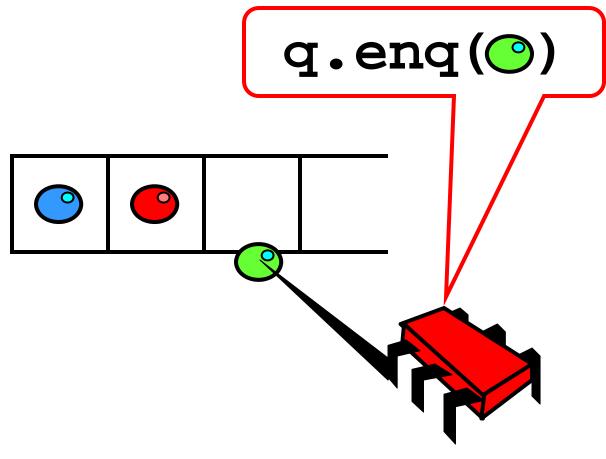


Objectivism

- What is a concurrent object?
 - How do we describe one?
 - How do we tell if we're right?

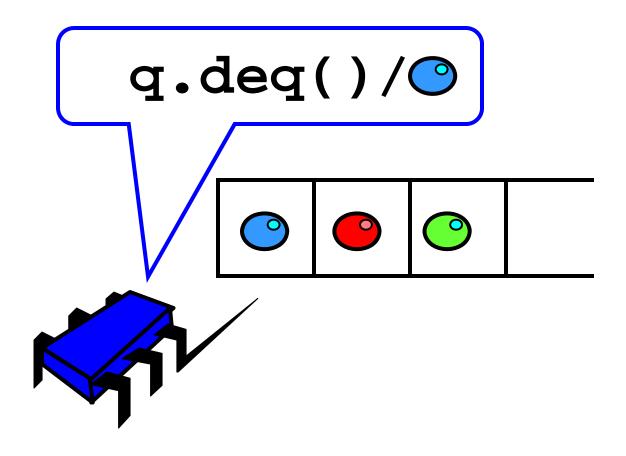


FIFO Queue: Enqueue Method



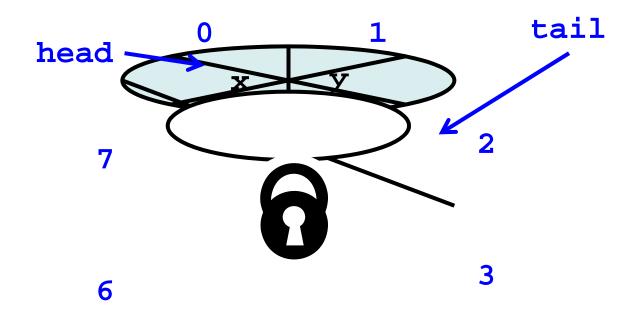


FIFO Queue: Dequeue Method





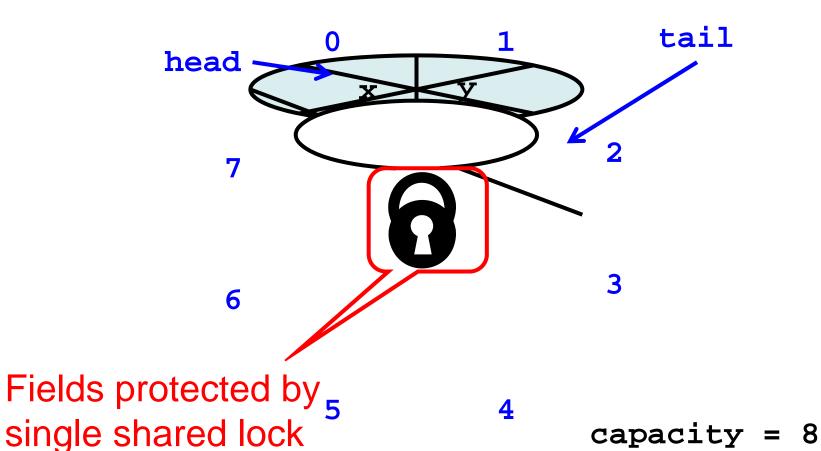
Lock-Based Queue



5 4 capacity = 8



Lock-Based Queue





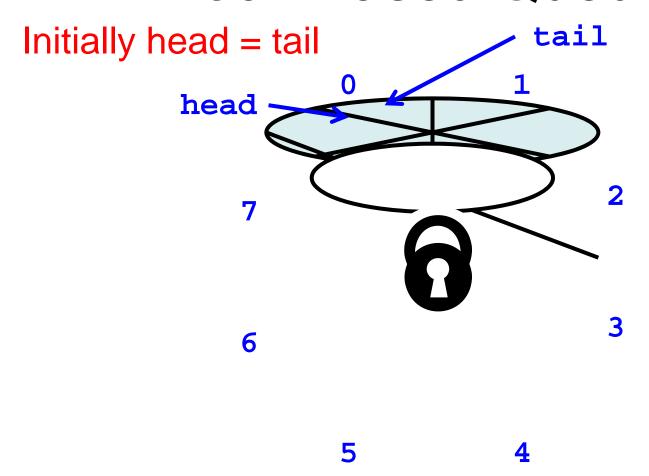
Art of Multiprocessor Programming

A Lock-Based Queue

Fields protected by single shared lock



Lock-Based Queue





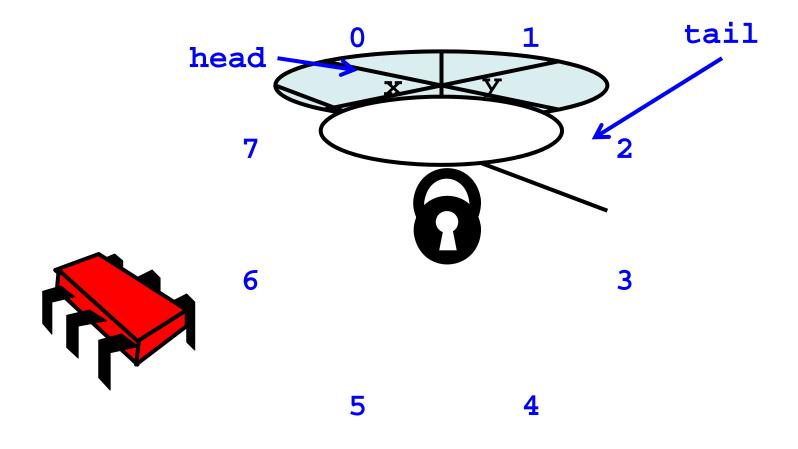
10

A Lock-Based Queue

```
head
                                           tail
                            capacity-1 YZ/
class LockBasedQueue<T> {
  int head, tail;
  T[] items;
  Lock lock;
  public LockBasedQueue(int capacity)
    head = 0; tail = 0;
    lock = new ReentrantLock();
    items = (T[]) new Object[capacity];
                     Initially head = tail
```

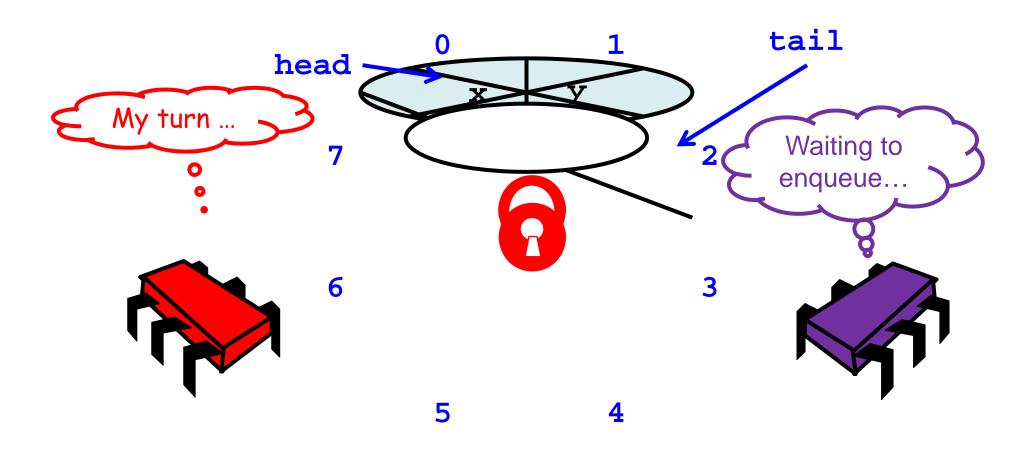


Lock-Based deq()





Acquire Lock

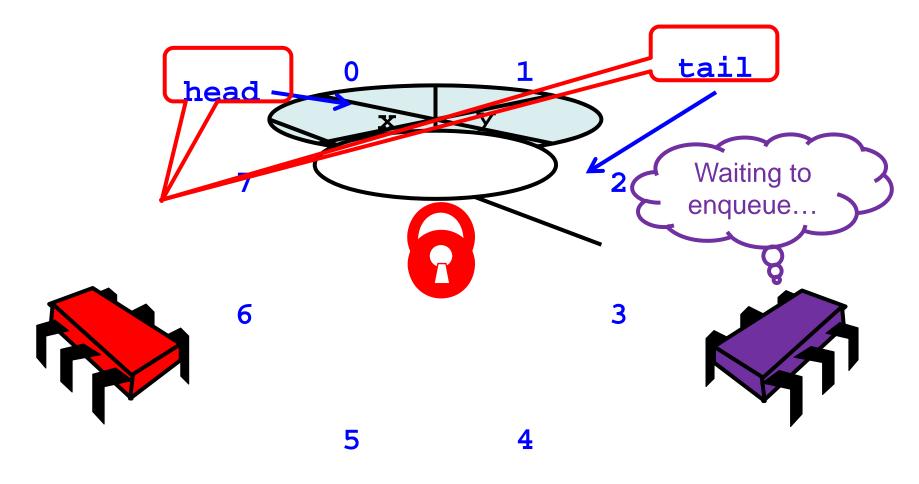




Implementation: deq()



Check if Non-Empty





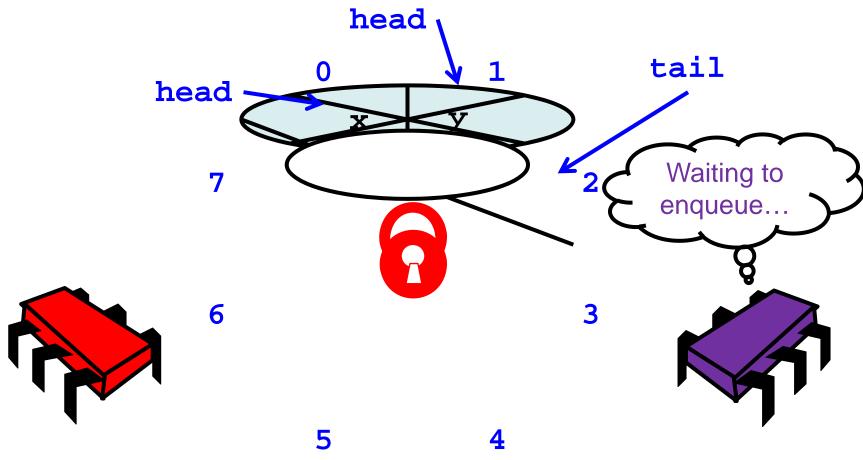
Implementation: deq()

```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
        throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  } finally {
    lock.unlock();
  }
}
```

If queue empty throw exception



Modify the Queue





Implementation: deq()

```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
        throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  } finally {
    lock.unlock();
  }
  Queue not empty?
```

Remove item and update head



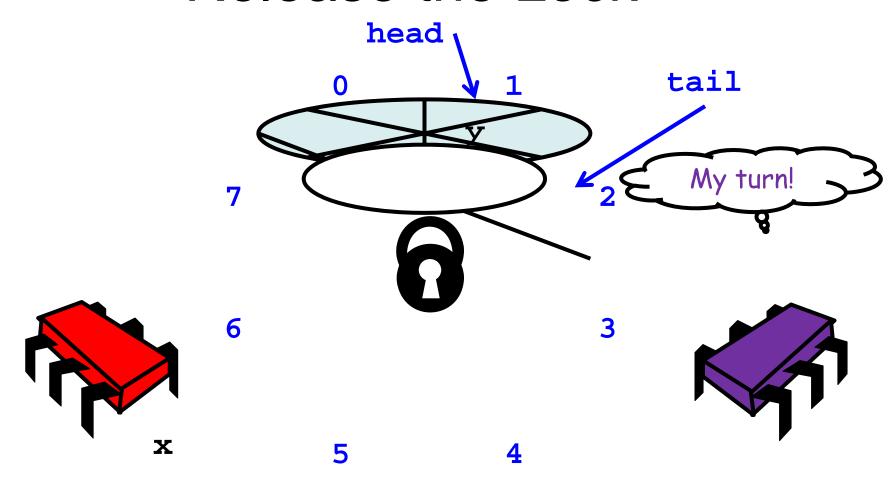
Implementation: deq()

```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
        throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  } finally
    lock.unlock();
}

Return result
```



Release the Lock





Implementation: deq()

```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
        throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  } finally {
    lock.unlock();
  }
}
Release lock no
```



matter what!

Implementation: deq()

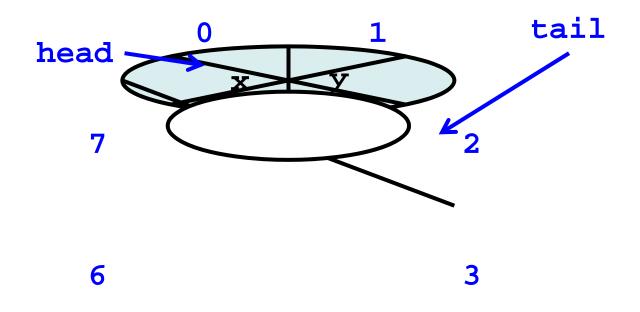
```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
       throw new EmptyException();
    T x = items[head % items.length];
                   modifications are mutually exclusive...
    head++;
                  Should be correct because
    return x;
  } finally {
    lock.unlock();
```



Now consider the following implementation

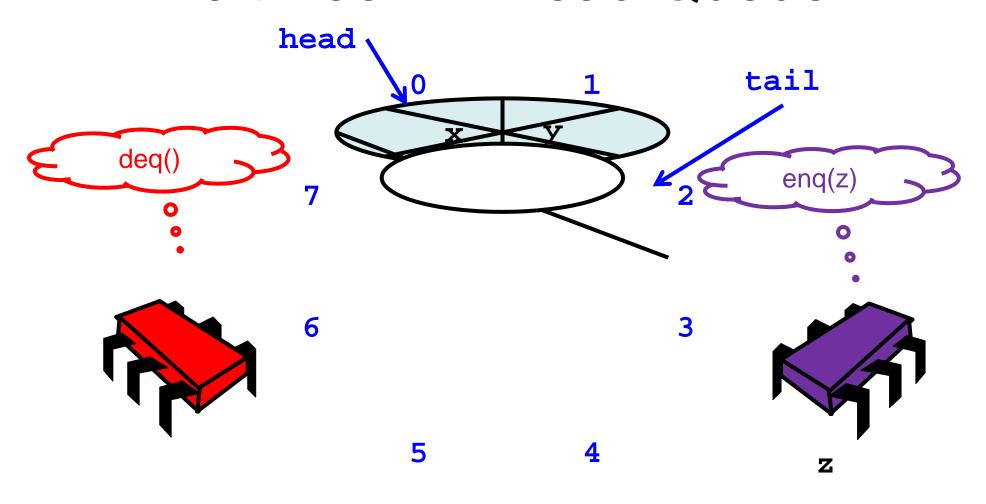
- The same thing without mutual exclusion
- For simplicity, only two threads
 - One thread enq only
 - The other deq only



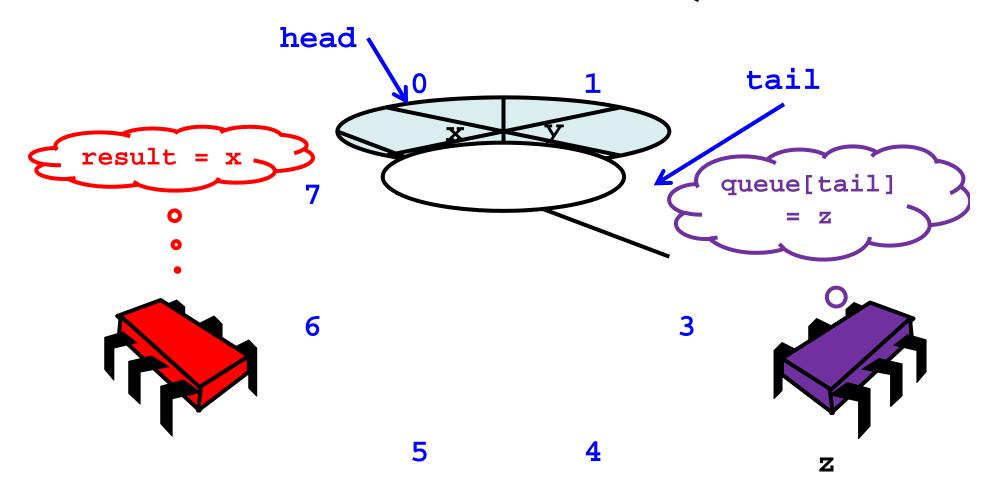


5 4 capacity = 8

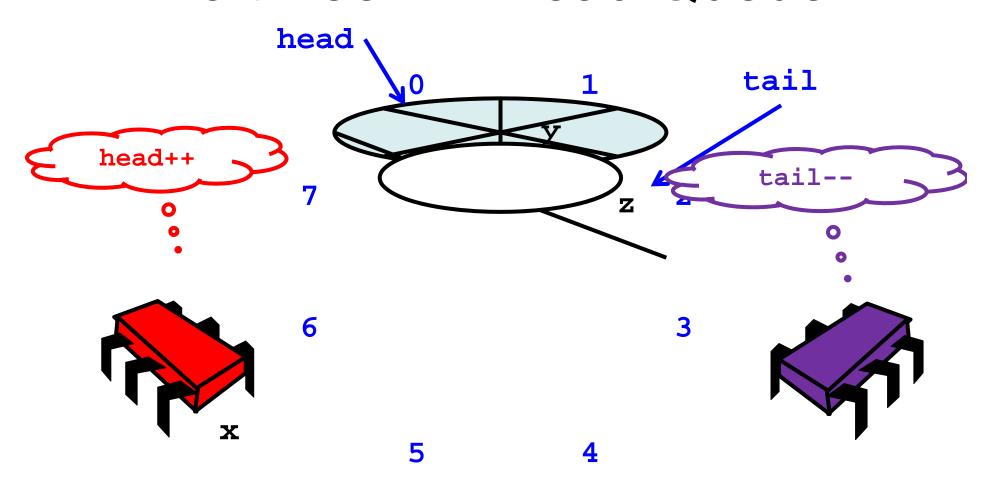














```
public class WaitFreeQueue {
                                     head
                                               tail
  int head = 0, tail = 0;
  items = (T[]) new Object[capacity
  public void enq(Item x) {
    if (tail-head == capacity) throw
         new FullEvgention():
    items[tail % capacity] = x; tail++
  public Item deq() {
     if (tail == head) throw
                                  No lock needed
         new EmptyException();
     Item item = items[head % capacity]; head++;
     return item;
                  Art of Multiprocessor
                                                 28
                    Programming
```

```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
        throw new EmptyExe "correct" when
        T x = items[headefine "correct" when
        head++;
        return How do we define not mutually
        head++;
        return How do we are not mutually
        lock.un exclusive?
    }
}
```



What is a Concurrent Queue?

- Need a way to specify a concurrent queue object
- Need a way to prove that an algorithm implements the object's specification
- Lets talk about object specifications ...

Correctness and Progress

- In a concurrent setting, we need to specify both the safety and the liveness properties of an object
- Need a way to define
 - when an implementation is correct
 - the conditions under which it guarantees progress

Lets begin with correctness

Sequential Objects

- Each object has a state
 - Usually given by a set of *fields*
 - Queue example: sequence of items
- Each object has a set of methods
 - Only way to manipulate state
 - Queue example: eng and deg methods

Sequential Specifications

- If (precondition)
 - the object is in such-and-such a state
 - before you call the method,
- Then (postcondition)
 - the method will return a particular value
 - or throw a particular exception.
- and (postcondition, con't)
 - the object will be in some other state
 - when the method returns,

Pre and PostConditions for Dequeue

- Precondition:
 - Queue is non-empty
- Postcondition:
 - Returns first item in queue
- Postcondition:
 - Removes first item in queue

Pre and PostConditions for Dequeue

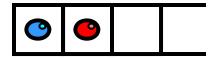
- Precondition:
 - Queue is empty
- Postcondition:
 - Throws Empty exception
- Postcondition:
 - Queue state unchanged

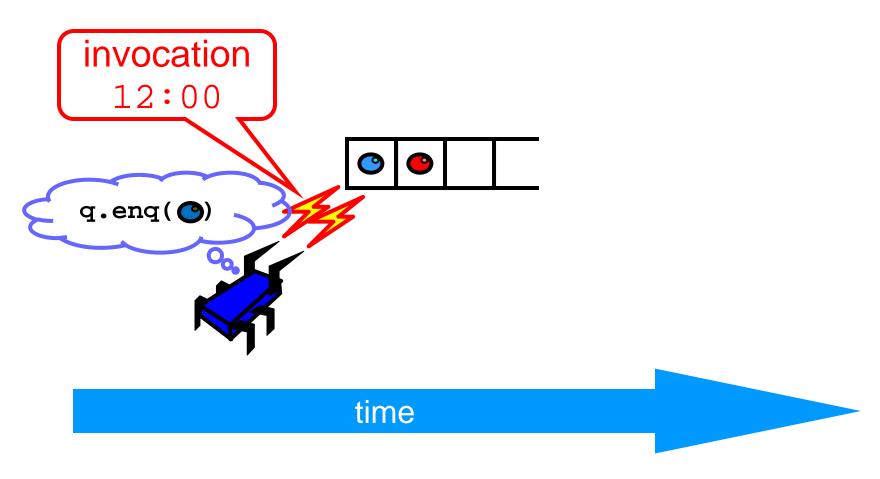
Why Sequential Specifications Totally Rock

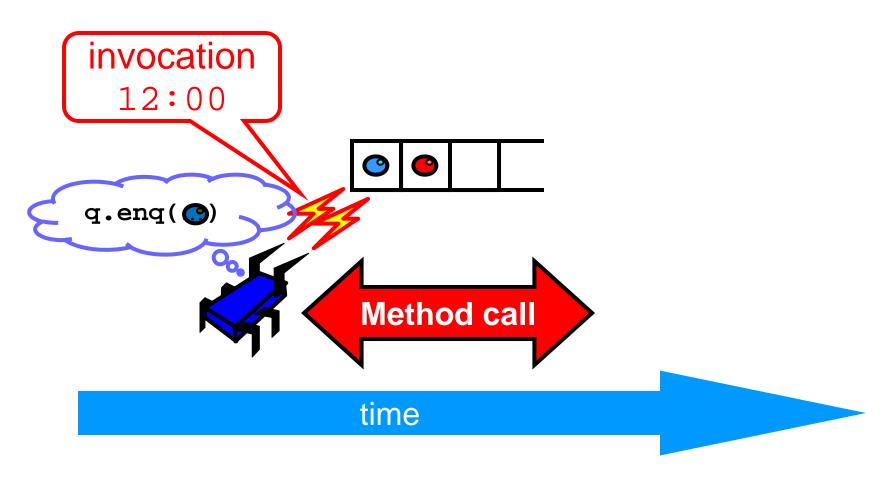
- Interactions among methods captured by sideeffects on object state
 - State meaningful between method calls
- Documentation size linear in number of methods
 - Each method described in isolation
- Can add new methods
 - Without changing descriptions of old methods

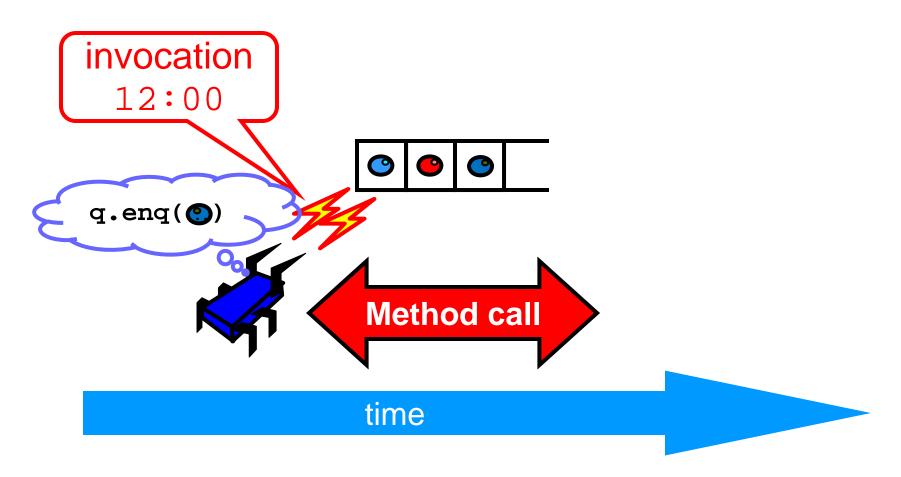
What About Concurrent Specifications?

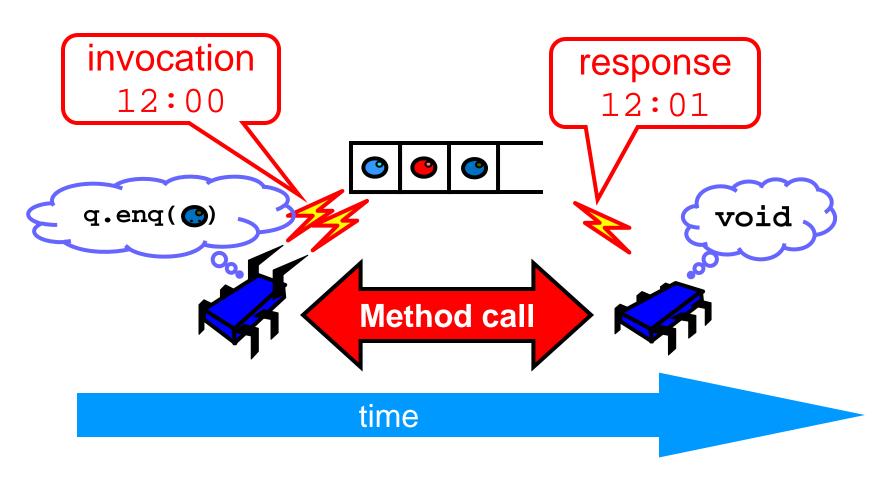
- Methods?
- Documentation?
- Adding new methods?



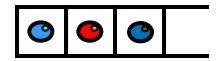


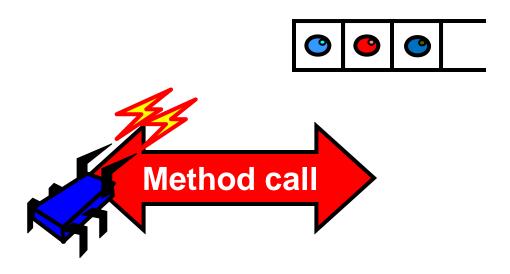


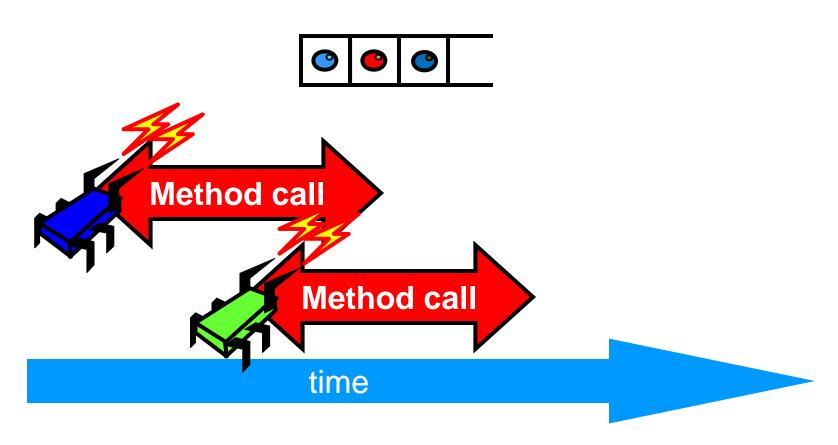


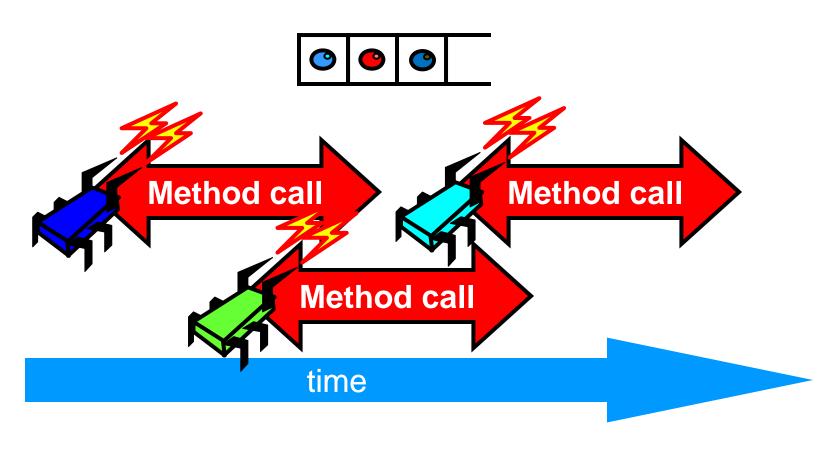


- Sequential
 - Methods take time? Who knew?
- Concurrent
 - Method call is not an event
 - Method call is an interval.









- Sequential:
 - Object needs meaningful state only between method calls
- Concurrent
 - Because method calls overlap, object might never be between method calls

- Sequential:
 - Each method described in isolation
- Concurrent
 - Must characterize all possible interactions with concurrent calls
 - What if two engs overlap?
 - Two deqs? enq and deq? ...

- Sequential:
 - Can add new methods without affecting older methods
- Concurrent:
 - Everything can potentially interact with everything else

- Sequential:
 - Can add new methods without affecting older methods
- Concurrent:

Everything can potentially interact with everything else

The Big Question

- What does it mean for a concurrent object to be correct?
 - What is a concurrent FIFO queue?
 - FIFO means strict temporal order
 - Concurrent means ambiguous temporal order

Intuitively...

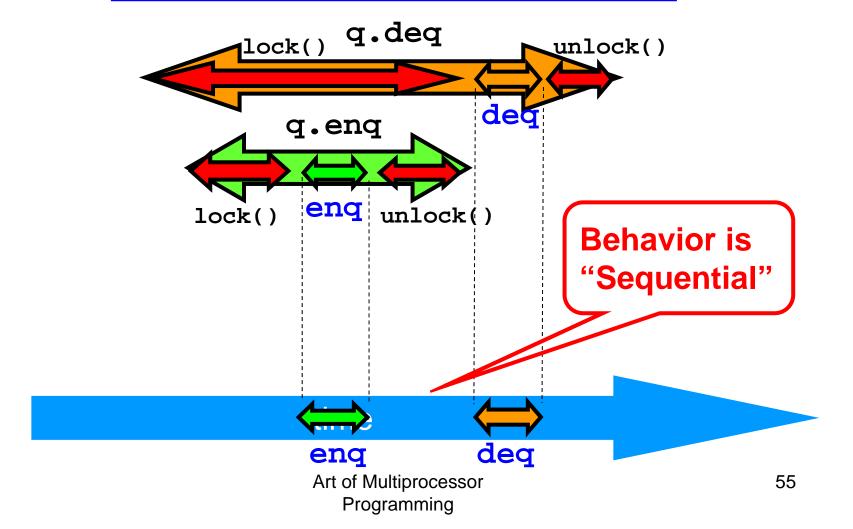
```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
        throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  } finally {
    lock.unlock();
  }
}
```

Intuitively...

```
public T deg() throws EmptyException {
   lock.lock();
   try {
      if (tail == head)
            throw New EmptyException();
      T x = items[head % items.length];
      head++;
      return x;
   }
   finally {
      lock.unlock();
      are mutually exclusive
   }
}
```

1.21...1...

Lets capture the idea of describing the concurrent via the sequential

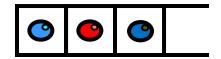


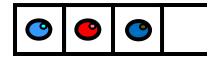
Linearizability

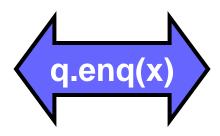
- Each method should
 - "take effect"
 - Instantaneously
 - Between invocation and response events
- Object is correct if this "sequential" behavior is correct
- Any such concurrent object is
 - Linearizable™

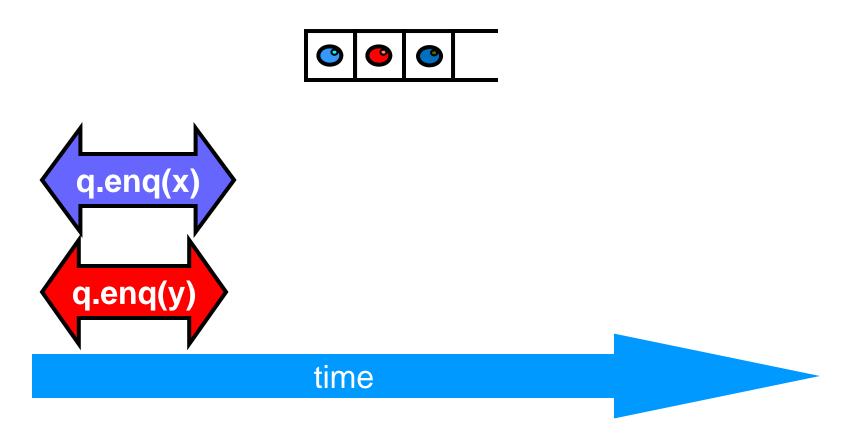
Is it really about the object?

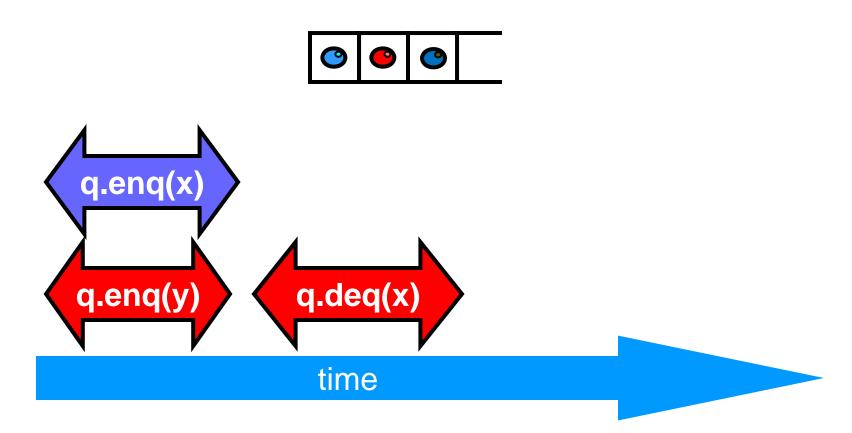
- Each method should
 - "take effect"
 - Instantaneously
 - Between invocation and response events
- Sounds like a property of an execution...
- A linearizable object: one all of whose possible executions are linearizable



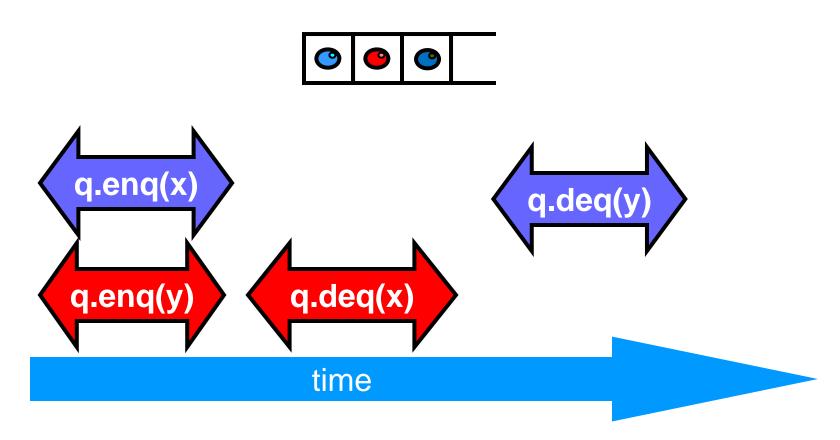


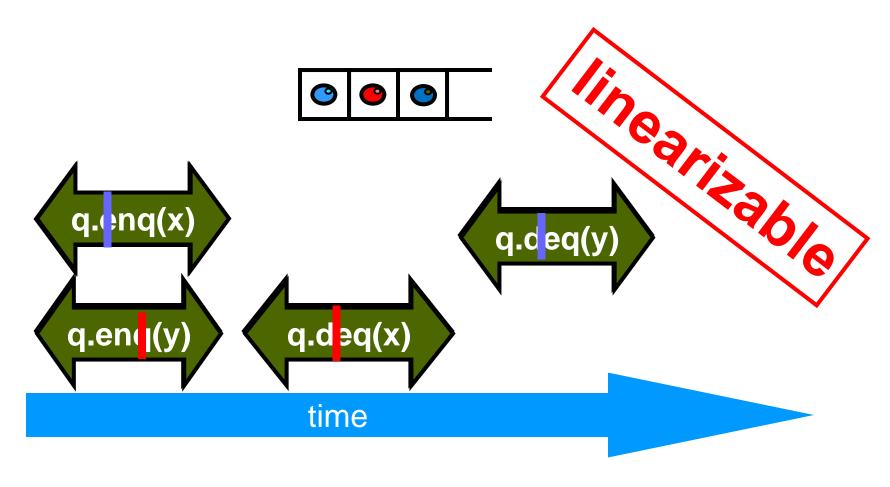


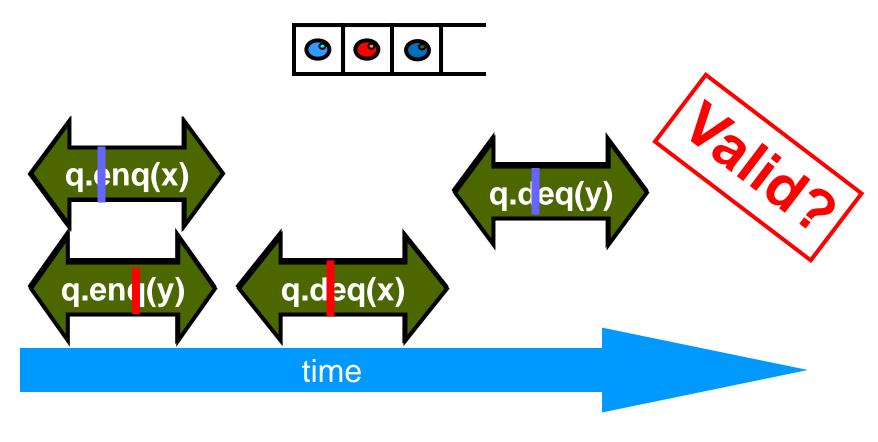


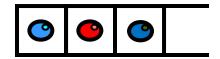


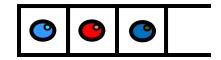


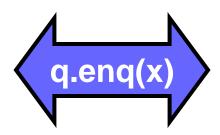


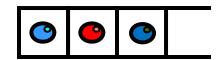


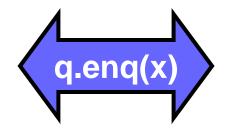


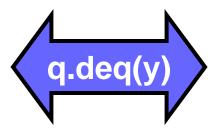




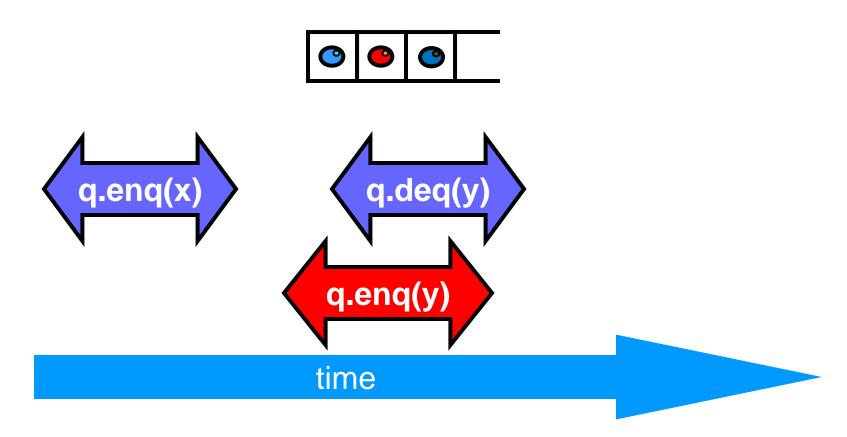




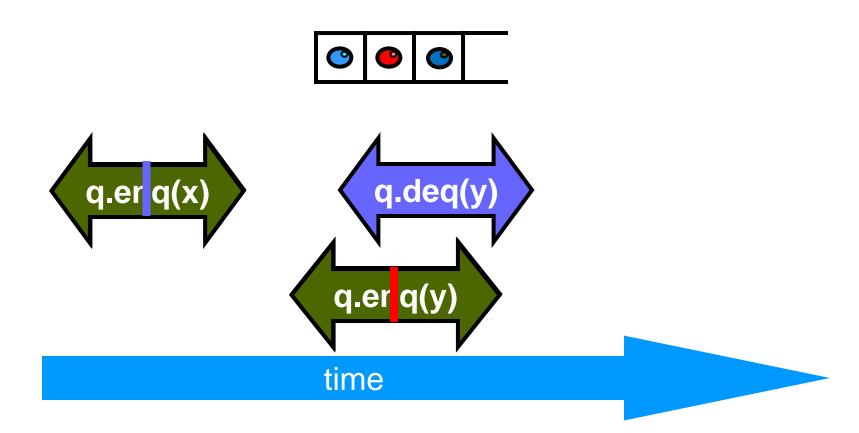


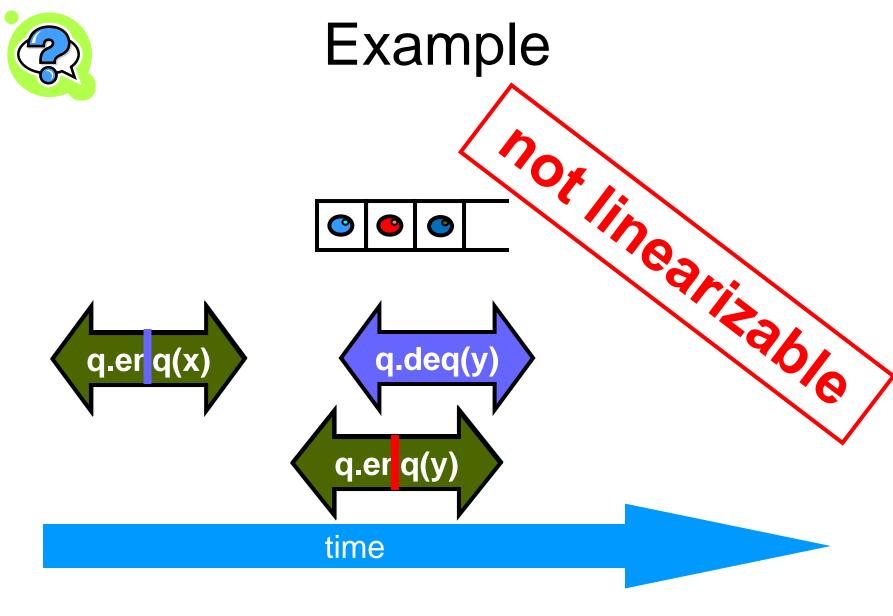


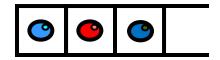


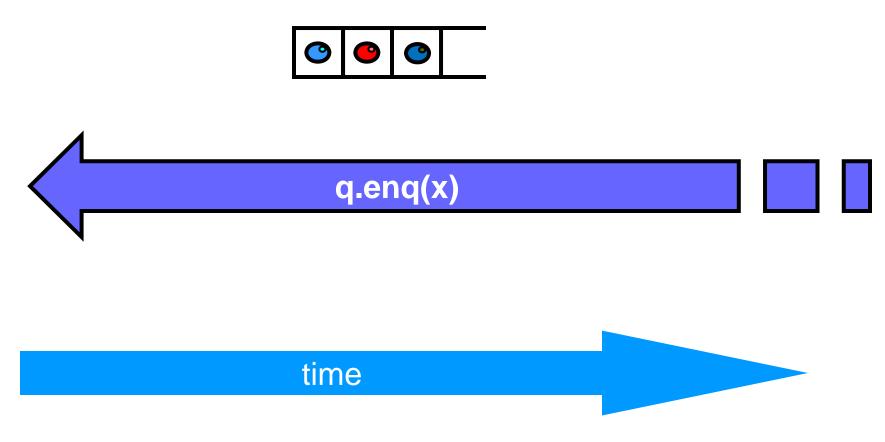




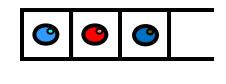


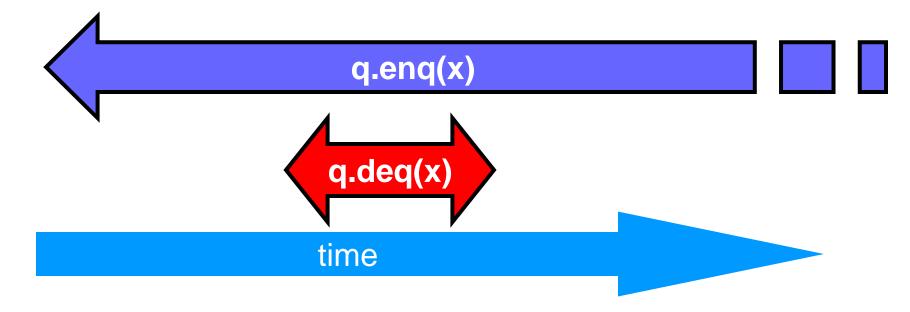




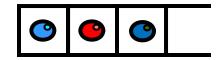


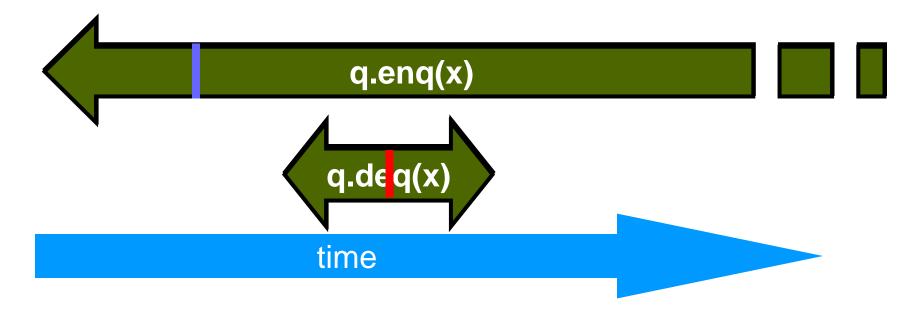




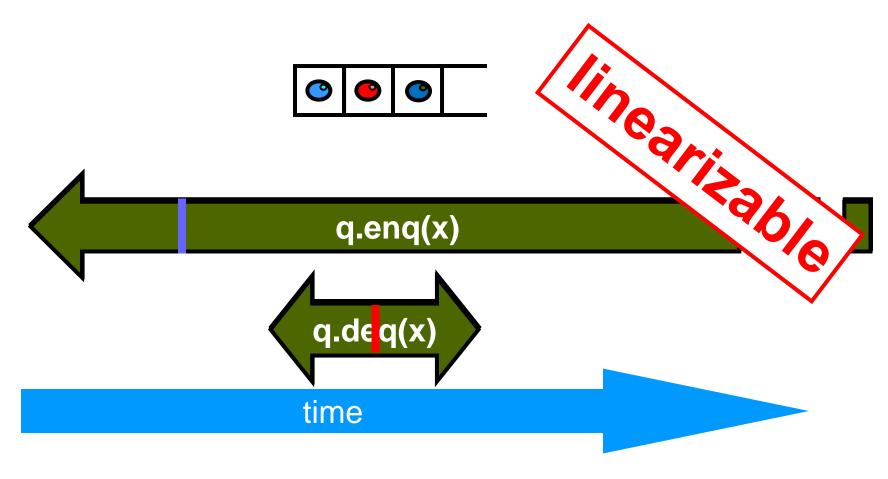


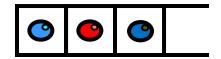


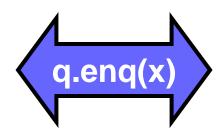




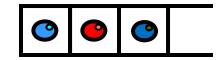


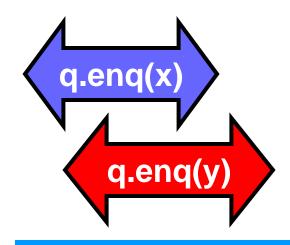




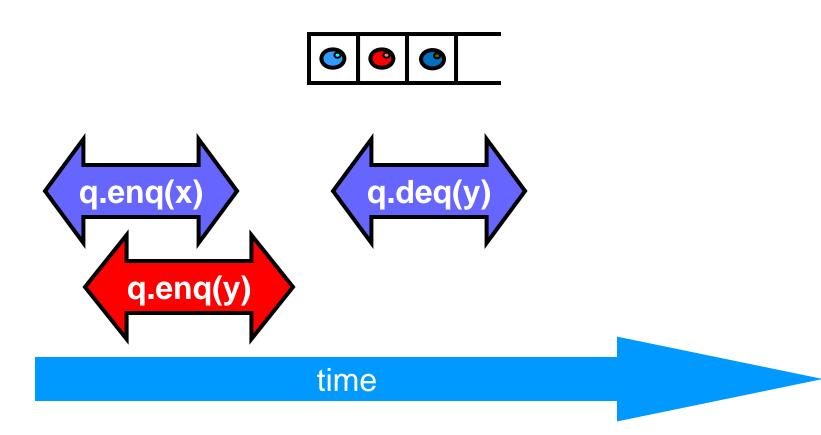


time

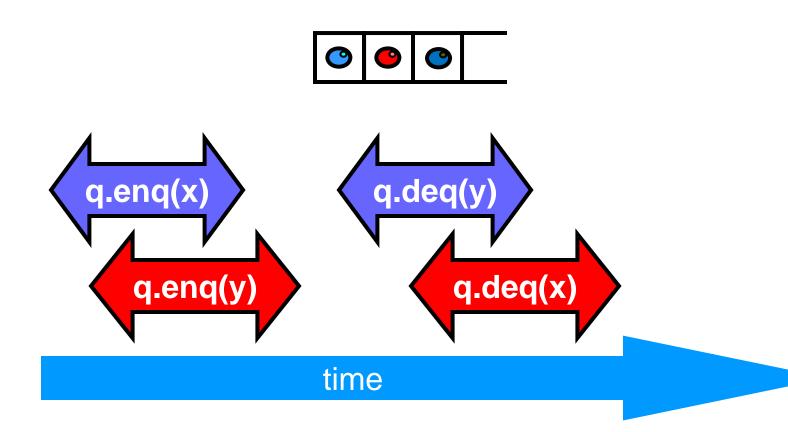


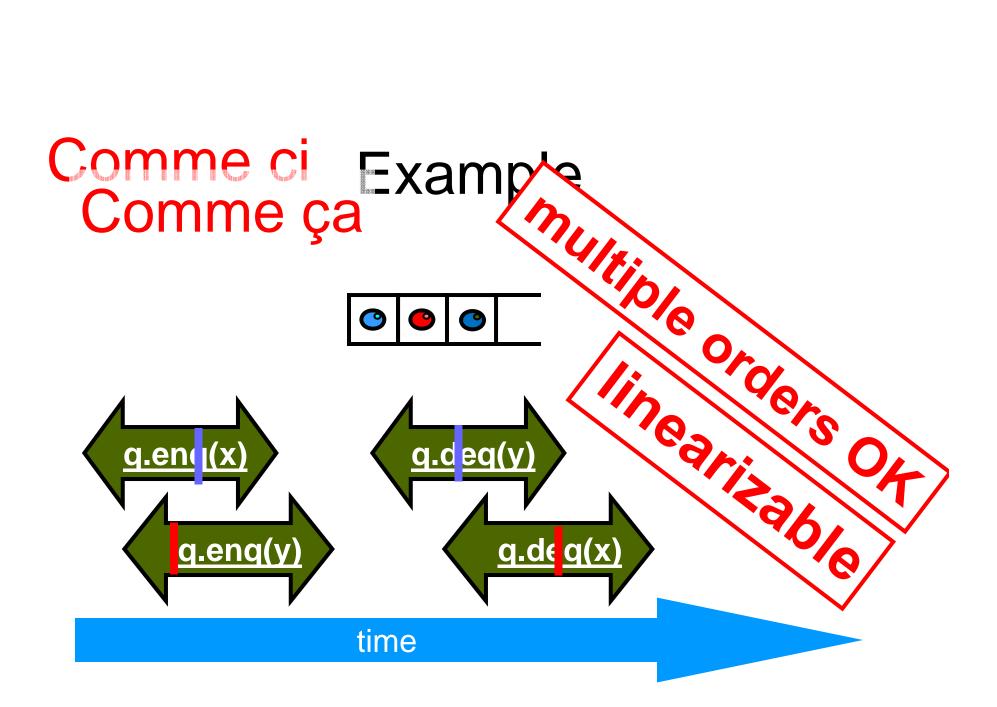


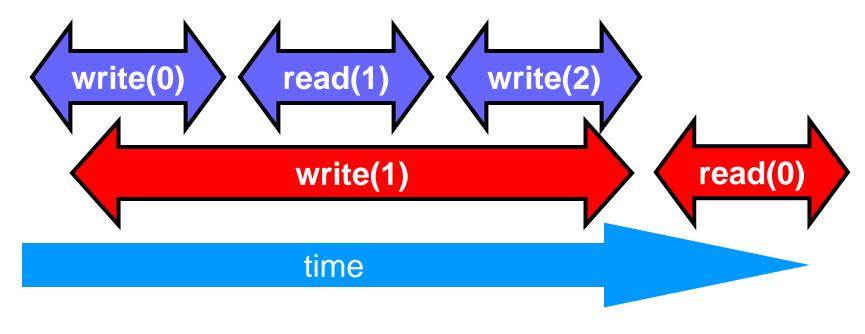
time

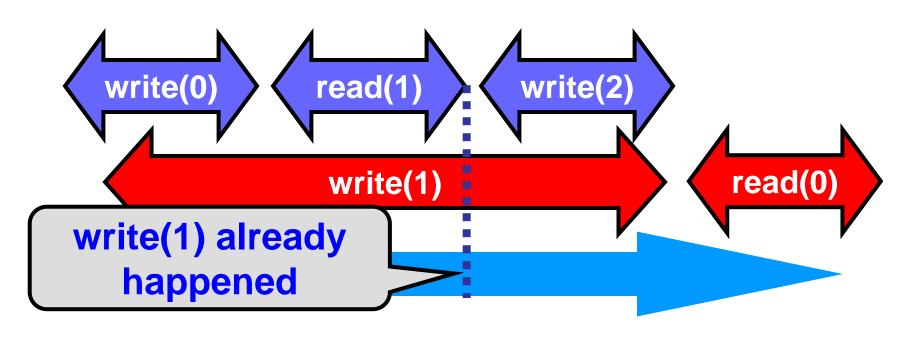


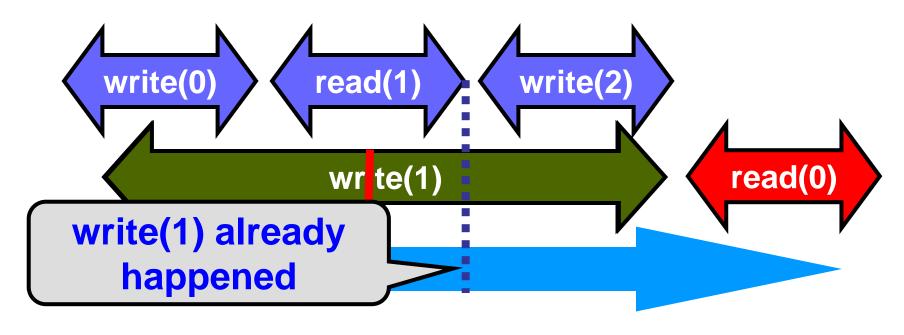


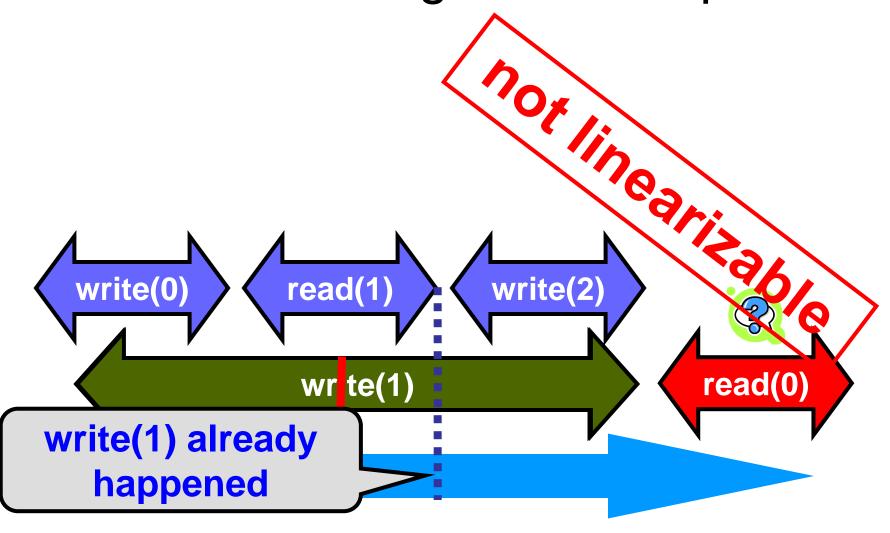


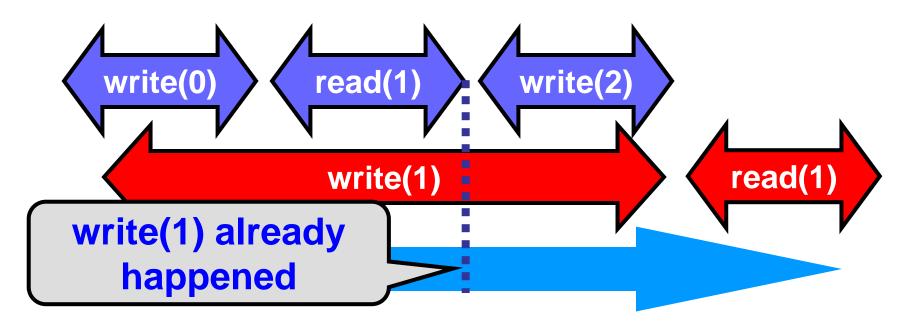


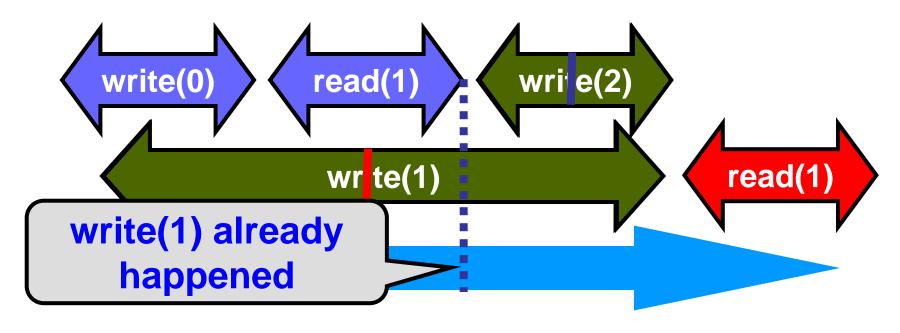


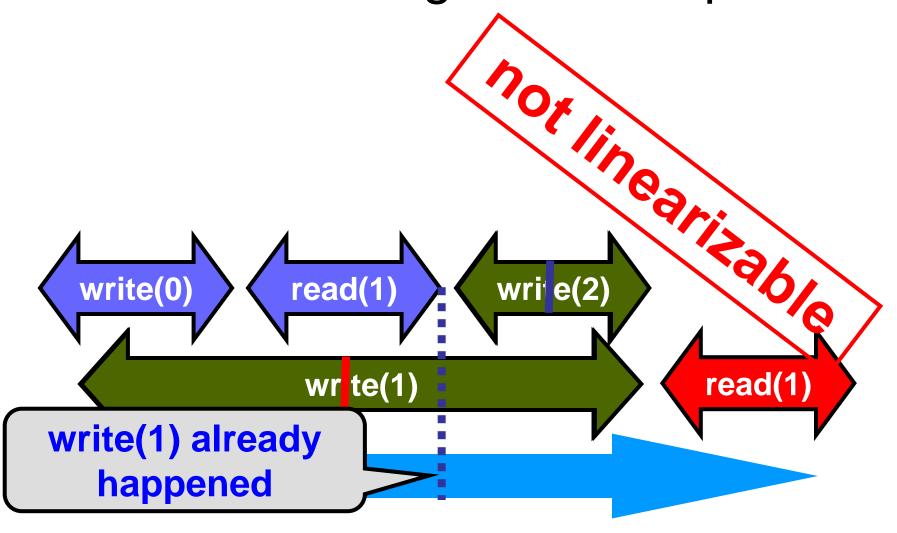


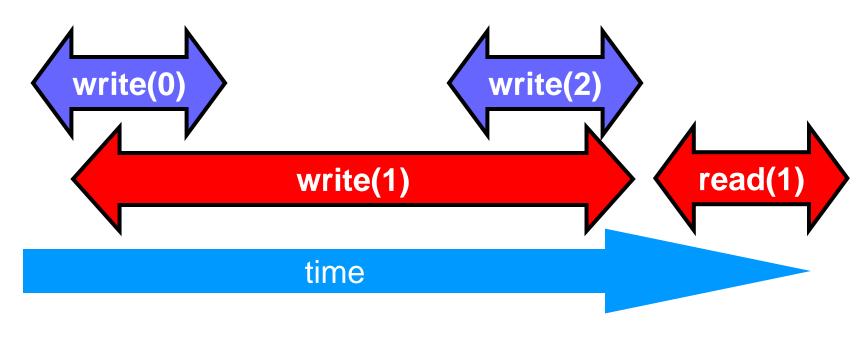


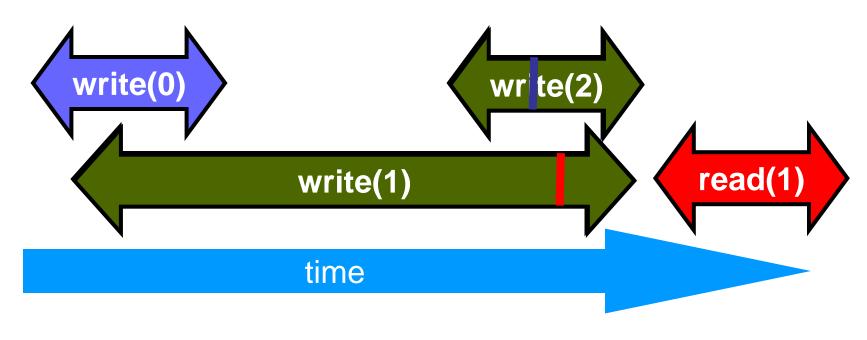


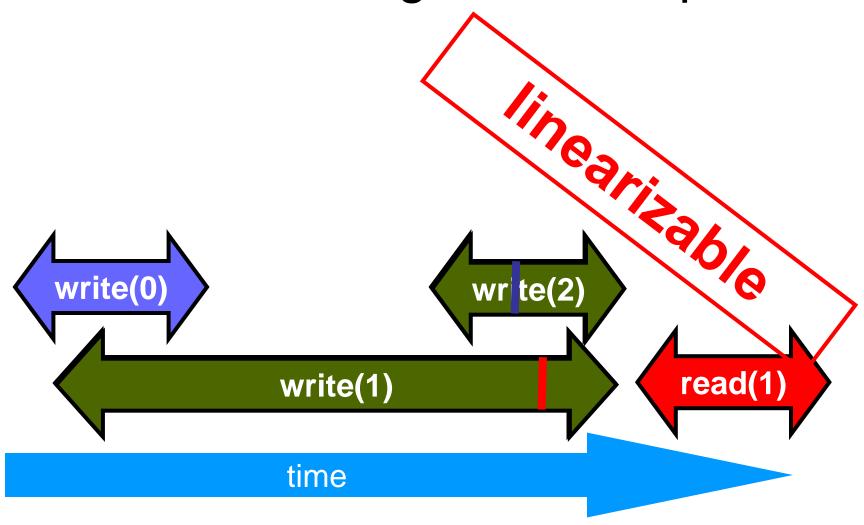


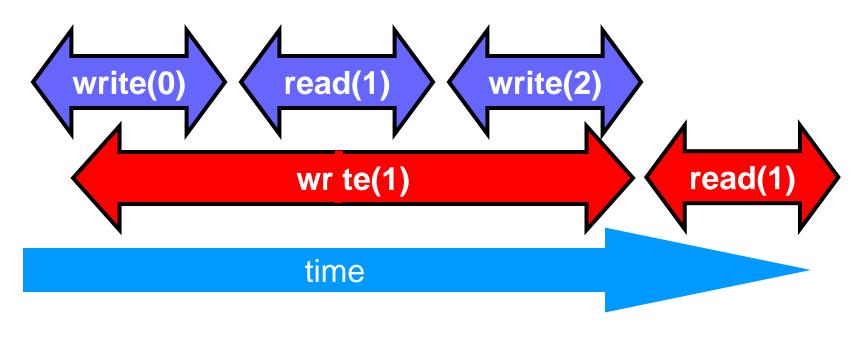


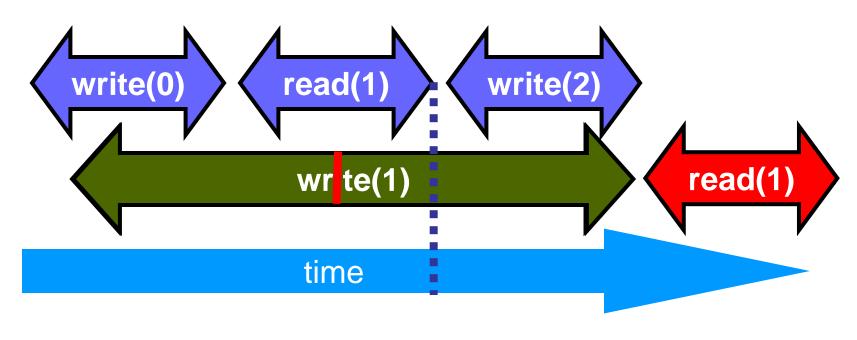


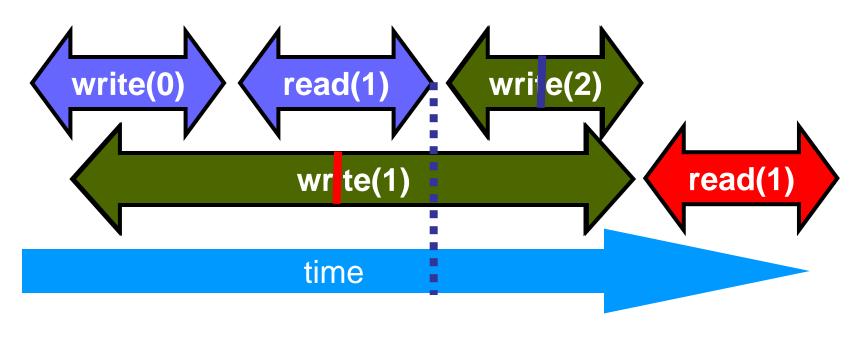


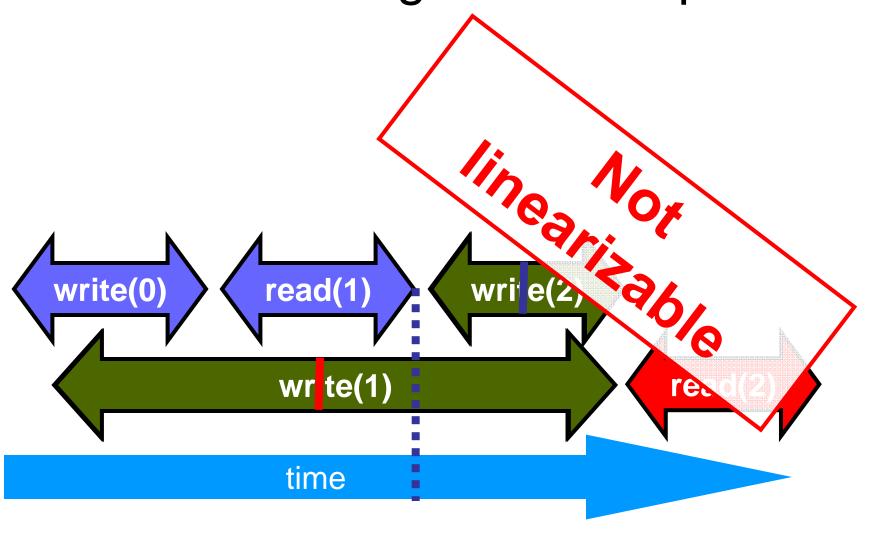












Talking About Executions

- Why?
 - Can't we specify the linearization point of each operation without describing an execution?
- Not Always
 - In some cases, linearization point depends on the execution

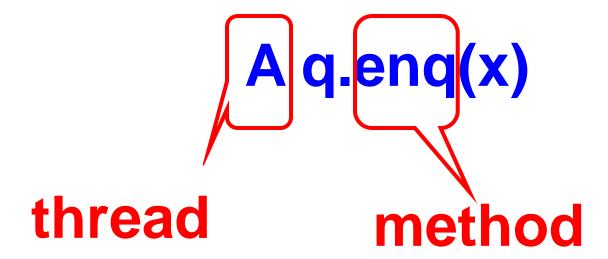
Formal Model of Executions

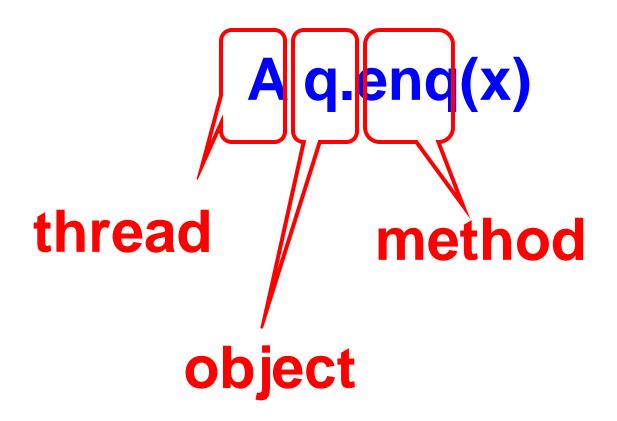
- Define precisely what we mean
 - Ambiguity is bad when intuition is weak
- Allow reasoning
 - Formal
 - But mostly informal
 - In the long run, actually more important
 - Ask me why!

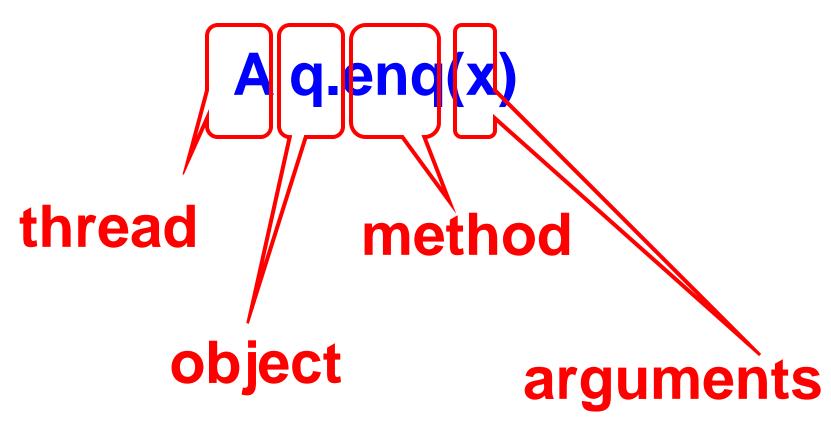
Split Method Calls into Two Events

- Invocation
 - method name & args
 - -q.enq(x)
- Response
 - result or exception
 - -q.enq(x) returns void
 - -q.deq() returns x
 - -q.deq() throws empty

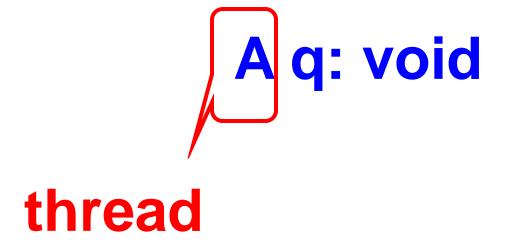
A q.enq(x)

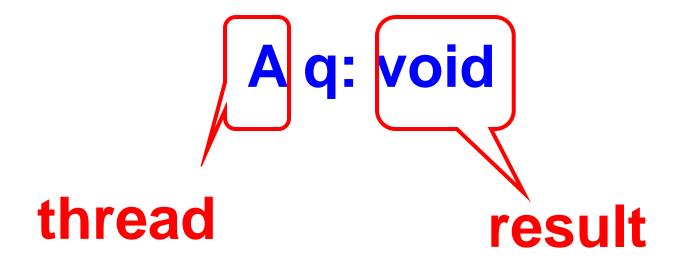


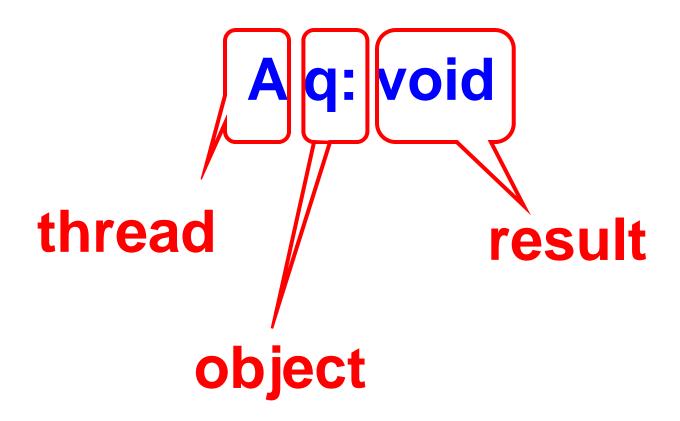


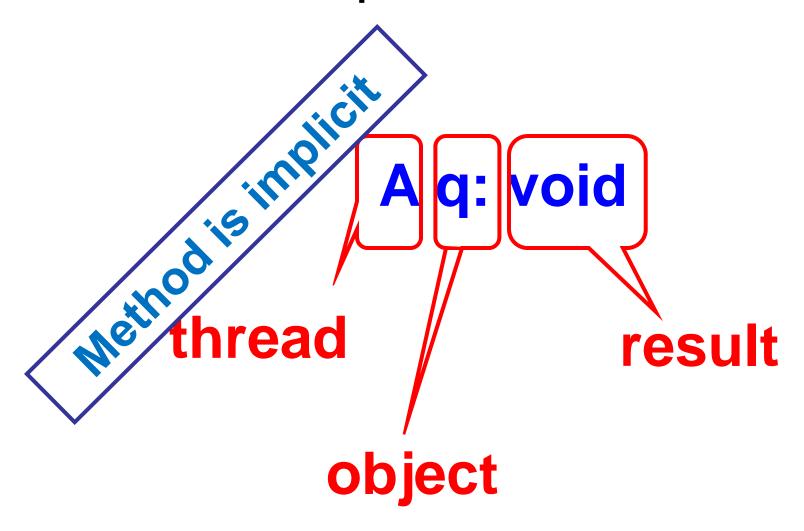


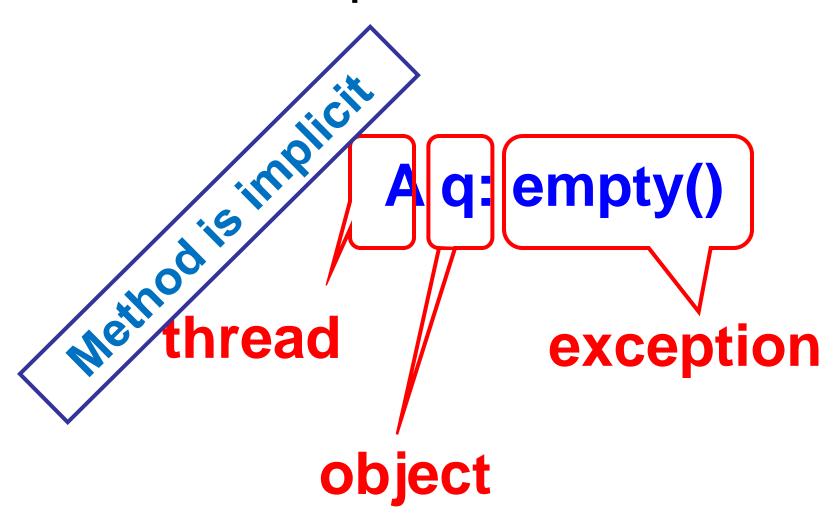
A q: void









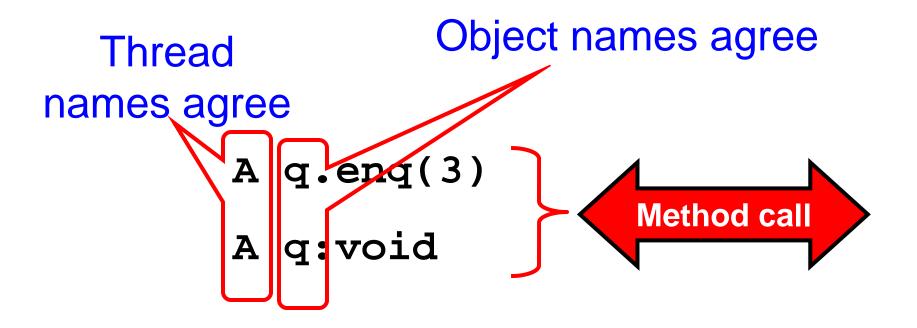


History - Describing an Execution

```
A q.enq(3)
A q:void
A q.enq(5)
H = B p.enq(4)
B p:void
B q.deq()
Sequence of invocations and responses
```

Definition

Invocation & response match if



Object Projections

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
```

Object Projections

```
A q.enq(3)
A q:void
H|q = B p.void
B q.deq()
B q:3
```

Thread Projections

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
```

Thread Projections

```
HIB = B p.enq(4)
B p:void
B q.deq()
B q:3
```

```
A q.enq(3)
A q:void
A q.enq(5)
H = B p.enq(4)
B p:void
B q.deq() An invocation is pending if it has no matching respnse
```

```
A q.enq(3)
A q:void
A q.enq(5)
H = B p.enq(4)
B p:void
B q.deq()
May or may not
have taken effect
```

```
A q.enq(3)
A q:void
A q.enq(5)
H = B p.enq(4)
B p:void
B q.deq()
B q:3 invocations
```

```
A q.enq(3)
A q:void

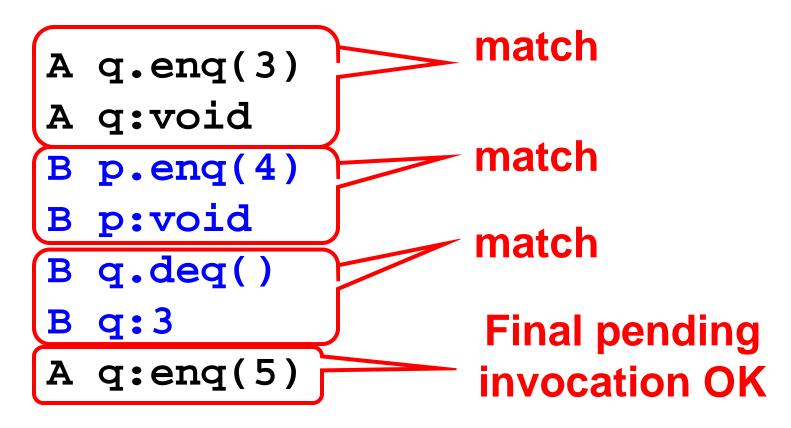
Complete(H) = B p.enq(4)
B p:void
B q.deq()
B q:3
```

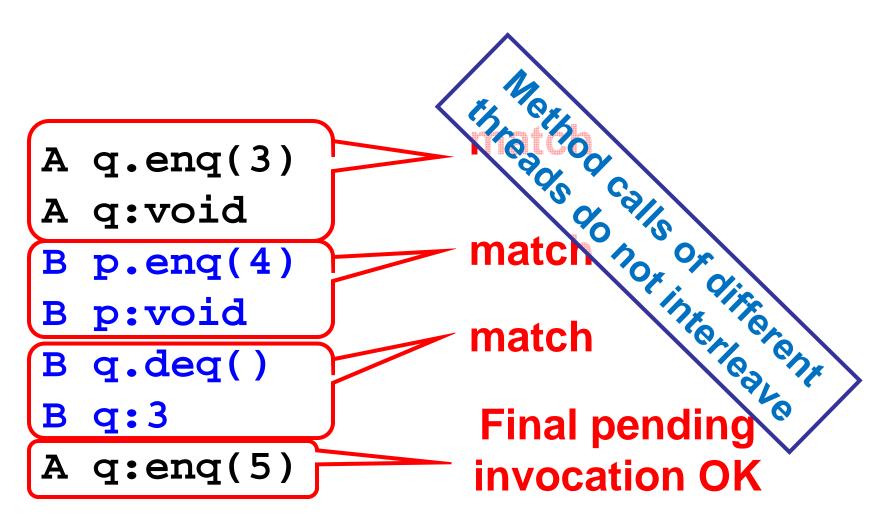
```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
A q:enq(5)
```





Well-Formed Histories

```
A q.enq(3)
B p.enq(4)
B p:void
B q.deq()
A q:void
B q:3
```

Well-Formed Histories

```
Per-thread projections
sequential

A q.enq(3)
B p.enq(4)
B q.deq()

B p.enq(4)
B p.enq(4)
B p.enq(4)
B q.deq()
```

A q:void

B q:3

Well-Formed Histories

```
Per-thread projections
sequential

A q.enq(3)
B p.enq(4)
B p.enq(4)
B p.enq(4)
B p:void
H= B q.deq()
A q:void
B q:3

H | A= A q.enq(3)
A q:void
B q:3
```

Equivalent Histories

```
Threads see the same \begin{bmatrix} H & A & = G & A \\ H & B & = G & B \end{bmatrix}
```

```
H=
A q.enq(3)
B p.enq(4)
B p:void
B q.deq()
A q:void
B q:3
```

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
```

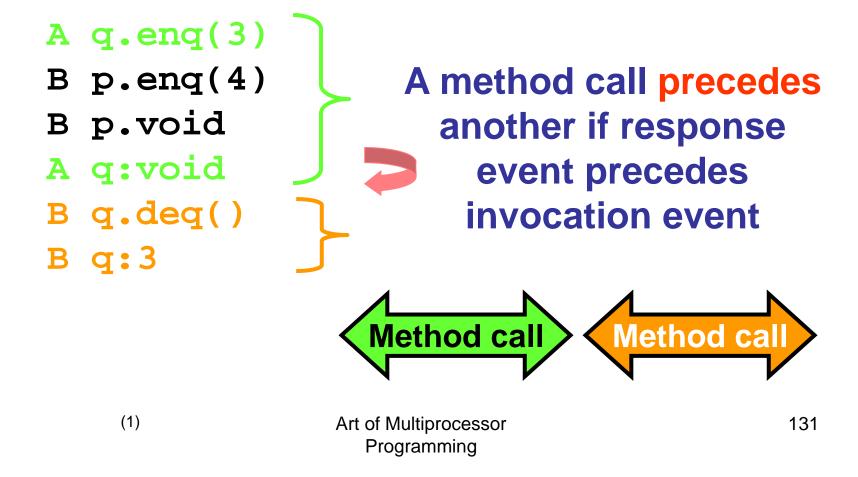
Sequential Specifications

- A sequential specification is some way of telling whether a
 - Single-thread, single-object history
 - Is legal
- For example:
 - Pre and post-conditions
 - But plenty of other techniques exist ...

Legal Histories

- A sequential (multi-object) history H is legal if
 - For every object x
 - H|x is in the sequential spec for x

Precedence



Non-Precedence

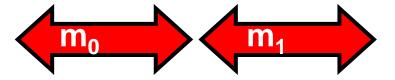
```
A q.enq(3)
B p.enq(4)
B p.void
B q.deq()
A q:void
B q:3

Method call

Method call
```

Notation

- Given
 - History H
 - method executions m₀ and m₁ in H
- We say $m_0 \rightarrow H m_1$, if
 - m₀ precedes m₁
- Relation m₀ → H m₁ is a

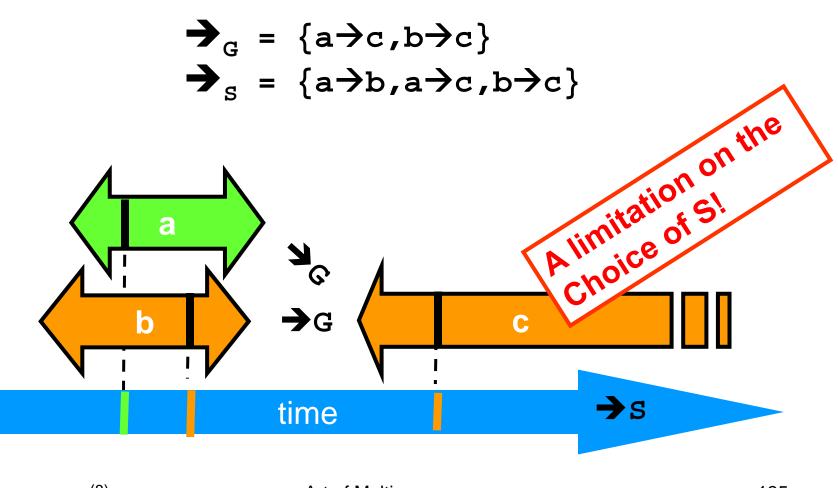


- Partial order
- Total order if H is sequential

Linearizability

- History H is *linearizable* if it can be extended to G by
 - Appending zero or more responses to pending invocations
 - Discarding other pending invocations
- So that G is equivalent to
 - Legal sequential history S
 - where $\rightarrow_{\mathsf{G}} \subset \rightarrow_{\mathsf{S}}$

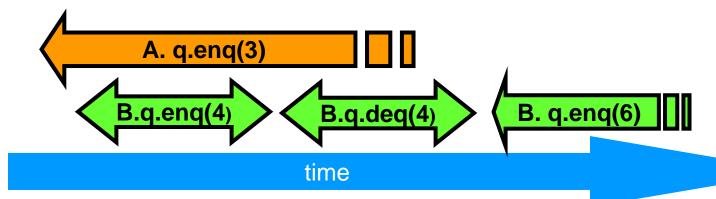
Ensuring $\rightarrow_{G} \subset \rightarrow_{S}$



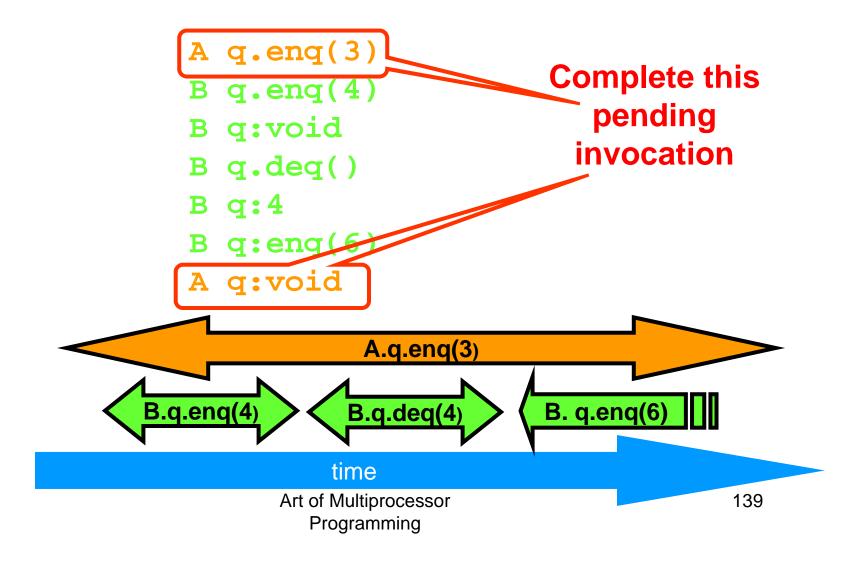
Remarks

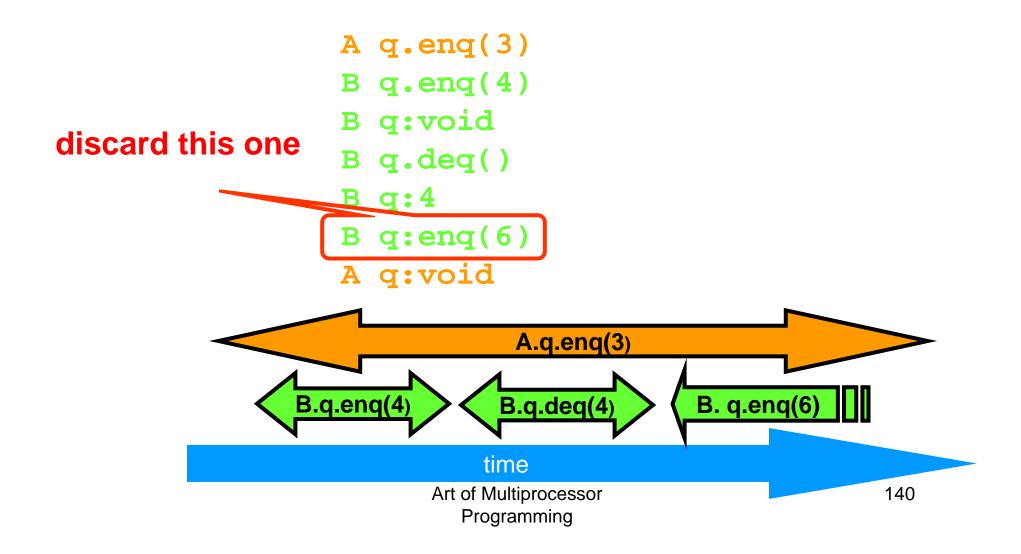
- Some pending invocations
 - Took effect, so keep them
 - Discard the rest
- Condition $\rightarrow_{\mathsf{G}} \subset \rightarrow_{\mathsf{S}}$
 - Means that S respects "real-time order" of G

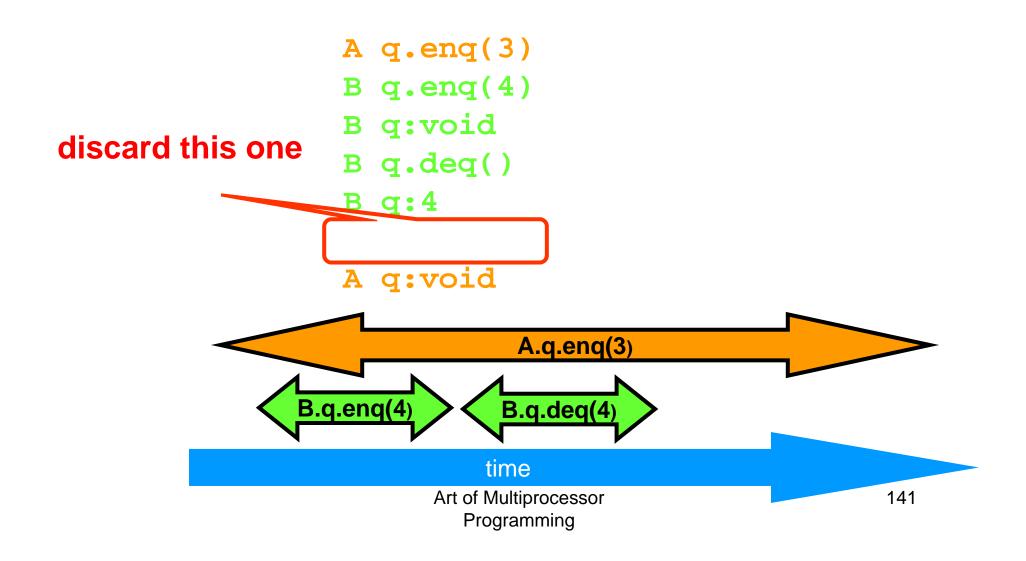
```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
```



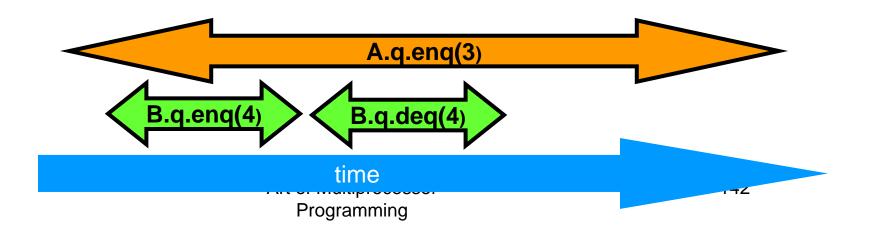
```
A q.enq(3)
                              Complete this
     q.enq(4)
                                 pending
     q:void
                                invocation
   B q.deq()
   B q:4
   B q:enq(6)
A. q.enq(3)
B.q.enq(4)
               B.q.deq(3)
                              B. q.enq(6)
              time
          Art of Multiprocessor
                                            138
            Programming
```







```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void
```



```
A q.enq(3)

B q.enq(4)

B q.enq(4)

B q:void

A q.enq(3)

B q.deq()

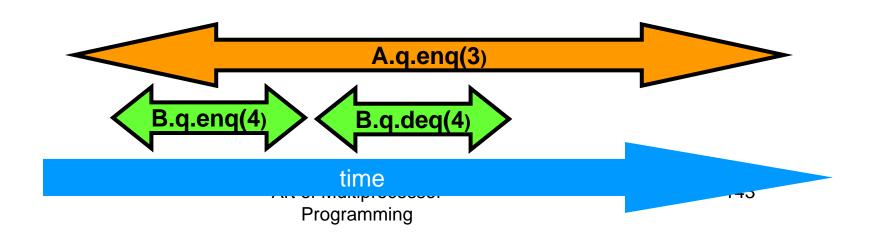
A q:void

B q:4

B q.deq()

A q:void

B q:4
```



```
Equivalent sequential history
                             q.enq(4)
A q.enq(3)
                           B q:void
B q.enq(4)
B q:void
                           A q.enq(3)
                           A q:void
B q.deq()
                           B q.deq()
B q:4
A q:void
                           B q:4
                     A.q.enq(3)
       B.q.er q(4
                    B.q. deq(4)
                   time
                 Programming
```

Concurrency

- How much concurrency does linearizability allow?
- When must a method invocation block?

Concurrency

- Focus on total methods
 - Defined in every state
- Example:
 - deq() that throws Empty exception
 - Versus deq() that waits
- Why?
 - Otherwise, blocking unrelated to synchronization

Concurrency

- Question: When does linearizability require a method invocation to block?
- Answer: never.
- Linearizability is non-blocking

Non-Blocking Theorem

```
If method invocation
  A q.inv(...)
is pending in history H, then there exists a
 response
  A q:res(...)
such that
  H + A q:res(...)
is linearizable
```

Proof

- Pick linearization S of H
- If S already contains
 - Invocation A q.inv(...) and response,
 - Then we are done.
- Otherwise, pick a response such that
 - -S + A q.inv(...) + A q:res(...)
 - Possible because object is total.

Composability Theorem

- History H is linearizable if and only if
 - For every object x
 - H|x is linearizable
- We care about objects only!
 - (Materialism?)

Why Does Composability Matter?

- Modularity
- Can prove linearizability of objects in isolation
- Can compose independently-implemented objects

Reasoning About Linearizability: Locking

```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
        throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  } finally {
    lock.unlock();
  }
}
```

Reasoning About Linearizability: Locking

```
public T deq() throws EmptyException {
  lock.lock();
  try {
    if (tail == head)
        throw new EmptyException();
    T x = items[head % items.length];
    head++;
    return x;
  }
  finally {
  lock.unlock();
  }
  Linearization points
  are when locks are
  released
```

More Reasoning: Wait-free

```
public class WaitFreeQueue {
                                       head
                                     capacity-1
  int head = 0, tail = 0;
  items = (T[]) new Object[capacity];
  public void enq(Item x) {
    if (tail-head == capacity) throw
         new FullException();
    items[tail % capacity] = x; tail++;
  public Item deq() {
     if (tail == head) throw
         new EmptyException();
     Item item = items[head % capacity]; head++;
     return item;
```

More Reasoning: Wait-free

```
tFreeQueue {
                       Linearization order is
                       order head and tail
                          fields modified
         1d enq(Item x) {
      lail-head == capacity) throw
  items[tail % capacity] = x;
public Item deq() {
   if (tail == head) throw
       new EmptyException();
   Item item = items[head % capacity];
   return item;
                 Art of Multiprocessor
                                                155
                   Programming
```

Strategy

- Identify one atomic step where method "happens"
 - Critical section
 - Machine instruction
- Doesn't always work
 - Might need to define several different steps for a given method

Linearizability: Summary

- Powerful specification tool for shared objects
- Allows us to capture the notion of objects being "atomic"
- Don't leave home without it

Alternative: Sequential Consistency

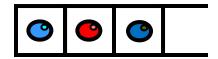
- History H is Sequentially Consistent if it can be extended to G by
 - Appending zero or more responses to pending invocations
 - Discarding other pending invocations
- So that G is equivalent to a
 - Legal sequential history S

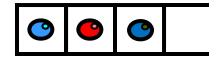
Differs from linearizability

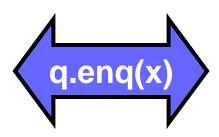


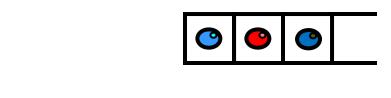
Sequential Consistency

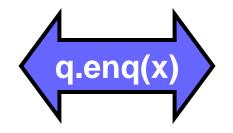
- No need to preserve real-time order
 - Cannot re-order operations done by the same thread
 - Can re-order non-overlapping operations done by different threads
- Often used to describe multiprocessor memory architectures

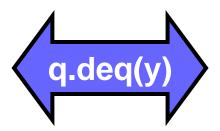




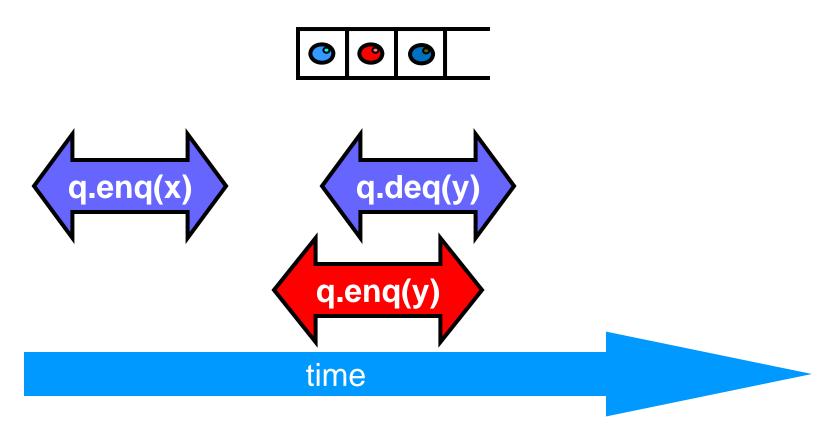




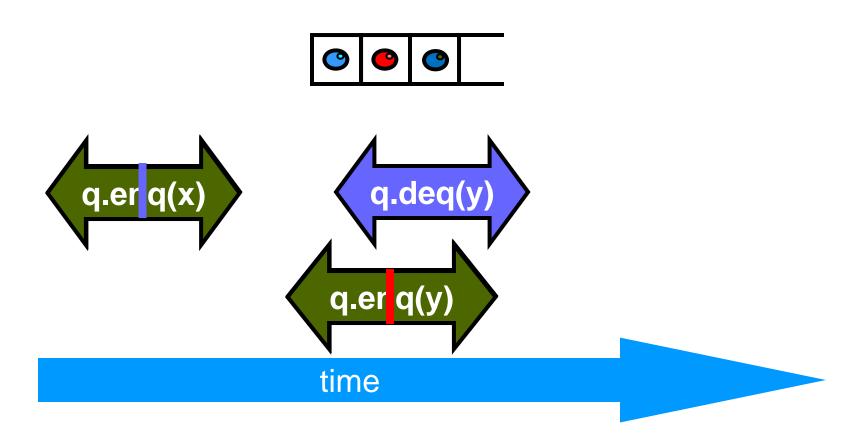


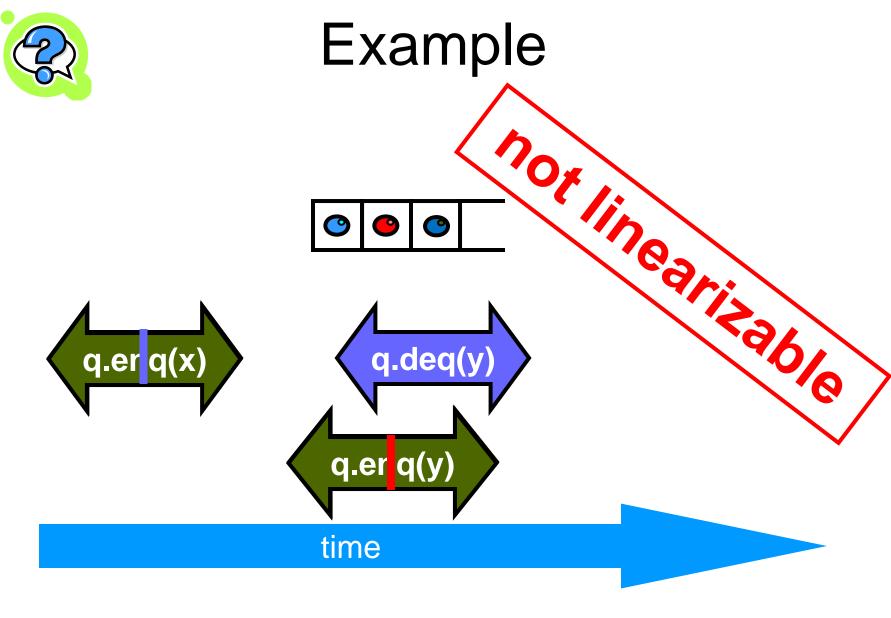


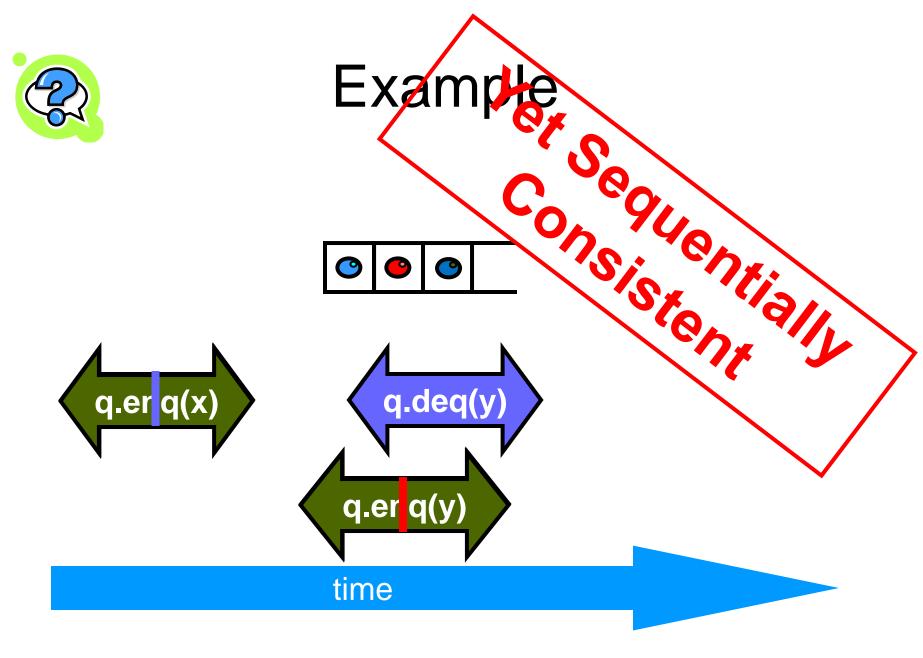








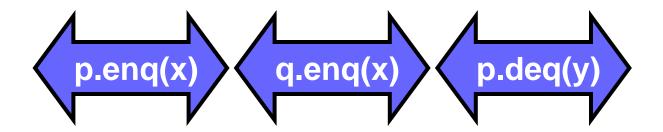




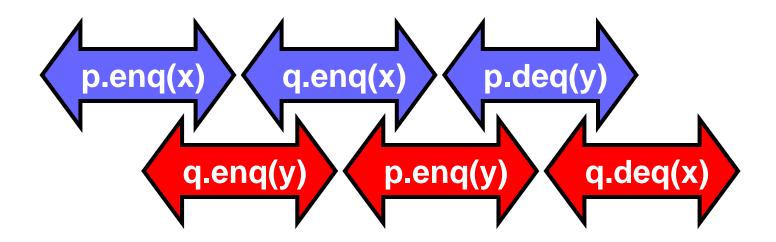
Theorem

Sequential Consistency is not composable

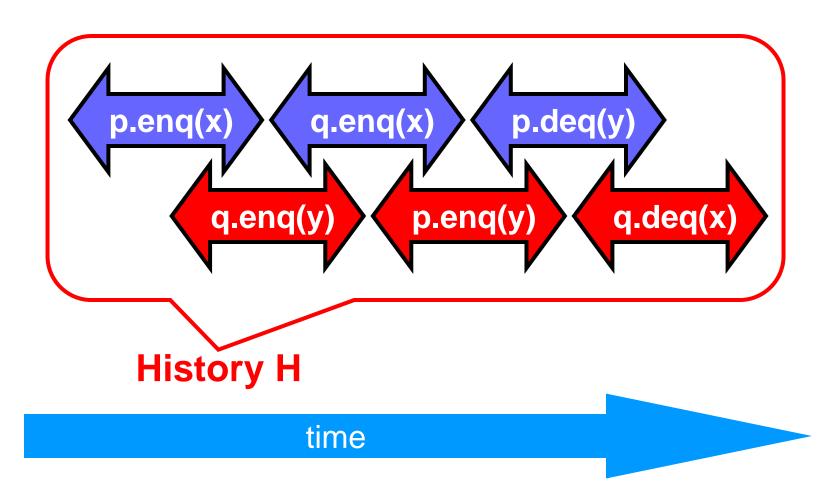
FIFO Queue Example



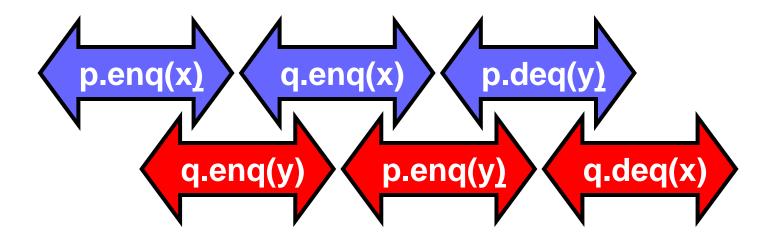
FIFO Queue Example



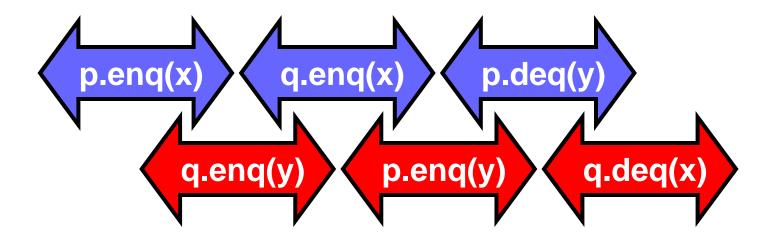
FIFO Queue Example



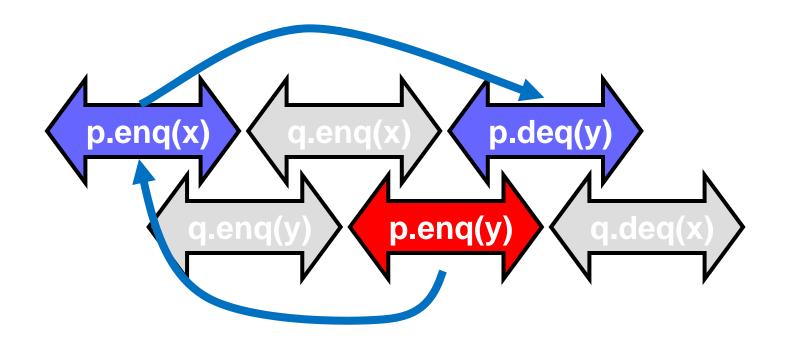
H|p Sequentially Consistent



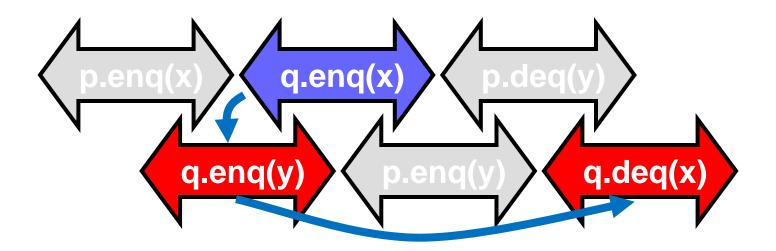
H|q Sequentially Consistent



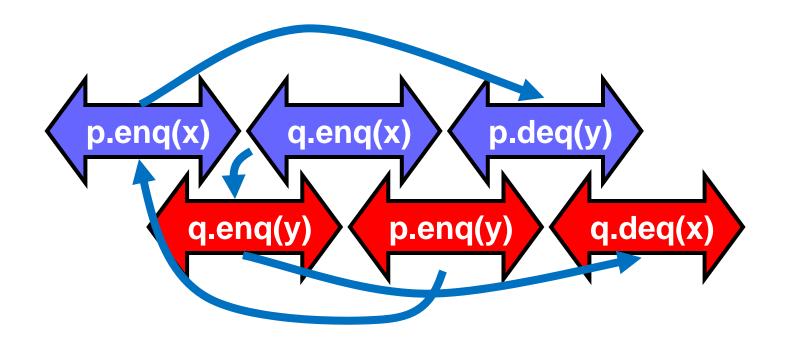
Ordering imposed by p



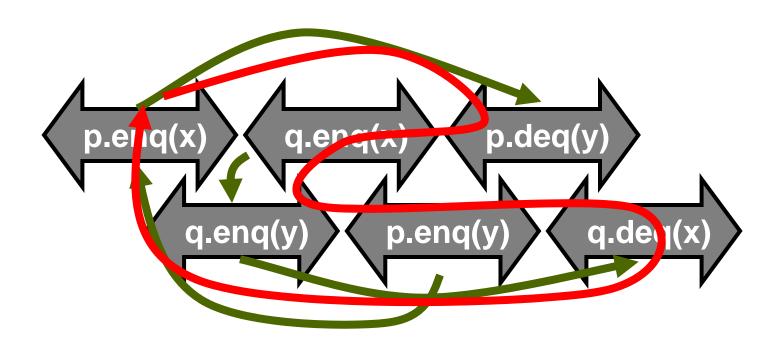
Ordering imposed by q



Ordering imposed by both



Combining orders

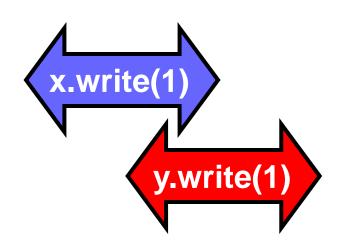


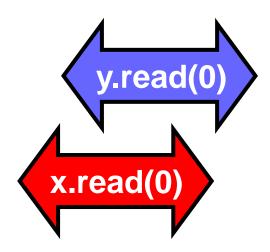


Fact

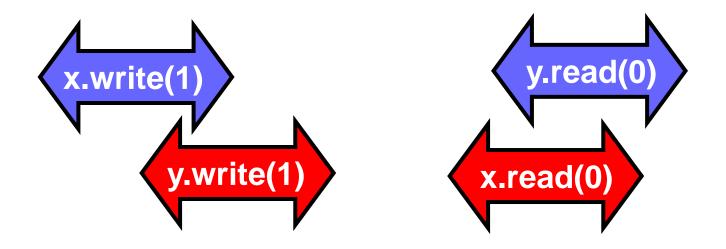
- Most hardware architectures don't support sequential consistency
- Because they think it's too strong
- Here's another story ...

The Flag Example



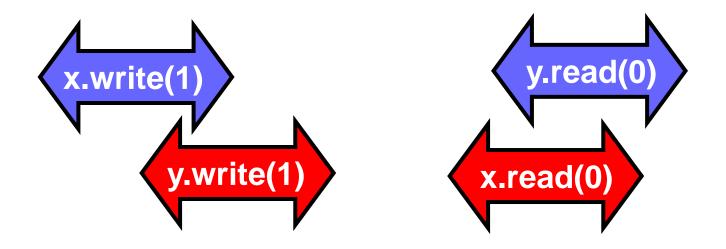


The Flag Example



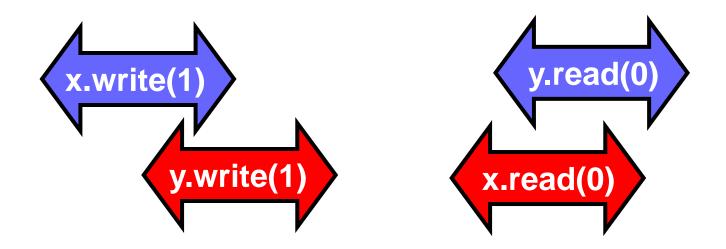
- Each thread's view is sequentially consistent
 - It went first

The Flag Example



- Entire history isn't sequentially consistent
 - Can't both go first

The Flag Example



- Is this behavior really so wrong?
 - We can argue either way ...

Opinion1: It's Wrong

- This pattern
 - Write mine, read yours
- Is exactly the flag principle
 - Beloved of Alice and Bob
 - Heart of mutual exclusion
 - Peterson
 - Bakery, etc.
- It's non-negotiable!

Opinion2: But It Feels So Right ...

- Many hardware architects think that sequential consistency is too strong
- Too expensive to implement in modern hardware
- OK if flag principle
 - violated by default
 - Honored by explicit request

Memory Hierarchy

- On modern multiprocessors, processors do not read and write directly to memory.
- Memory accesses are very slow compared to processor speeds,
- Instead, each processor reads and writes directly to a cache

Memory Operations

- To read a memory location,
 - load data into cache.
- To write a memory location
 - update cached copy,
 - lazily write cached data back to memory

While Writing to Memory

- A processor can execute hundreds, or even thousands of instructions
- Why delay on every memory write?
- Instead, write back in parallel with rest of the program.

Revisionist History

- Flag violation history is actually OK
 - processors delay writing to memory
 - until after reads have been issued.
- Otherwise unacceptable delay between read and write instructions.
- Who knew you wanted to synchronize?

Who knew you wanted to synchronize?

- Writing to memory = mailing a letter
- Vast majority of reads & writes
 - Not for synchronization
 - No need to idle waiting for post office
- If you want to synchronize
 - Announce it explicitly
 - Pay for it only when you need it

Explicit Synchronization

- Memory barrier instruction
 - Flush unwritten caches
 - Bring caches up to date
- Compilers often do this for you
 - Entering and leaving critical sections
- Expensive

Volatile

- In Java, can ask compiler to keep a variable up-to-date with volatile keyword
- Also inhibits reordering, removing from loops, & other "optimizations"

Real-World Hardware Memory

- Weaker than sequential consistency
- But you can get sequential consistency at a price
- OK for expert, tricky stuff
 - assembly language, device drivers, etc.
- Linearizability more appropriate for highlevel software

Linearizability

- Linearizability
 - Operation takes effect instantaneously between invocation and response
 - Uses sequential specification, locality implies composablity
 - Good for high level objects

Correctness: Linearizability

- Sequential Consistency
 - Not composable
 - Harder to work with
 - Good way to think about hardware models
- We will use *linearizability* as in the remainder of this course unless stated otherwise

Progress

- We saw an implementation whose methods were lock-based (deadlock-free)
- We saw an implementation whose methods did not use locks (lock-free)
- How do they relate?

Progress Conditions

- Deadlock-free: some thread trying to acquire the lock eventually succeeds.
- Starvation-free: every thread trying to acquire the lock eventually succeeds.
- Lock-free: some thread calling a method eventually returns.
- Wait-free: every thread calling a method eventually returns.

Progress Conditions

Non-Blocking

Blocking

Everyone makes progress

Someone makes progress

Wait-free	Starvation-free
Lock-free	Deadlock-free

Summary

 We will look at *linearizable blocking* and non-blocking implementations of objects.



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