Using Proofs-from-Tests to Verify Higher-Order Programs

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Joint work with He Zhu

Introduction

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    - ★ Enrich the class of *provably* correct optimizations
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  - *Modular*
  - *Precise*
    - Incorporate notions of {path, context} - sensitivity
  - *Scalable and lightweight*
    - Use off-the-shelf verification tools
  - *Understandable*
    - Analysis over a high-level intermediate representation
    - Useful for error checking

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Failure to infer a "rich" specification only implies a missed optimization opportunity, not a violation of compiler correctness

Challenges

Consider

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fun arraymax a g =
  let fun am h j m =
       let val k = h j
           val a = assert (k>=0 /\ k<len a)
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p is a maximal element

Expressive assertion language

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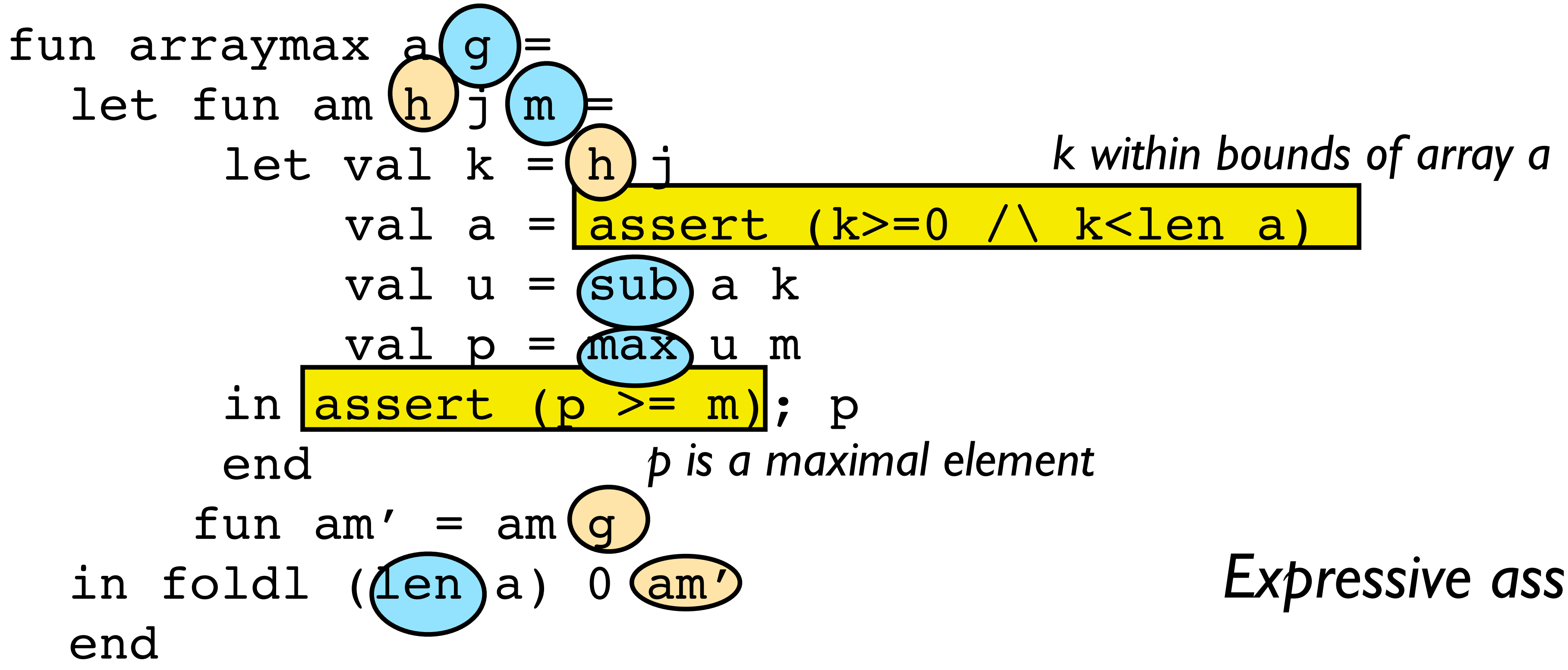

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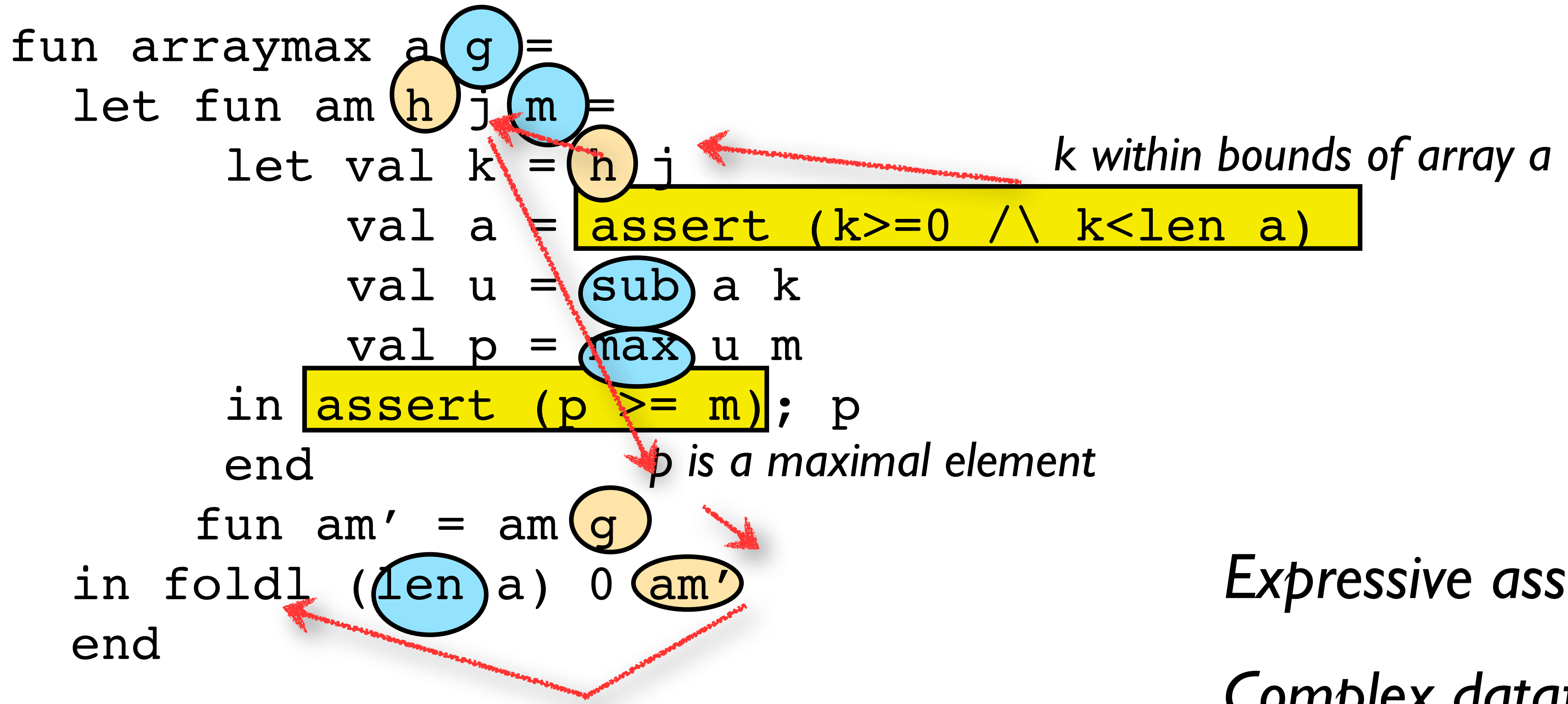

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Specifications must propagate across procedure boundaries

Liquid Types

```
fun max x y  =
  if x > y
      then x
      else y



val r =
   max a b

val _ = assert (r >= a)
```

Liquid Types

```
fun max x y  = {x : {ν: int | true} → y : {ν : int | true} →{ν : int | ν >= x /\ ν >= y}}
  if x > y
     then x {ν : int | ν = x}
     else y {ν : int | ν = y}
  {ν : int | ν = x}

val r =  {ν >= a /\ ν >= b}
   max a b

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Extend standard types with refinement predicates that refer to program variables and primitive functions

Well-typed program implies correctness

Refinement Predicates

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    m : {true} → true} =
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Set of logical qualifiers is potentially quite large

Would like to infer the potential set of qualifiers from context and refine them as appropriate

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materialize quantifiers based on constraints introduced in non-lexical scope

Basic Idea

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- Analyze higher-order programs using first-order verification engine
  - ★ Use a modular (inter-procedural) analysis to abstract dataflow through higher-order procedures.
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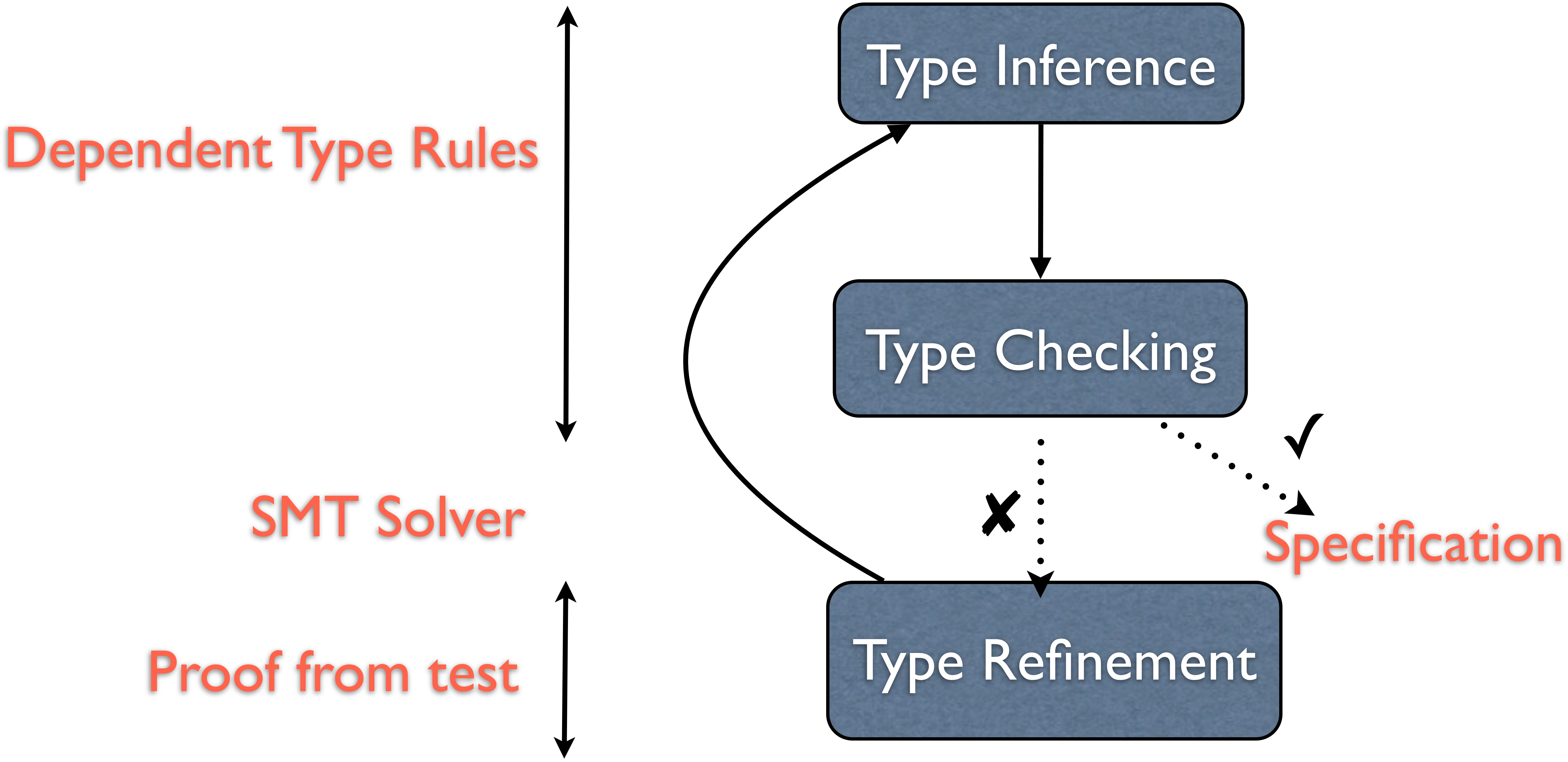
- Integrate these steps as a separate compiler phase

Framework



Dependent Type Rules

SMT Solver

Proof from test

Type Inference

Type Checking

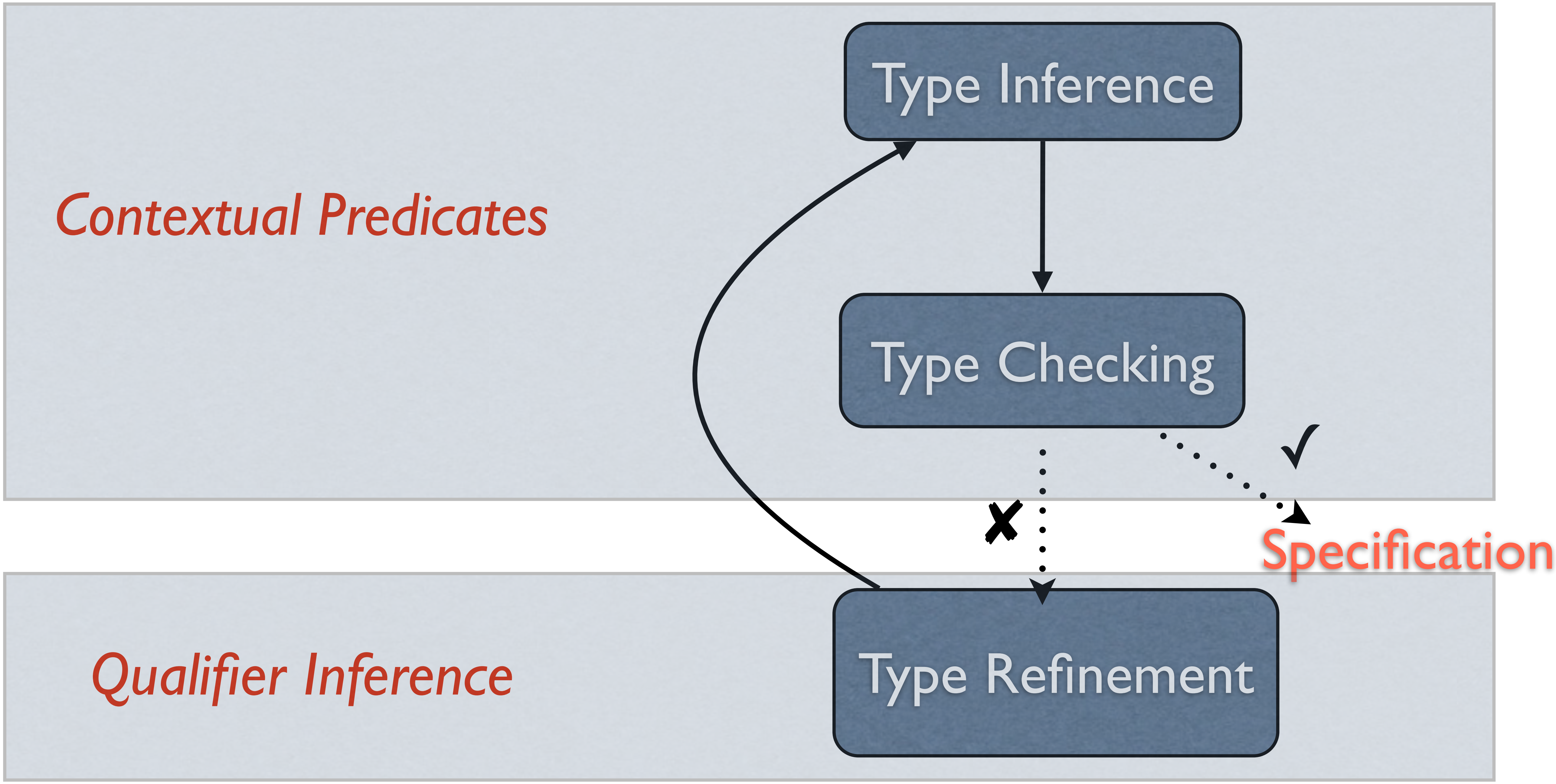
✗

✓

Specification

Type Refinement

Framework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propagate dependent type constraints for
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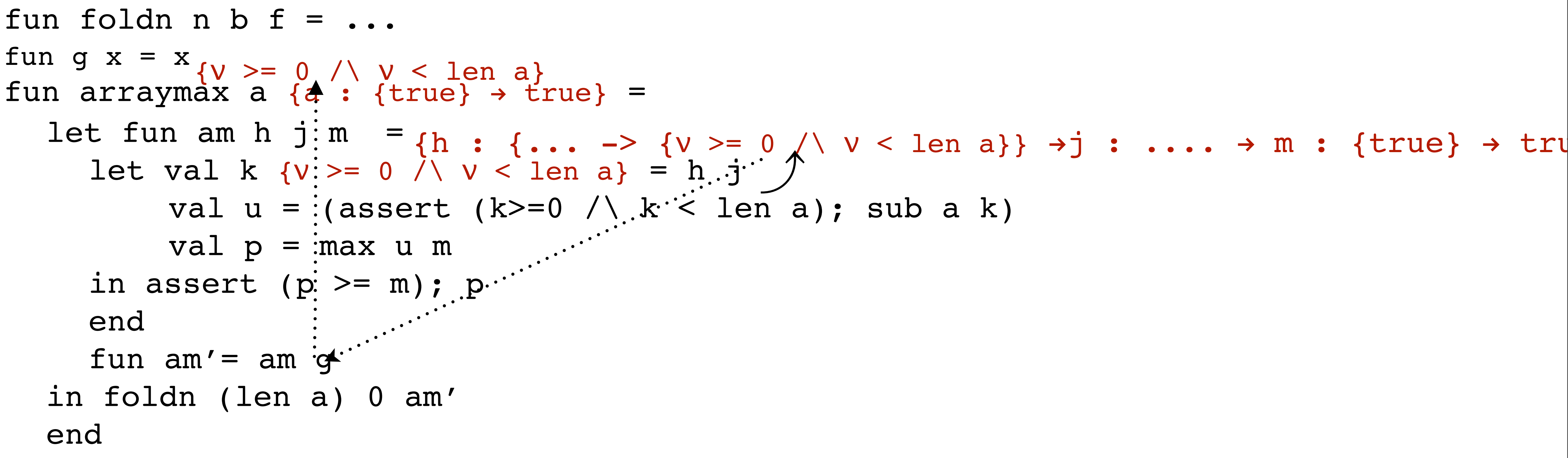

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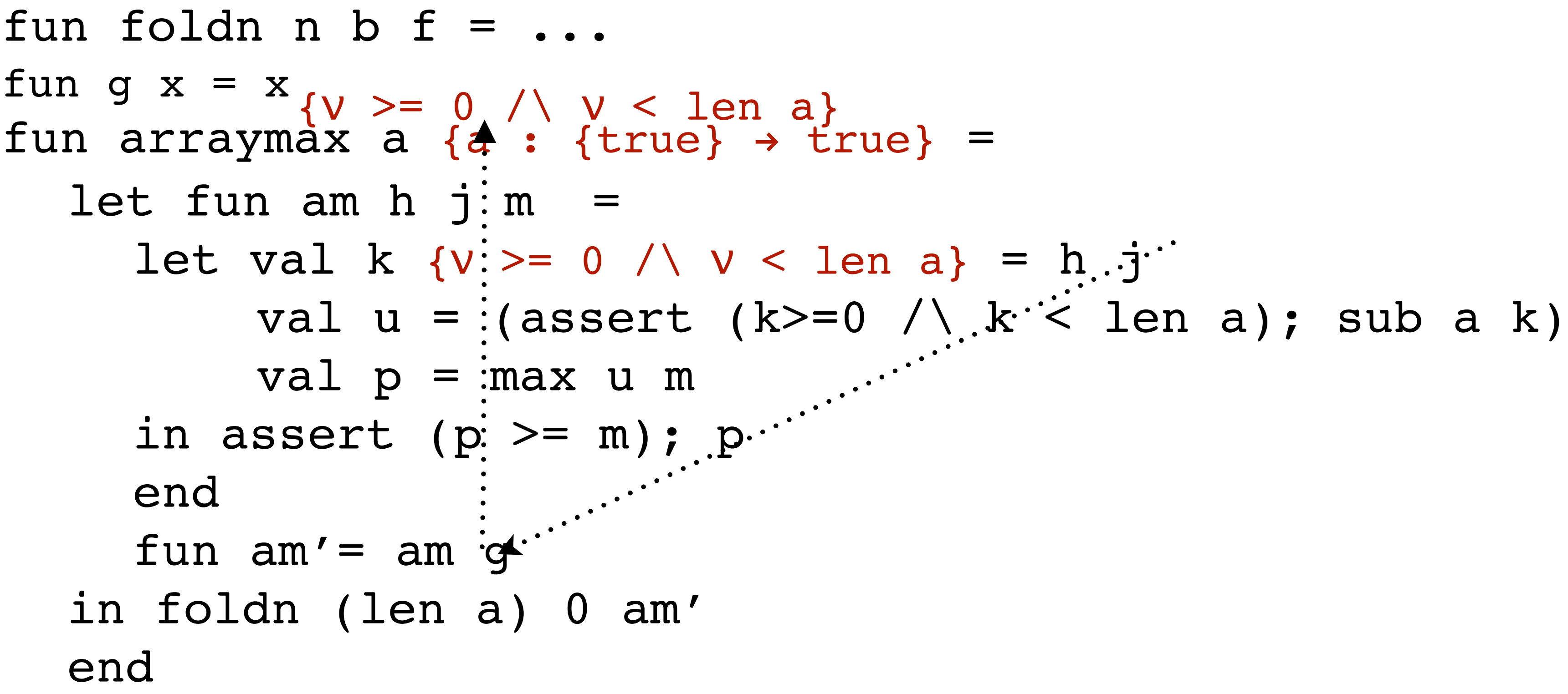

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Type Checking

- Construct a verification condition (VC) as a first-order formula from inferred dependent types
  - ★ Typing rules track path conditions that are encoded in the structure of the VC
- The condition to be verified by an SMT solver is the negation of the VC
  - ★ unsat => type checking successful
  - ★ sat => additional strengthening required to derive a consistent specification

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App

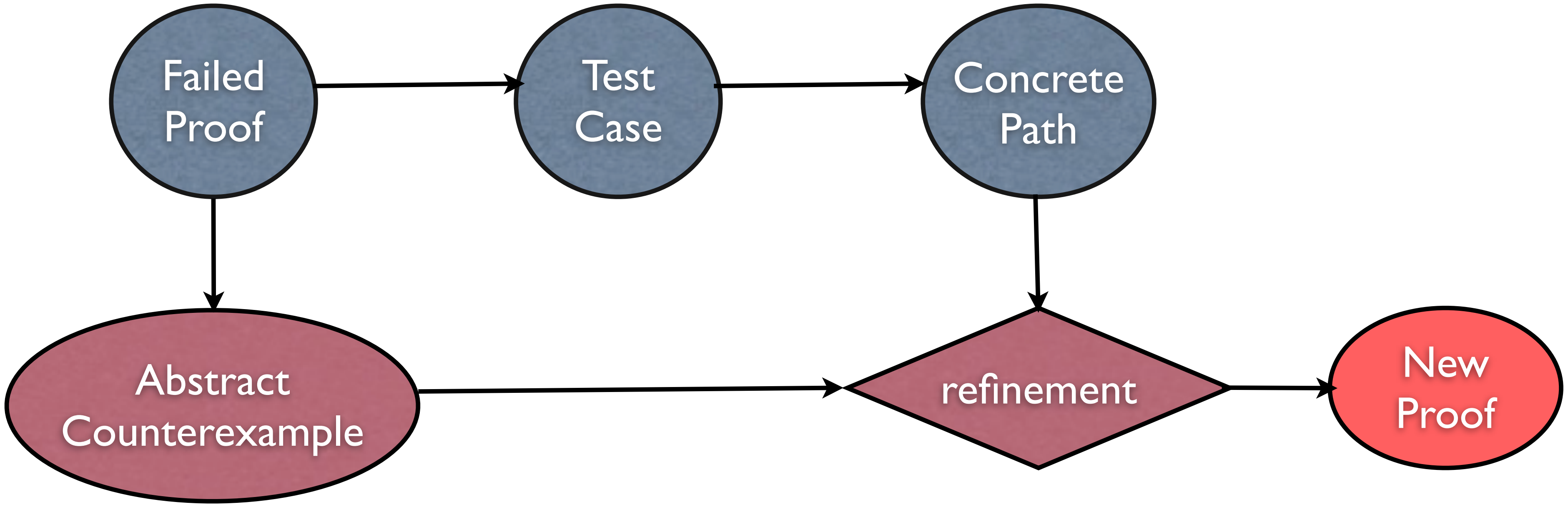
$$\frac{\Gamma \vdash v_2 : P_x \quad \Gamma \vdash e_f : (x : P_x \rightarrow P)}{\Gamma \vdash e_f(v_2) : [v_2/x]P}$$

IF

$$\frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash P \quad \Gamma; e_1 \vdash e_2 : P \quad \Gamma; \neg e_1 \vdash e_3 : P}{\Gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : P}$$

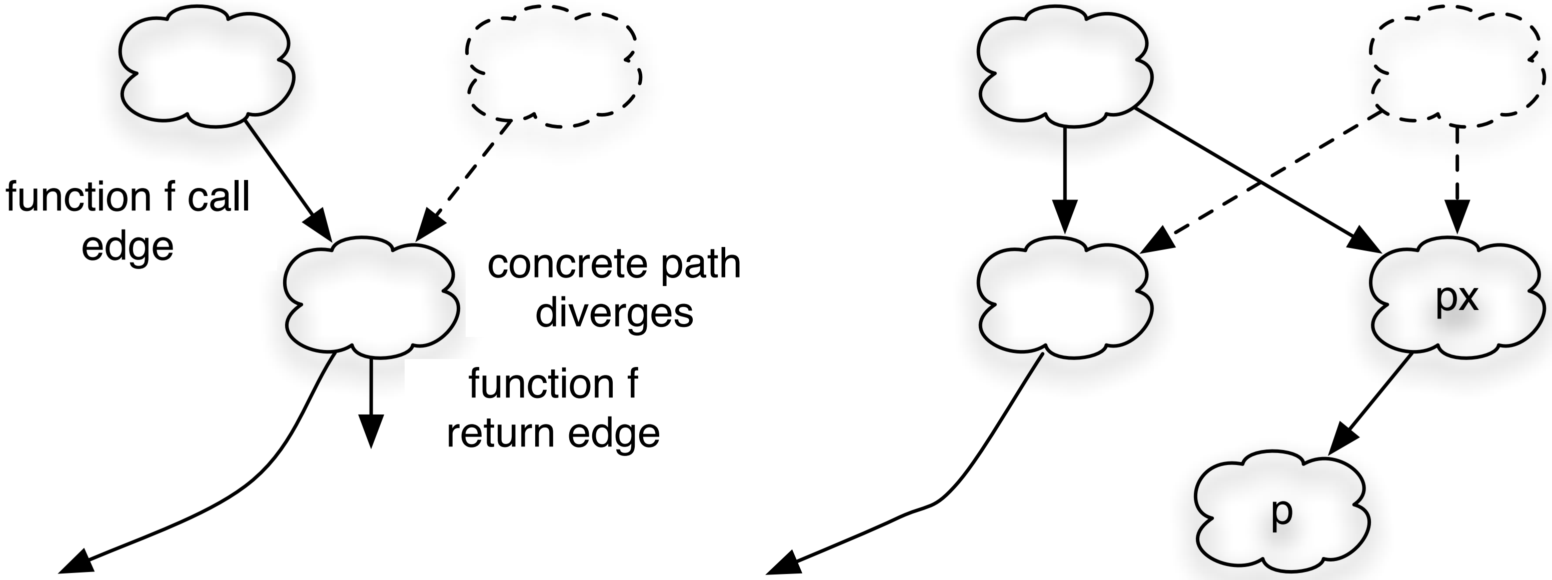
Type Refinement

- Type Refinement is used to augment the set of qualifiers
  - ★ We could re-analyze the body of called functions along a counterexample (inter-procedural) path from a call-site.
  - ★ But, within a compiler, can explore concrete paths fairly easily
    - ✦ Internally, build a compile-test-run loop over an optimized well-typed IR
- Use lightweight testing to determine where and how to refine the type system. (Proofs-from-tests aka Dash)



- Testing provides a concrete witness to guide how existing invariants can be strengthened.

Directed Testing



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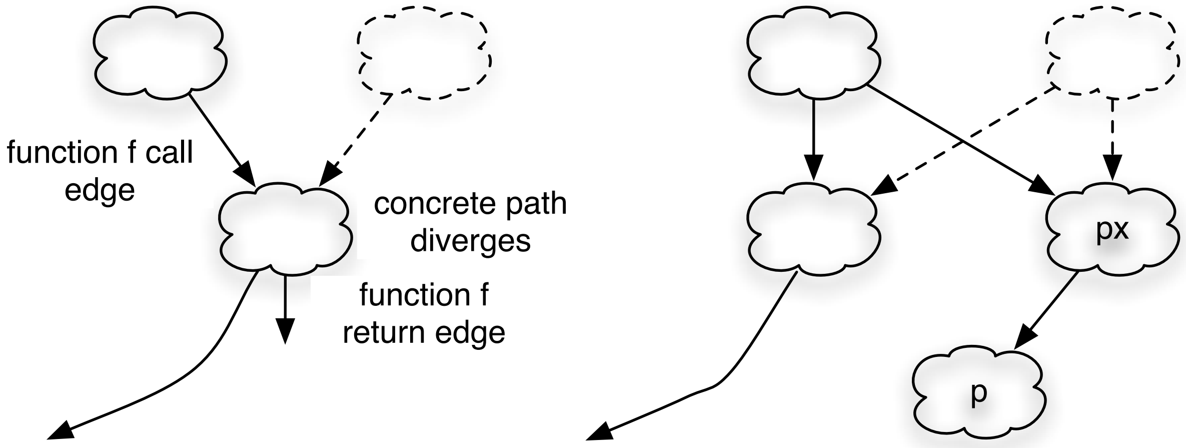

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- If environment at a call-site
  - ★ implies the called function’s pre-condition, then original counterexample will no longer be reported if we strengthen called function’s post-condition and compute weakest precondition.
  - ★ does not imply the called function’s precondition, then must strengthen function’s pre-condition to force divergence.
- Goal: either eliminate the counterexample by strengthening callee’s post-condition, or direct test case execution to converge to counterexample, strengthening caller’s pre-condition

Example

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let fun g x = x
    fun f x = if x < 2
                 then let s = g x
                      in assert (s <= 0)
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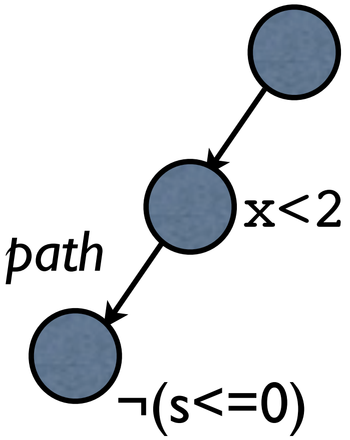

counterexample path

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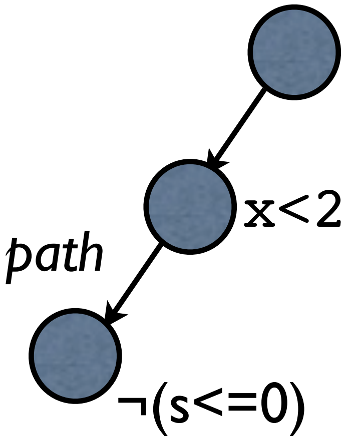

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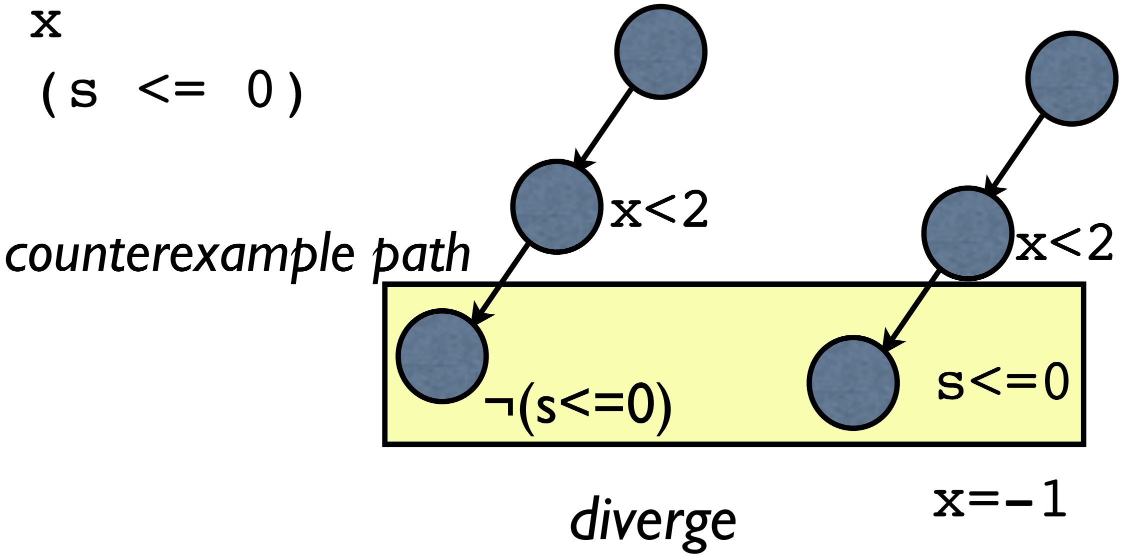

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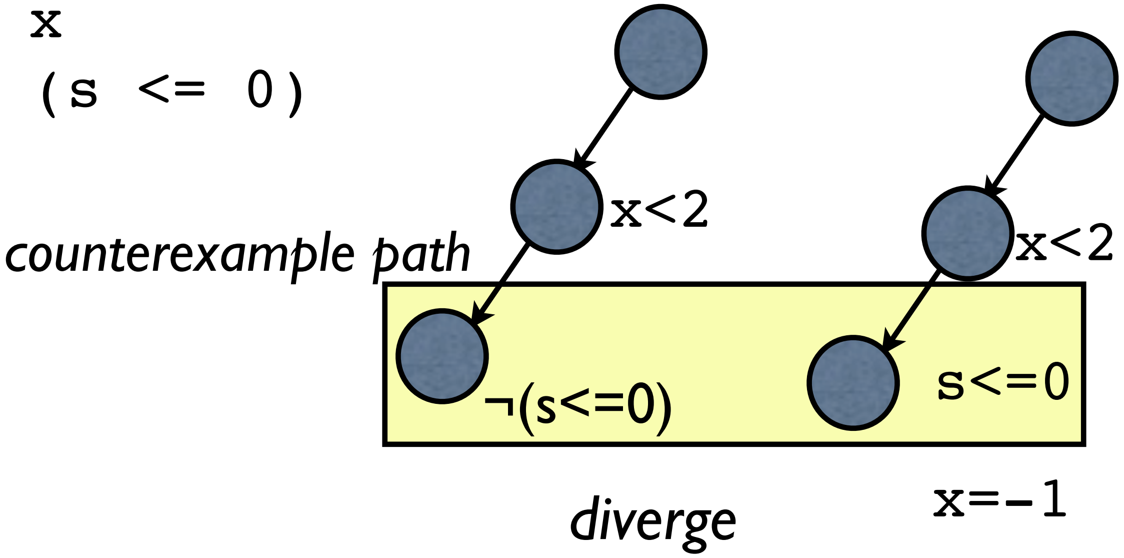

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Strengthen g's type which is initially $\{\mathtt{true}\} \rightarrow \{\mathtt{true}\}$

$$\{\nu \leq 0\} \rightarrow \{\nu \leq 0\}$$

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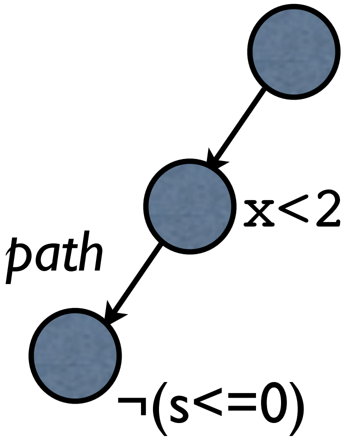

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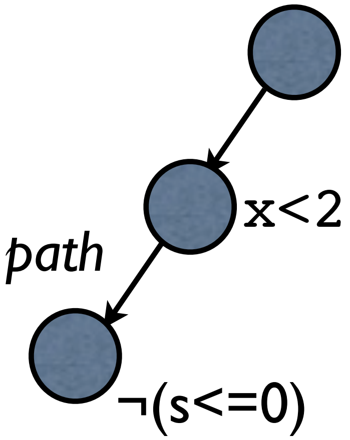
Strengthen g's type which is initially $\{\mathtt{true}\} \rightarrow \{\mathtt{true}\}$

$\{\nu \leq 0\} \rightarrow \{\nu \leq 0\}$

Example

```
let fun g x = x
    fun f x = if x < 2
                 then let s = g x
                      in assert (s <= 0)
                      end
                 else ()
  ...
```



Initially, solve:

$$\neg VC : \neg(\mathrm{x} < 2 \Rightarrow s \leq 0)$$

Generate a concrete test supplying

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$$\{\nu \leq 0\} \rightarrow \{\nu \leq 0\}$$

Generate new verification condition based on g's refinement

$$\neg VC : \neg((\mathrm{x} < 2 \wedge (x \leq 0 \Rightarrow s \leq 0) \Rightarrow (s \leq 0))$$

Example

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    fun f x = if x < 2
                 then let s = g x
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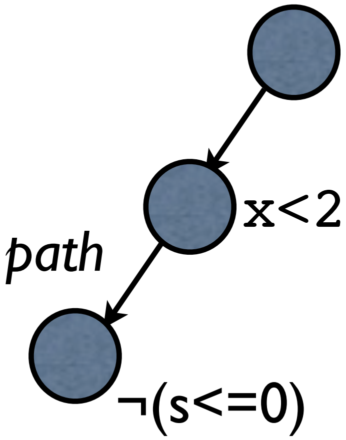

counterexample path

Initially, solve:

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Generate new verification condition based on g's refinement

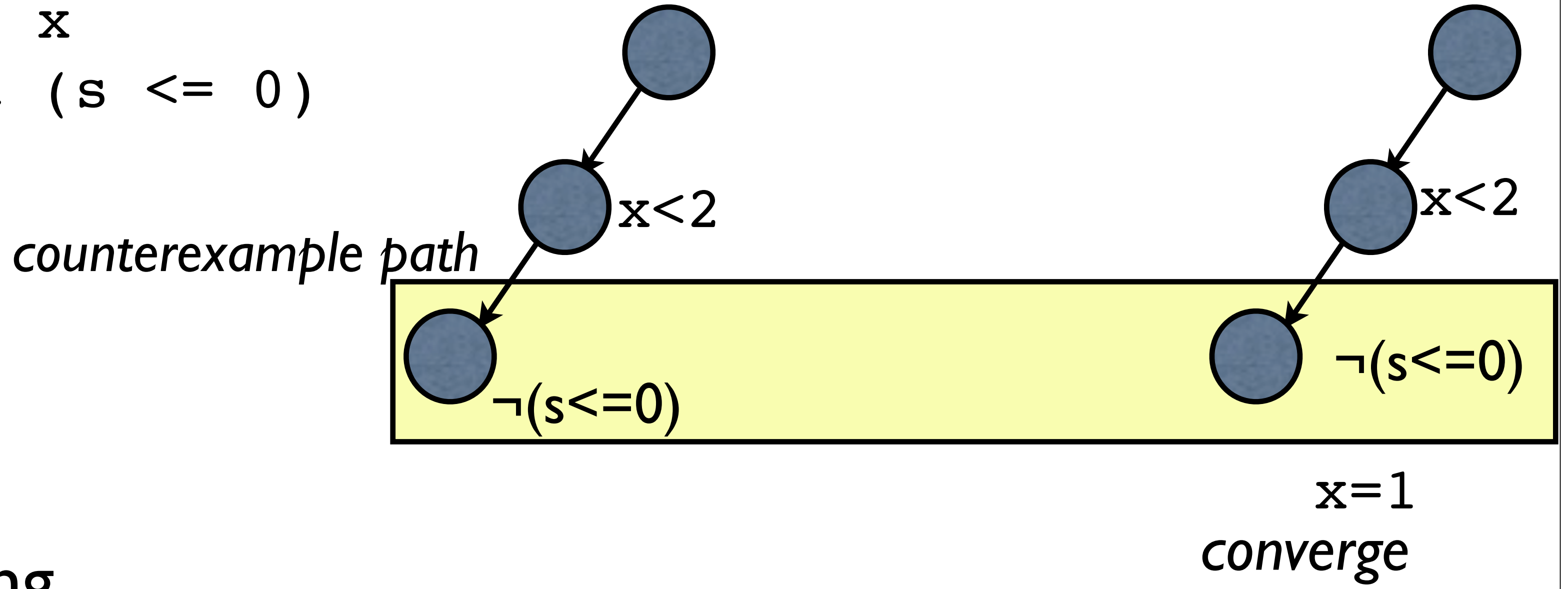
$$\neg VC : \neg((\mathrm{x} < 2 \wedge (x \leq 0 \Rightarrow s \leq 0) \Rightarrow (s \leq 0))$$

Generate new concrete test supplying

$$\mathrm{x} = 1$$

Example

```
let fun g x = x
    fun f x = if x < 2
                 then let s = g x
                      in assert (s <= 0)
                      end
                 else ()
  ...
```



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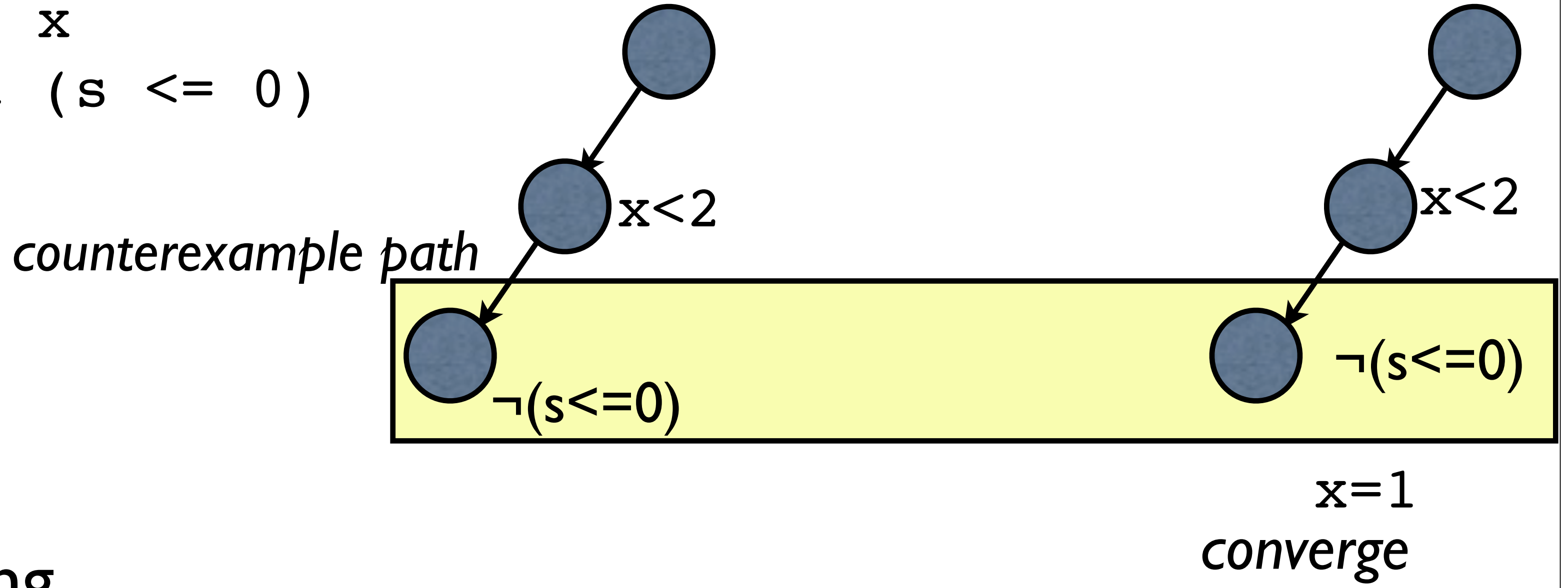

Initially, solve:

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Generate a concrete test supplying

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$$\{\nu \leq 0\} \rightarrow \{\nu \leq 0\}$$

Generate new verification condition based on g's refinement

$$\neg VC : \neg((\mathrm{x} < 2 \wedge (x \leq 0 \Rightarrow s \leq 0) \Rightarrow (s \leq 0))$$

Generate new concrete test supplying

$$\mathrm{x} = 1$$

Strengthen pre-condition for f to $\{\mathrm{x} : \nu \leq 0\}$to eliminate the counter-example

Pre-Condition Refinement

```
fun foldn n b f
  {n : true → b : true → f : {x1: {v >= 0 /\ v < n} → x2 : true → true} → true} =
  let fun loop i c {i : true → c : true → true} =
    if (i < n) then loop (i+1) (f i c) else c
  in loop 0 b
  end
```

Pre-Condition Refinement

```
fun foldn n b f
  {n : true → b : true → f : {x1: {ν >= 0 /\ ν < n} → x2 : true → true} → true} =
  let fun loop i c {i : true → c : true → true} =
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  in loop 0 b
  end
```

Type-checking loop involves solving:

$$\neg VC : \neg\{(\mathtt{i} < \mathtt{n} \wedge (\nu = \mathtt{i}) \Rightarrow (\nu \geq 0 \wedge \nu < \mathtt{n})\}$$

Pre-Condition Refinement

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fun foldn n b f
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A concrete test case: { i = -1, n = 0 }

Test case run does not diverge from counterexample path

Pre-Condition Refinement

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fun foldn n b f
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Weakest pre-condition generation: $\mathtt{i} < \mathtt{n} \Rightarrow (\mathtt{i} \geq 0)$

Pre-Condition Refinement

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fun foldn n b f
  {n : true → b : true → f : {x1: {ν >= 0 /\ ν < n} → x2 : true → true} → true} =
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A concrete test case: { i = -1, n = 0 }

Test case run does not diverge from counterexample path

Weakest pre-condition generation: $\mathtt{i} < \mathtt{n} \Rightarrow (\mathtt{i} \geq 0)$

Strengthen dependent type for loop:

$$\{\mathtt{i} : \{(\nu < \mathtt{n}) \Rightarrow (\nu \geq 0)\} \rightarrow \mathtt{c} : \mathrm{true} \rightarrow \mathrm{true}$$

Post-Condition Refinement

```
fun arraymax a g =
  let fun am h j m
    {h : {x1 : {true}→{ν >= 0 /\ ν < len a}} →j : {true} → m : {true} → true} =
    let val k {ν >= 0 /\ ν < len a} = h j
        val u {true} = sub a k
        val p {true}     = max u m
    in assert (p >=m); p {ν=p} end {true}
    fun am’ = am g
  in fold (len a) 0 am’ end
```

Post-Condition Refinement

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fun arraymax a g =
  let fun am h j m
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    fun am’ = am g
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```

Type-checking function am involves solving:

$$\neg VC : \neg\{(\mathtt{k} \geq 0) \wedge (\mathtt{k} < \mathtt{len\ a}) \Rightarrow (\mathtt{p} \geq \mathtt{m})\}$$

Post-Condition Refinement

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fun arraymax a g =
  let fun am h j m
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A possible test case generated by the solver:

$$\{\texttt{h} = \lambda x.0, \texttt{j} = 0, \texttt{m} = 0, \texttt{a} = \text{Array}[0]\}$$

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fun arraymax a g =
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        val u {true} = sub a k
        val p          = max u m
    in assert (p >=m); p {ν=p} end {true}
    fun am′ = am g
  in fold (len a) 0 am′ end
```

Type-checking function am involves solving:

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Concrete test diverges at call to max

Strengthen post-condition for max to $\{\nu \geq \mathtt{y}\}$ *where* $\mathtt{y}$ *is the second formal parameter to max*

Post-Condition Refinement

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fun arraymax a g =
  let fun am h j m
    {h : {x1 : {true}→{ν >= 0 /\ ν < len a}} →j : {true} → m : {true} → true} =
    let val k {ν >= 0 /\ ν < len a} = h j
        val u {true} = sub a k
        val p { ν >= m }= max u m
    in assert (p >=m); p {ν=p} end {true}
    fun am’ = am g
  in fold (len a) 0 am’ end
```

Type-checking function am involves solving:

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Algorithmic Type Inference

- Type inference
  - *Rule Generation*
    - Encode first-order formula from liquid type rules
  - *Constraint Propagation*
    - Subtyping chains
- Type checking
  - Verification condition generation and solve
- Type Refinement
  - Directed testing to generate functional precondition and postcondition constraints

Algorithmic Type Inference

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Language

Expressions

similar to Core ML intermediate representation in MLton

$$
\begin{aligned}
e' ::= & \\
& |\nu, v, x \in Var \\
& |c \\
& |\lambda x.e' \\
& |\text{if } e' \text{ then } e' \text{ else } e' \\
& |\text{let } x = e' \text{ in } e' \\
& |\text{assert } e'; e' \\
e_f ::= & \\
& |v \\
& |e_f \ v \\
e ::= & \\
& |[\Lambda\alpha]e' \\
& |[\gamma]e'
\end{aligned}
$$

Base Types

$$
\begin{aligned}
B ::= & \\
& |int \\
& |bool
\end{aligned}
$$

Dependent Types, Schemas

$$
\begin{aligned}
P ::= & \\
& |\{\nu : B|e\} \\
& |P \rightarrow P \\
& |\alpha \\
& |\forall\alpha.\forall x.P \\
& |P \wedge P
\end{aligned}
$$

Intersection types to distinguish predicates at different call-sites

Materialization to express non-lexical constraints

Derived from weakest precondition generation and propagated along subtyping chains

Dependent type encoding

- Liquid type rules provide well-formedness and subtyping constraints on dependent types
- Encode dependent types of local variables and terms using program terms and pre/post-conditions of functions
  - Local dependent types encode intra-procedural path information
    - Extract program error path from a (weak) dependent type system
- Abstract dependent types of functions
  - Initially, all argument and return of functions are abstracted to true
  - Strengthen pre- and post-conditions based on the program error path

Encoding Type Rules

- Encode type rules as constraints over first-order formula
- Encoding should preserve {path,context} sensitivity
- Facilitate test generation and predicate refinement

Example:

$$\frac{\Gamma \vdash e_1 : \texttt{bool} \qquad \Gamma \vdash P \qquad \Gamma; e_1 \vdash e_2 : P \qquad \Gamma; \neg e_1 \vdash e_3 : P}{\Gamma \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 : P}$$

Generate constraint for P from this rule as:

$$P = \mathcal{C}(e_1) \Rightarrow \mathcal{C}(e_2) \wedge \neg\mathcal{C}(e_1) \Rightarrow \mathcal{C}(e_3)$$

Constraint relates value of guard to specific branches

$\mathcal{C}(e)$ defines the constraints generated for e based on the structure of the liquid type rules

Verification Condition Generation

Verification Condition Generation

Fun

$$\frac{\Gamma \vdash x : P_x \rightarrow P \quad \Gamma; x : P_x \vdash e : P}{\Gamma \vdash \lambda x.e : (x : P_x \rightarrow P)}$$

Verify postcondition P with precondition Px

App

$$\frac{\Gamma \vdash v_2 : P_x \quad \Gamma \vdash e_f : (x : P_x \rightarrow P)}{\Gamma \vdash e_f(v_2) : [v_2/x]P}$$

Postcondition P is assumed if and only if its precondition Px is asserted. Encoding?

Verification Condition Generation

Fun

$$\frac{\Gamma \vdash x : P_x \rightarrow P \quad \Gamma ; x : P_x \vdash e : P}{\Gamma \vdash \lambda x.e : (x : P_x \rightarrow P)}$$

Verify postcondition P with precondition Px

App

$$\frac{\Gamma \vdash v_2 : P_x \quad \Gamma \vdash e_f : (x : P_x \rightarrow P)}{\Gamma \vdash e_f(v_2) : [v_2/x]P}$$

Postcondition P is assumed if and only if its precondition Px is asserted. Encoding?

ASSERT

$$\frac{\Gamma \vdash e' : bool \quad \Gamma \vdash \{\_|true\} <: \{\_|e'\} \quad \Gamma \vdash e : P}{\Gamma \vdash \text{assert } e'; e : P}$$

Prove [Γ@e']⇒e' where [Γ@e'] represent the conjunction of dependent types of variables in e' from Γ

Encoding Application

App

$$\frac{\Gamma \vdash v_2 : P_x \quad \Gamma \vdash e_f : (x : P_x \to P)}{\Gamma \vdash e_f(v_2) : [v_2/x]P}$$

Two issues:

1. Want to preserve some measure of context-sensitivity

   keep pre- and post-condition constraints at different call sites distinct
2. Make "must-hold" properties defined by assertions within the function body explicit

Address these issues using intersection types

$$P_x \to P \bigwedge_i P_{x_i} \to P_i$$

The left conjunct captures constraints induced by assertions on function arguments that occur within the function body:

These constraints must always hold

Verification condition Γ |- v_2 : Px $\mathcal{C}(P) = [v_2/x]P$

The right conjunct discriminates over different call-sites

Constraints deduced for post-conditions at a call inform structure of pre-conditions

$\mathcal{C}(P) = \wedge_i([v2/\nu)P_{x_i} \Rightarrow [v_2/x]P_i$ if v_2 is of base type

$\mathcal{C}(P) = \wedge_i[v2/x]P_i$ otherwise because implication is made to hold by subtyping relation for functions

Type Checking/Refinement Algorithm

```
verify f λx.e WL
   let vcs = vcgen ∅ λx.e (abs_ty (λ x. e))
   in
      foreach vc in vcs
         let (result, vc_assignment, abstract_ce) = solve vc
         in
            if result then ()
            else
               let
                  t = genTestcase vc_assignment
                  concrete\_path = run (f, t)
                  if(abstract_ce = concrete_path) then
                     genPrecondition f concrete_path WL
                     verify f e
                  else
                     pred = divergePred f abstract_ce concrete_path
                     genPostcondtion f pred WL
   end
```

Type Checking/Refinement Algorithm

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verify f λx.e WL
   let vcs = vcgen ∅ λx.e (abs_ty (λ x. e)) Generate verification conditions
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      foreach vc in vcs
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   end
```

Type Checking/Refinement Algorithm

```
verify f λx.e WL
   let vcs = vcgen ∅ λx.e (abs_ty (λ x. e)) Generate verification conditions
   in
      foreach vc in vcs  solve each condition using SMT
         let (result, vc_assignment, abstract_ce) = solve vc
         in
            if result then ()
            else
               let
                  t = genTestcase vc_assignment
                  concrete\_path = run (f, t)
                  if(abstract_ce = concrete_path) then
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   end
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      foreach vc in vcs  solve each condition using SMT
         let (result, vc_assignment, abstract_ce) = solve vc
         in
            if result then ()
            else
               let    examine counterexample
                  t = genTestcase vc_assignment
                  concrete\_path = run (f, t)
                  if(abstract_ce = concrete_path) then
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               let    examine counterexample
                  t = genTestcase vc_assignment
                  concrete\_path = run (f, t)   extract and run test case from assignment
                  if(abstract_ce = concrete_path) then
                     genPrecondition f concrete_path WL
                     verify f e
                  else
                     pred = divergePred f abstract_ce concrete_path
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   end
```

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                  t = genTestcase vc_assignment
                  concrete\_path = run (f, t)  extract and run test case from assignment
                  if(abstract_ce = concrete_path) then
                     genPrecondition f concrete_path WL  weakest precondition generation when
                     verify f e                          concrete and abstract paths converge
                  else
                     pred = divergePred f abstract_ce concrete_path
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Type Checking/Refinement Algorithm

```
verify f λx.e WL
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            if result then ()
            else
               let   examine counterexample
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                  concrete\_path = run (f, t)   extract and run test case from assignment
                  if(abstract_ce = concrete_path) then
                     genPrecondition f concrete_path WL   weakest precondition generation when
                     verify f e                           concrete and abstract paths converge
                  else
                     pred = divergePred f abstract_ce concrete_path
                     genPostcondtion f pred WL   strengthen post-condition when they diverge
   end
```

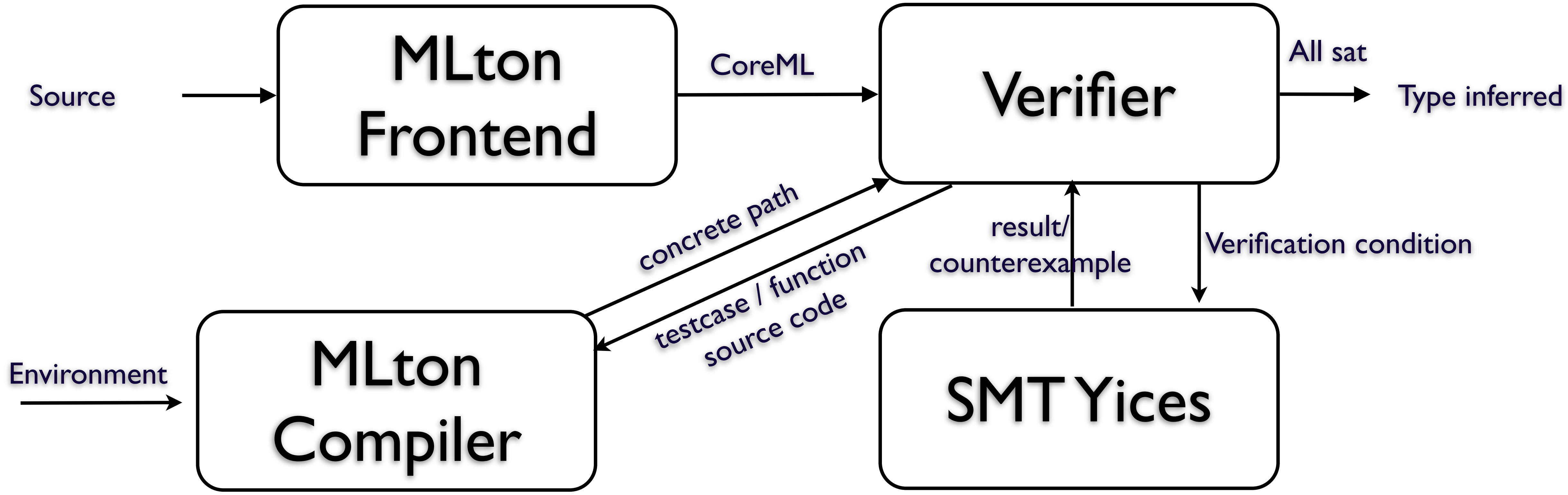
Type Checking/Refinement Algorithm

- Functions are put into worklist (WL) based on call relations
- Each iteration of the fixed-point picks a function from WL and verifies it using `verify`
- Generate pre/postcondition
  - ★ Can strengthen the dependent type of this function and callee function.
  - ★ Put all function whose dependent type has changed into WL (worklist)
- When precondition of a function is strengthened, invoke type constraint propagation algorithm

Implementation

- Implementation is based on MLton, an open-source optimizing Standard ML compiler
  - Incorporate verification mechanism as a front-end pass in Core ML
    - After defunctorization (no modules) and local simplification
      - Type information and original program variables still available
    - Before closure conversion, monomorphisation, and SSA translation
- Use theorem prover Yices as the verification engine
  - Use MLton's FFI interface to interact with a binary version of this prover
- A total of 7.5 KLOC

Implementation


Source

MLton Frontend

CoreML

Verifier

All sat

Type inferred

concrete path

testcase / function source code

result/ counterexample

Verification condition

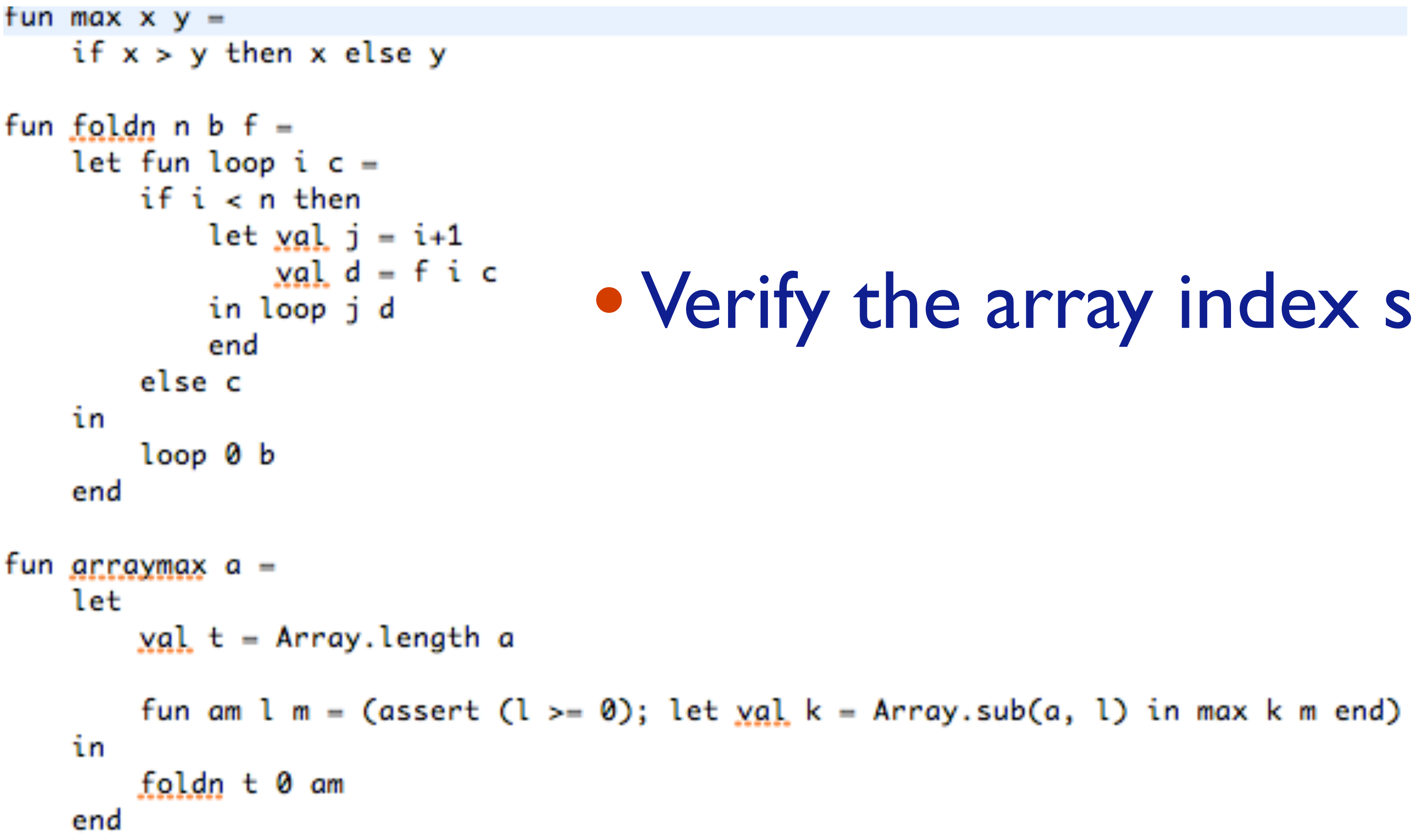
Environment

MLton Compiler

SMT Yices

- MLton frontend defunctorizes, type-checks, and does local simplification yielding CoreML, an intermediate representation which is polymorphic, higher-order and has nested patterns.
- Verifier generates verification conditions from typing rules for CoreML and feeds them to Yices.
- Obtain counterexample along with a testcase witness.
- Compile and run the function under verification with the testcase to obtain a concrete path
- Verifier uses the concrete path and counterexample to strengthen type system

Preliminary Results



```
fun max x y =
    if x > y then x else y

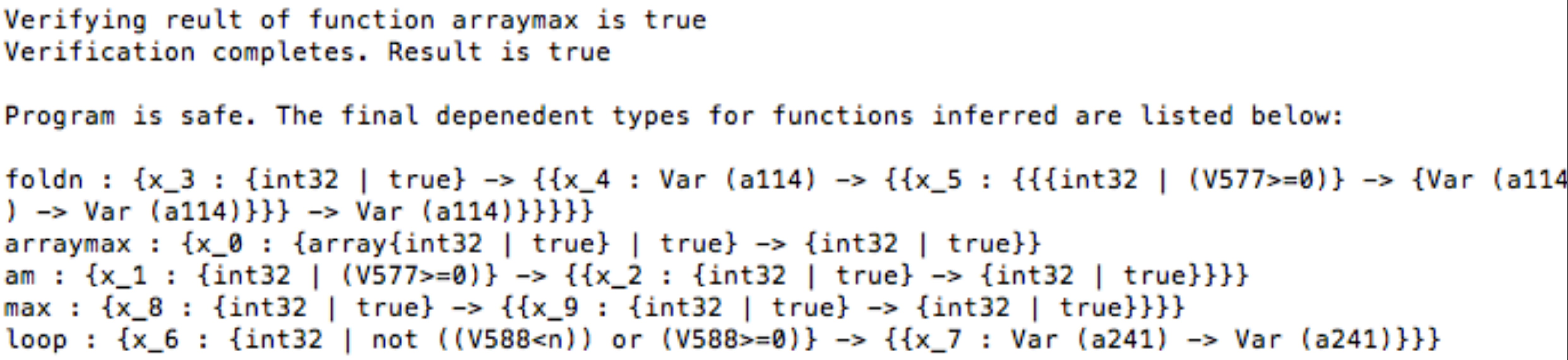
fun foldn n b f =
    let fun loop i c =
        if i < n then
            let val j = i+1
                val d = f i c
            in loop j d
            end
        else c
    in
        loop 0 b
    end

fun arraymax a =
    let
        val t = Array.length a

        fun am l m = (assert (l >= 0); let val k = Array.sub(a, l) in max k m end)
    in
        foldn t 0 am
    end
```

- Verify the array index should always be no less than 0

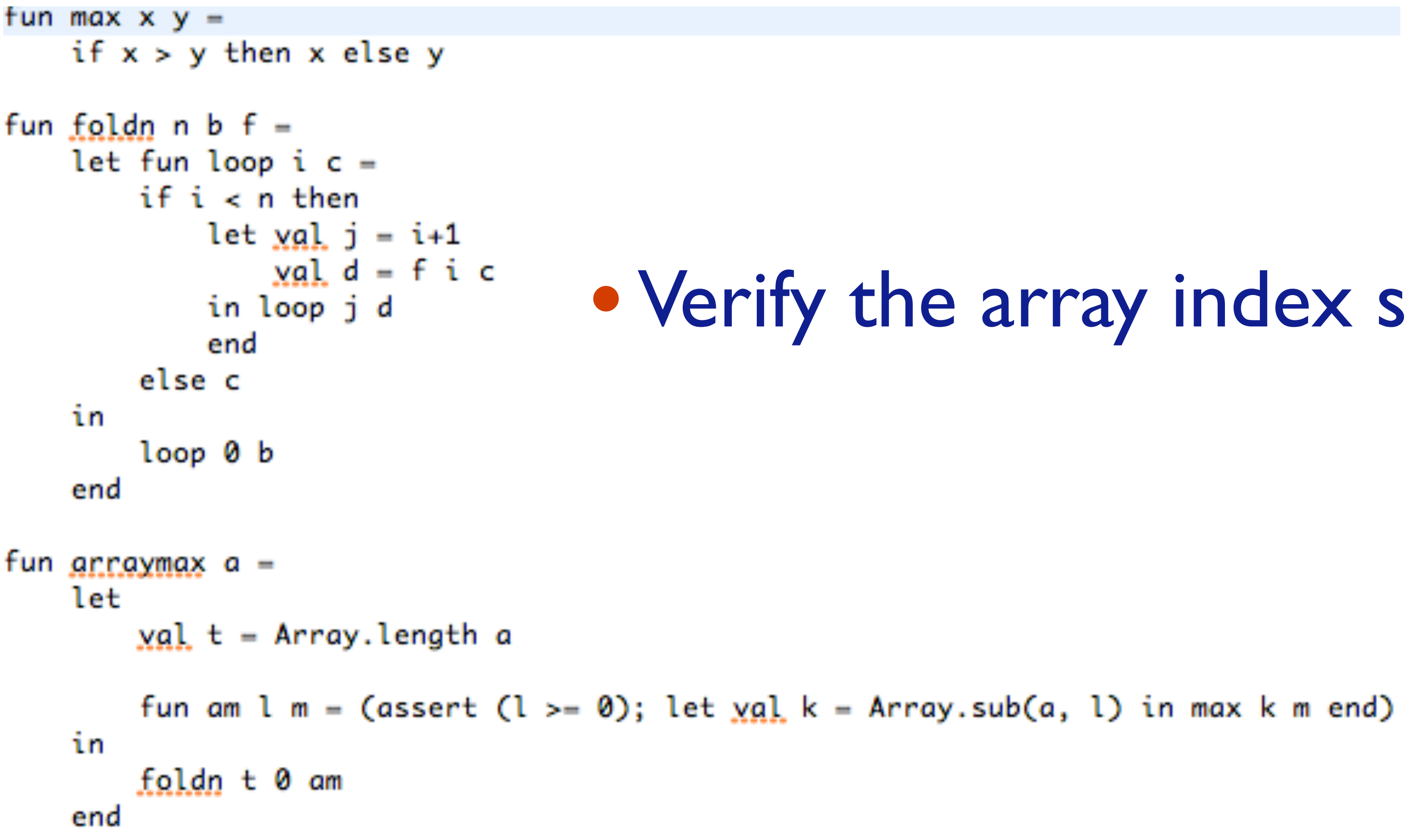
Preliminary Results



```
Verifying reult of function arraymax is true
Verification completes. Result is true

Program is safe. The final depenedent types for functions inferred are listed below:

foldn : {x_3 : {int32 | true} -> {{x_4 : Var (a114) -> {{x_5 : {{{int32 | (V577>=0)} -> {Var (a114
) -> Var (a114)}}} -> Var (a114)}}}}}
arraymax : {x_0 : {array{int32 | true} | true} -> {int32 | true}}
am : {x_1 : {int32 | (V577>=0)} -> {{x_2 : {int32 | true} -> {int32 | true}}}}
max : {x_8 : {int32 | true} -> {{x_9 : {int32 | true} -> {int32 | true}}}}
loop : {x_6 : {int32 | not ((V588<n)) or (V588>=0)} -> {{x_7 : Var (a241) -> Var (a241)}}}
```



```
fun max x y =
    if x > y then x else y

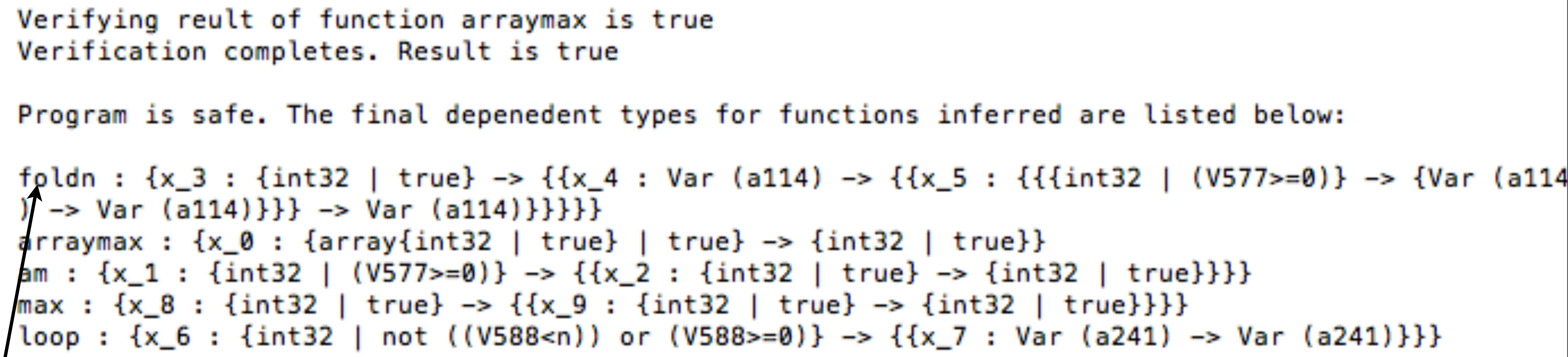
fun foldn n b f =
    let fun loop i c =
        if i < n then
            let val j = i+1
                val d = f i c
            in loop j d
            end
        else c
    in
        loop 0 b
    end

fun arraymax a =
    let
        val t = Array.length a

        fun am l m = (assert (l >= 0); let val k = Array.sub(a, l) in max k m end)
    in
        foldn t 0 am
    end
```

- Verify the array index should always be no less than 0

Preliminary Results



```
Verifying reult of function arraymax is true
Verification completes. Result is true

Program is safe. The final depenedent types for functions inferred are listed below:

foldn : {x_3 : {int32 | true} -> {{x_4 : Var (a114) -> {{x_5 : {{{int32 | (V577>=0)} -> {Var (a114
)} -> Var (a114)}}} -> Var (a114)}}}}}
arraymax : {x_0 : {array{int32 | true} | true} -> {int32 | true}}
am : {x_1 : {int32 | (V577>=0)} -> {{x_2 : {int32 | true} -> {int32 | true}}}}
max : {x_8 : {int32 | true} -> {{x_9 : {int32 | true} -> {int32 | true}}}}
loop : {x_6 : {int32 | not ((V588<n)) or (V588>=0)} -> {{x_7 : Var (a241) -> Var (a241)}}}
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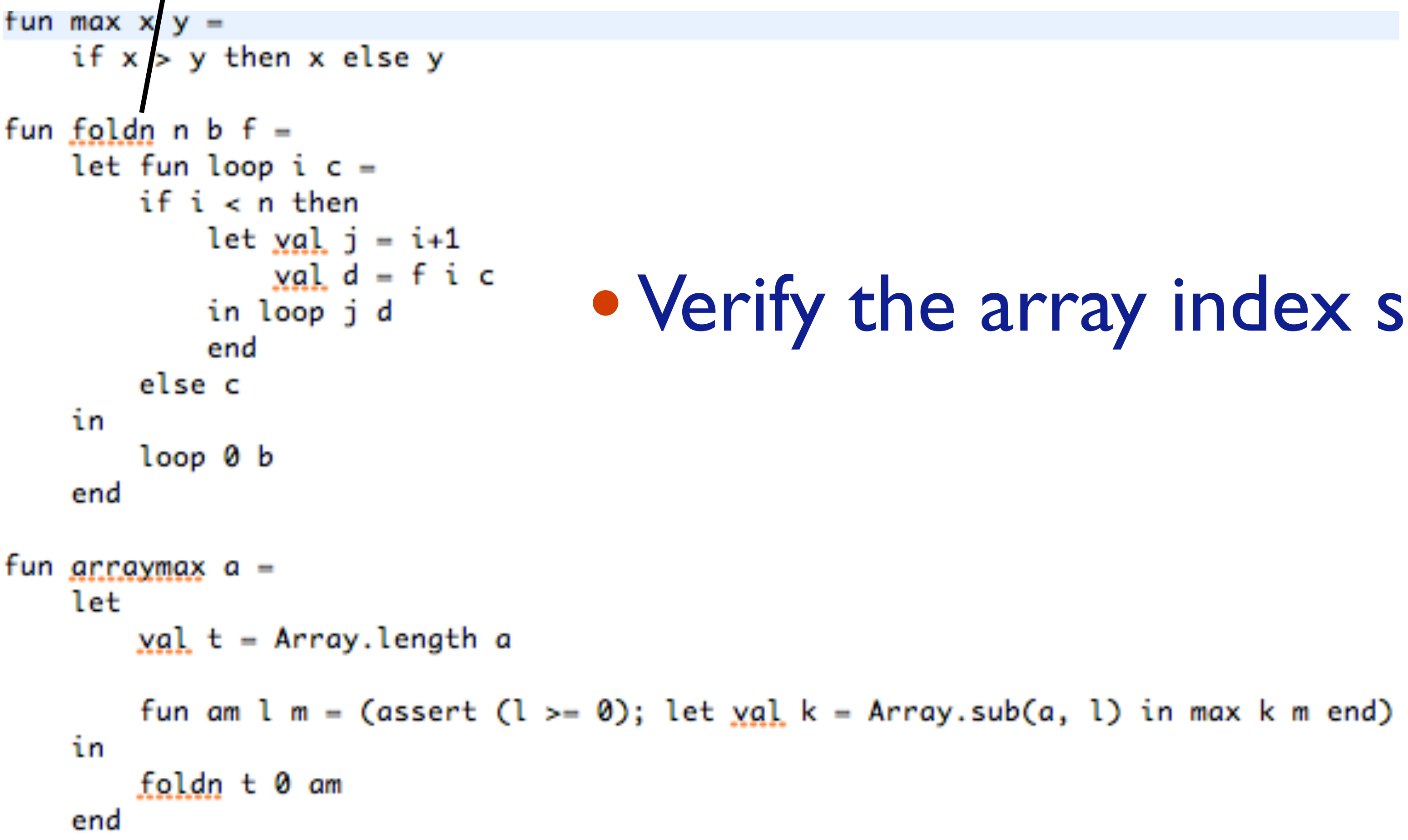

```
fun max x y =
    if x > y then x else y

fun foldn n b f =
    let fun loop i c =
        if i < n then
            let val j = i+1
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    let
        val t = Array.length a

        fun am l m = (assert (l >= 0); let val k = Array.sub(a, l) in max k m end)
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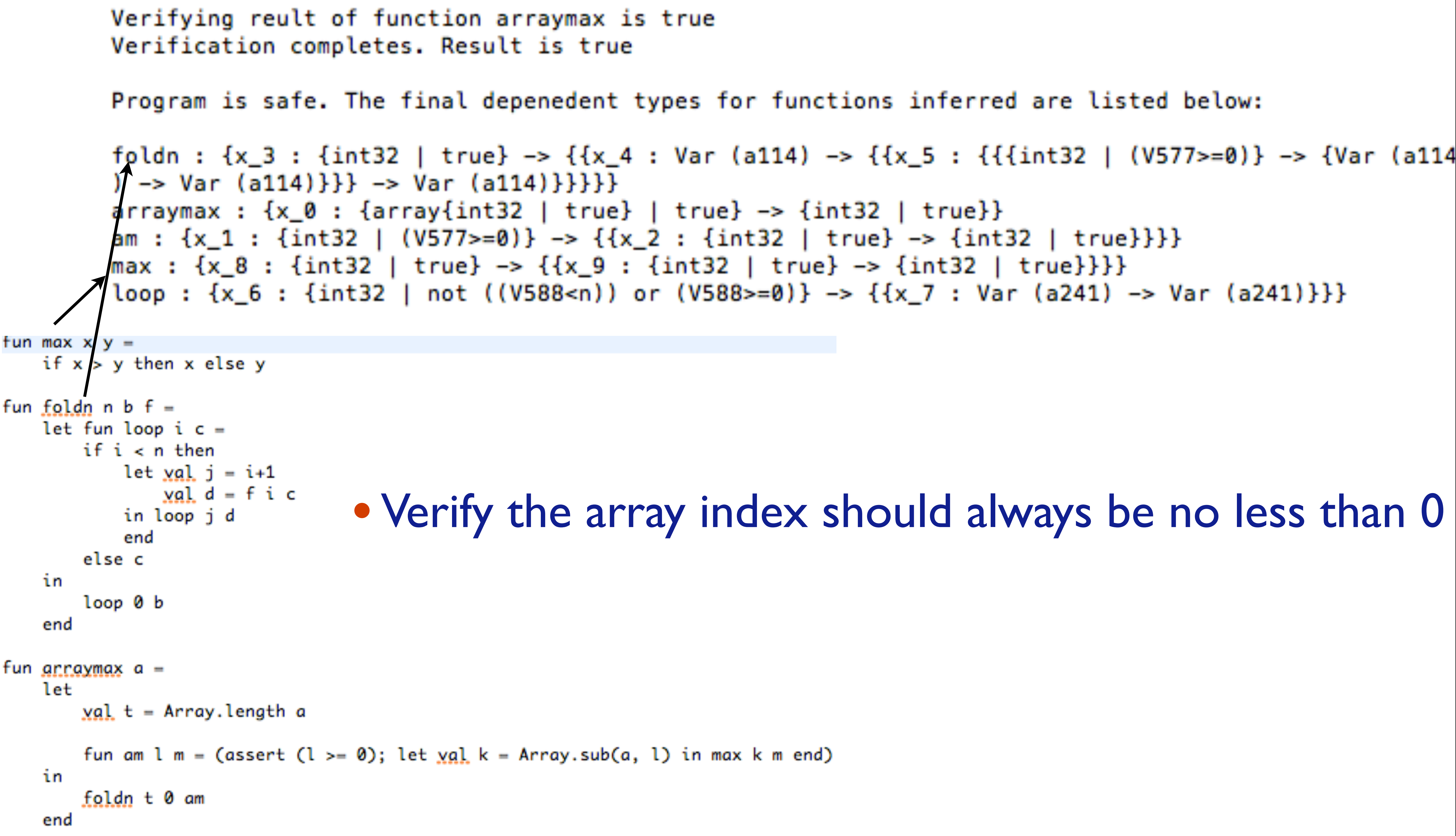
Preliminary Results



```
Verifying reult of function arraymax is true
Verification completes. Result is true

Program is safe. The final depenedent types for functions inferred are listed below:

foldn : {x_3 : {int32 | true} -> {{x_4 : Var (a114) -> {{x_5 : {{{int32 | (V577>=0)} -> {Var (a114
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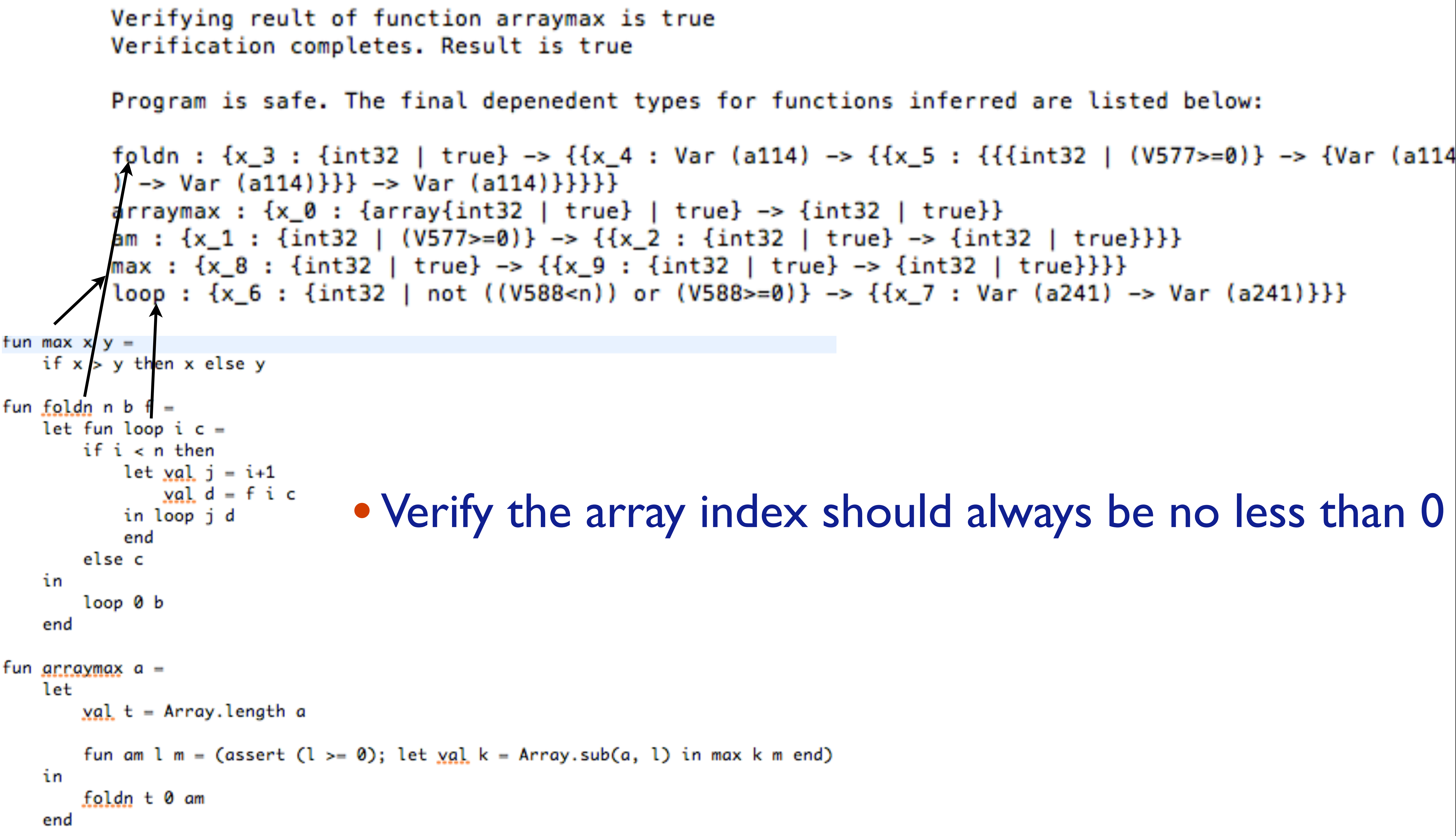

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Verification completes. Result is true

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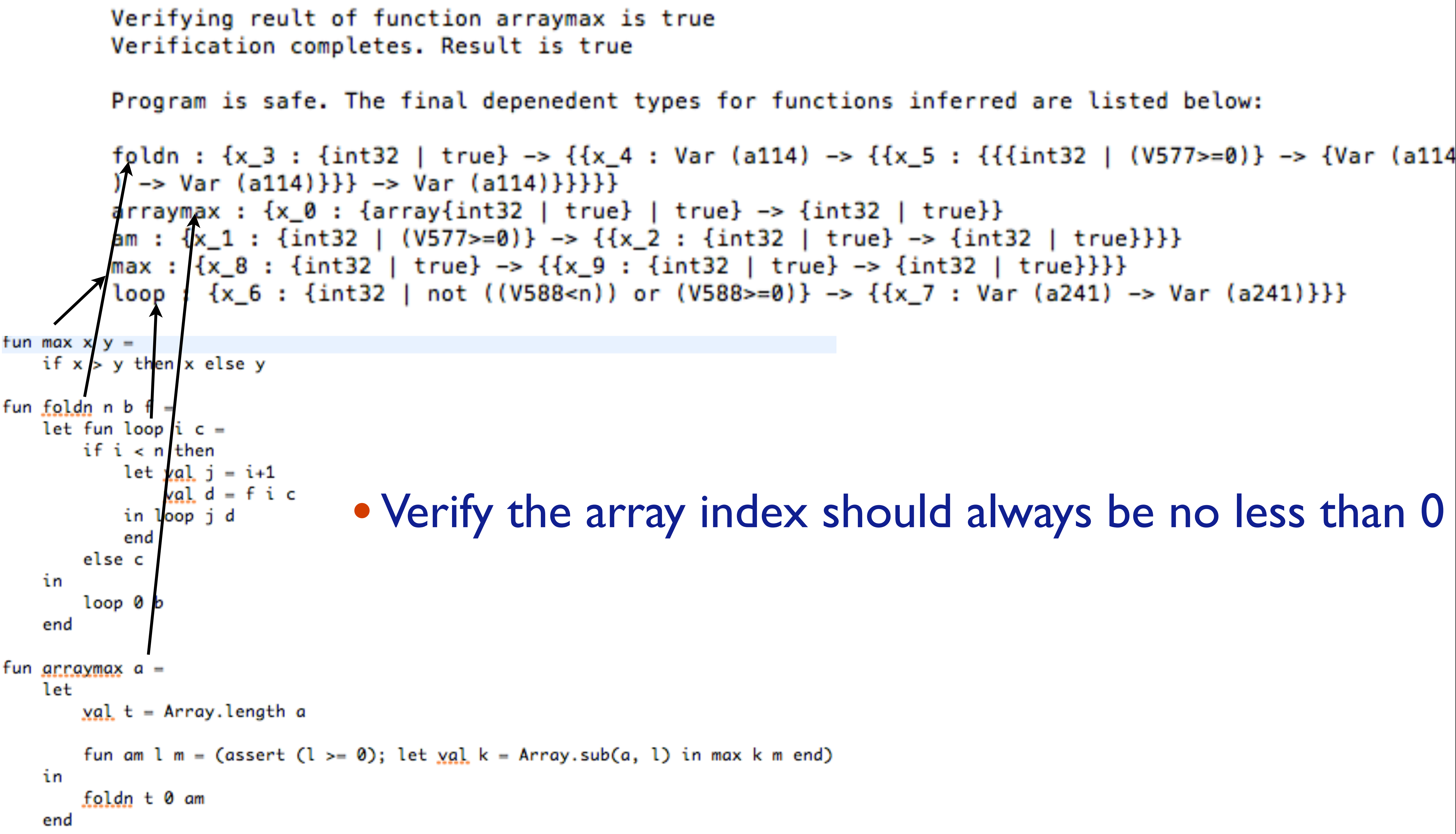

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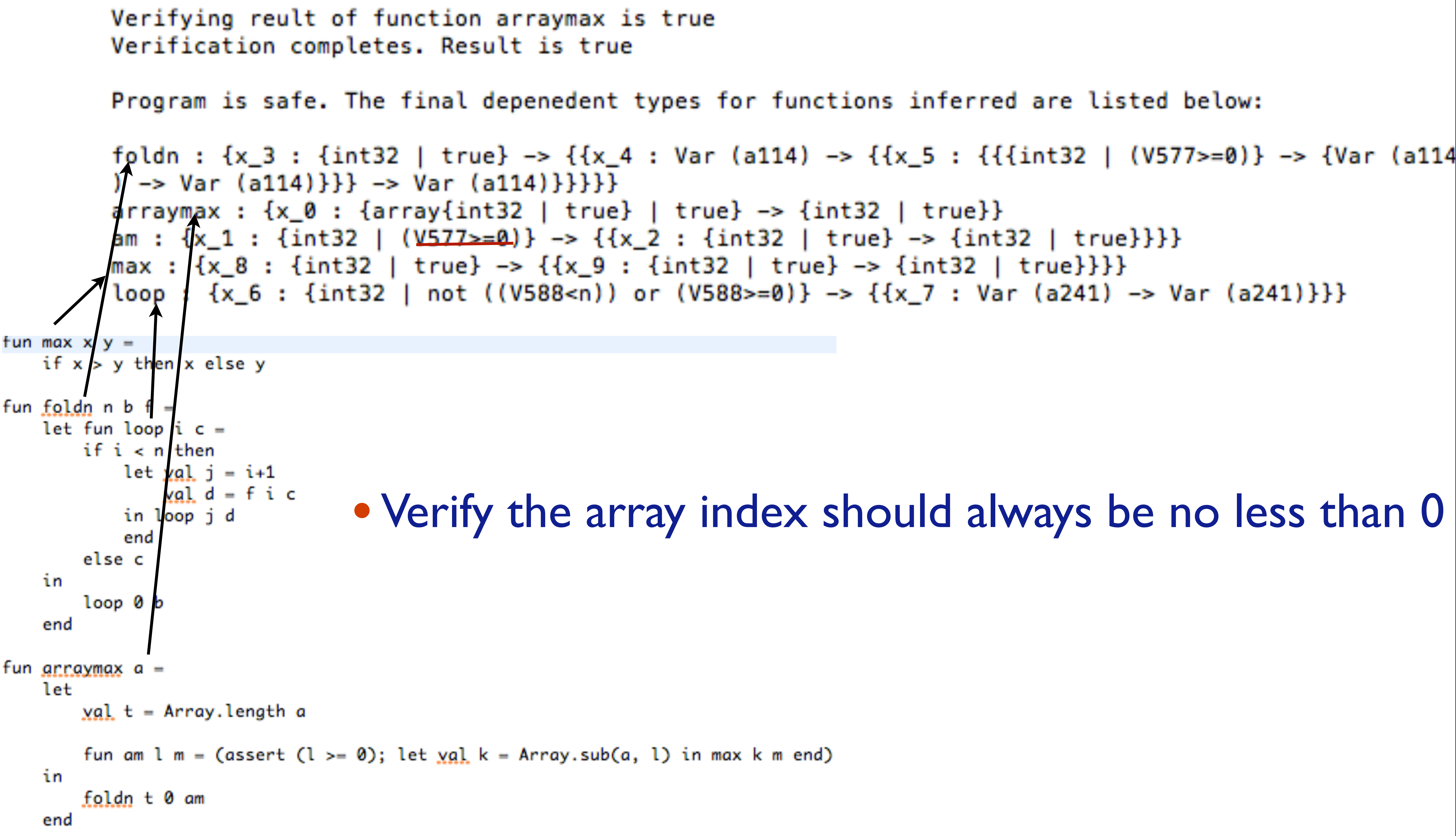
Preliminary Results



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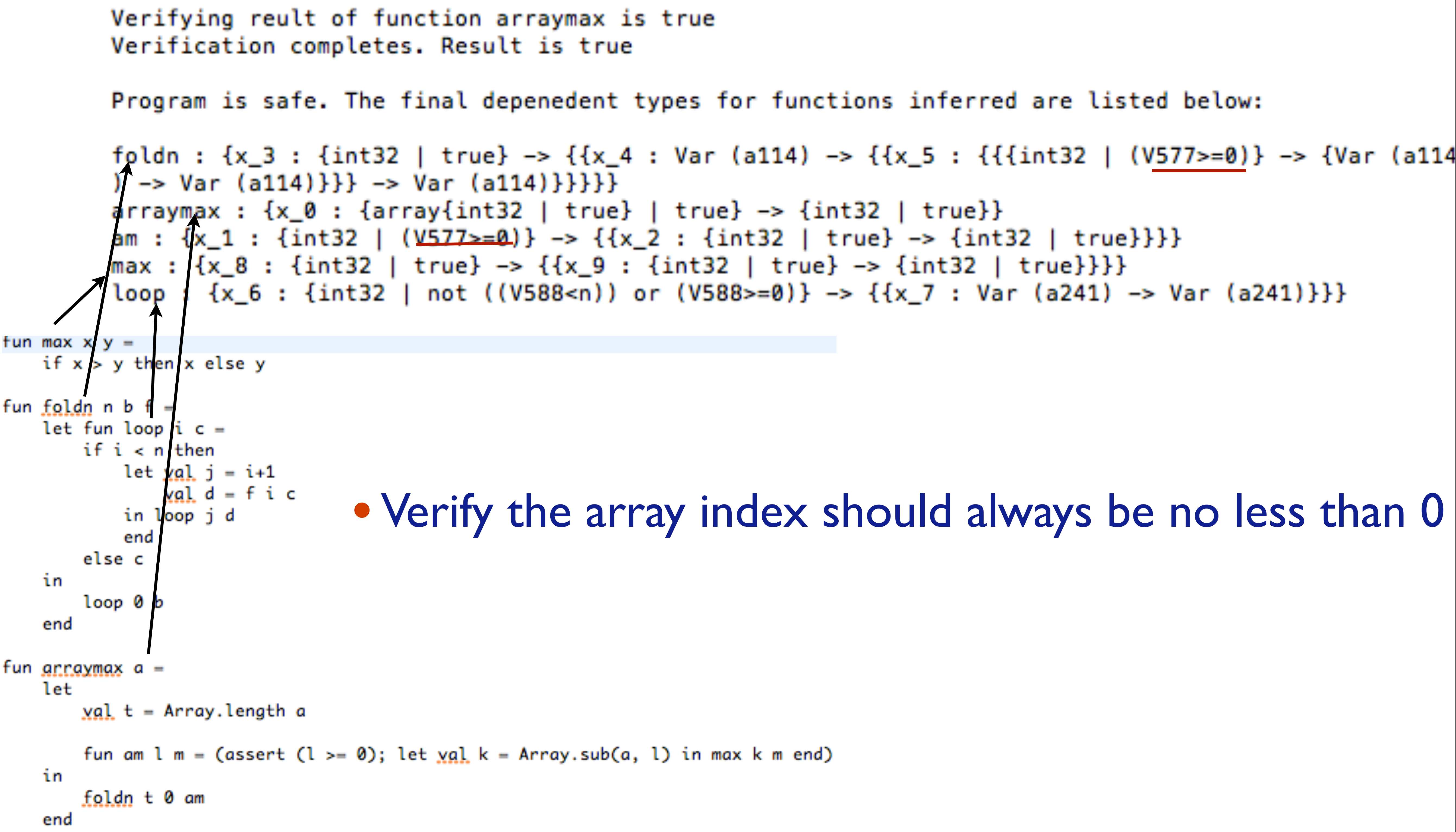
Preliminary Results



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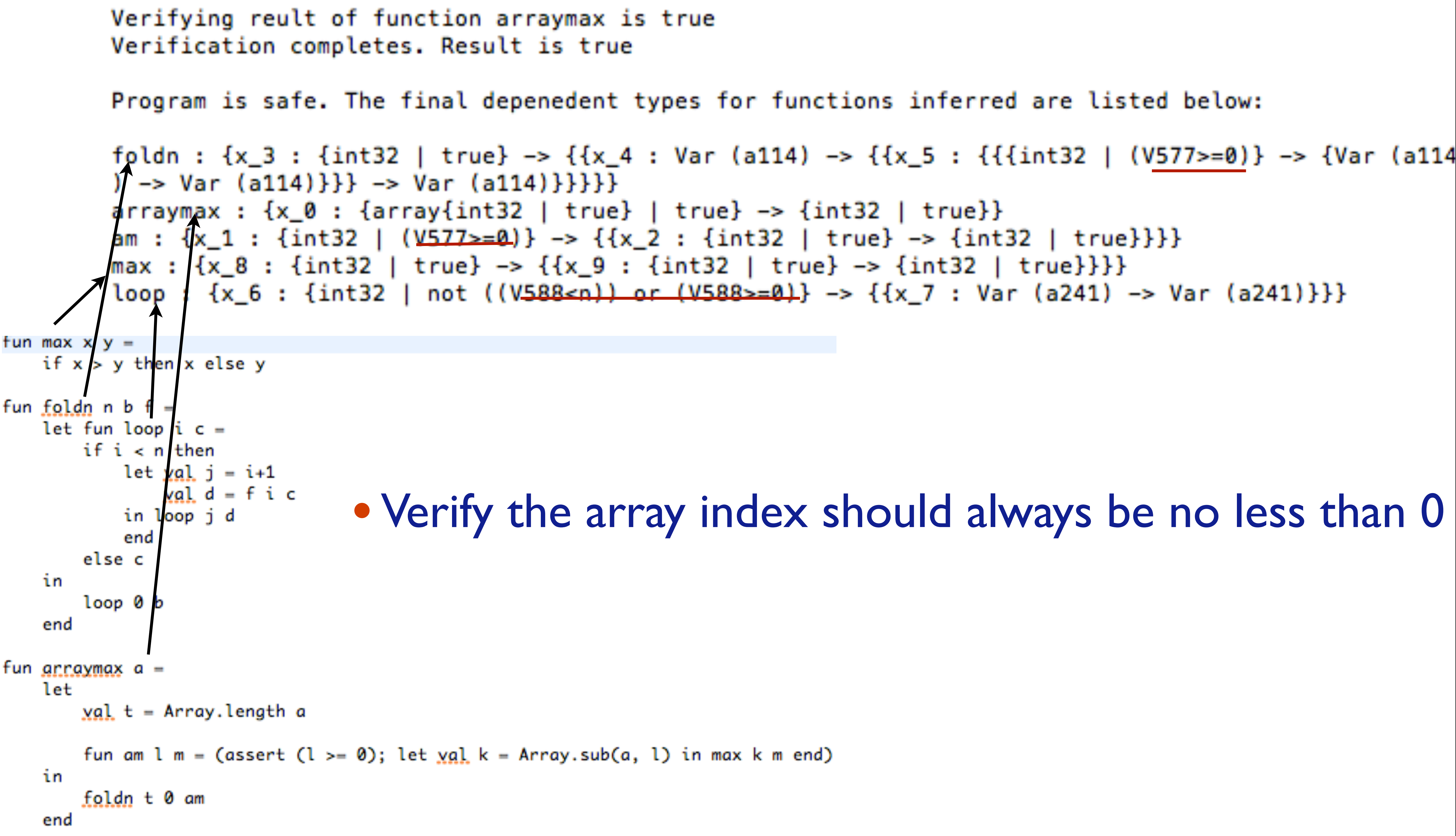
Preliminary Results



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)} -> Var (a114)}}} -> Var (a114)}}}}}
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max : {x_8 : {int32 | true} -> {{x_9 : {int32 | true} -> {int32 | true}}}}
loop : {x_6 : {int32 | not ((V588<n)) or (V588>=0)} -> {{x_7 : Var (a241) -> Var (a241)}}}
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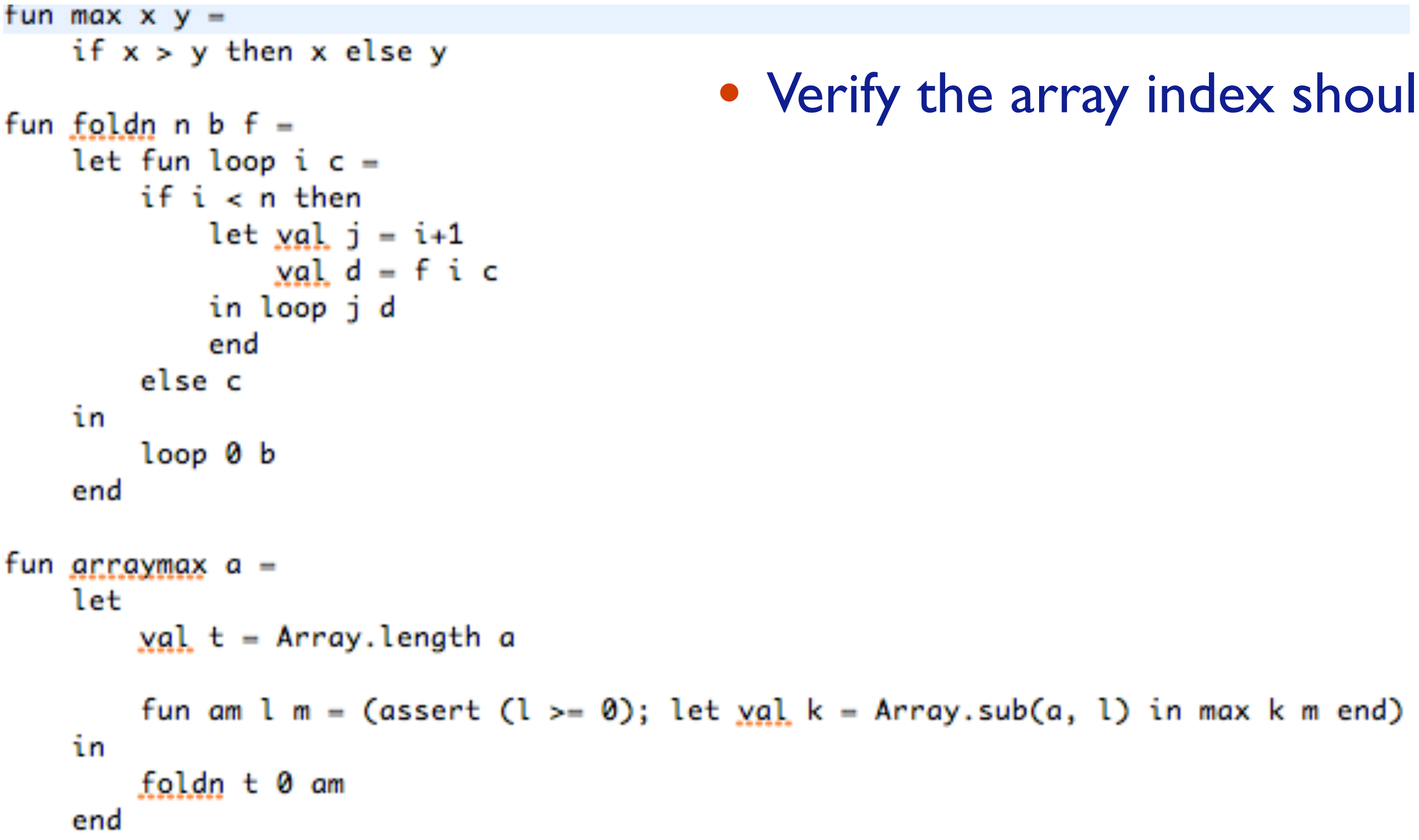
Preliminary Results



```
fun max x y =
    if x > y then x else y

fun foldn n b f =
    let fun loop i c =
        if i < n then
            let val j = i+1
                val d = f i c
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fun arraymax a =
    let
        val t = Array.length a

        fun am l m = (assert (l >= 0); let val k = Array.sub(a, l) in max k m end)
    in
        foldn t 0 am
    end
```
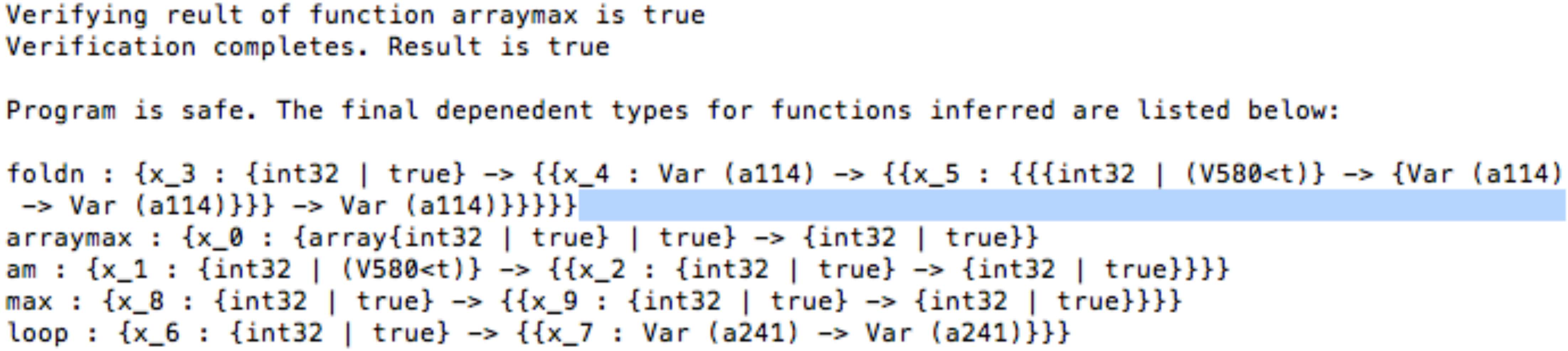
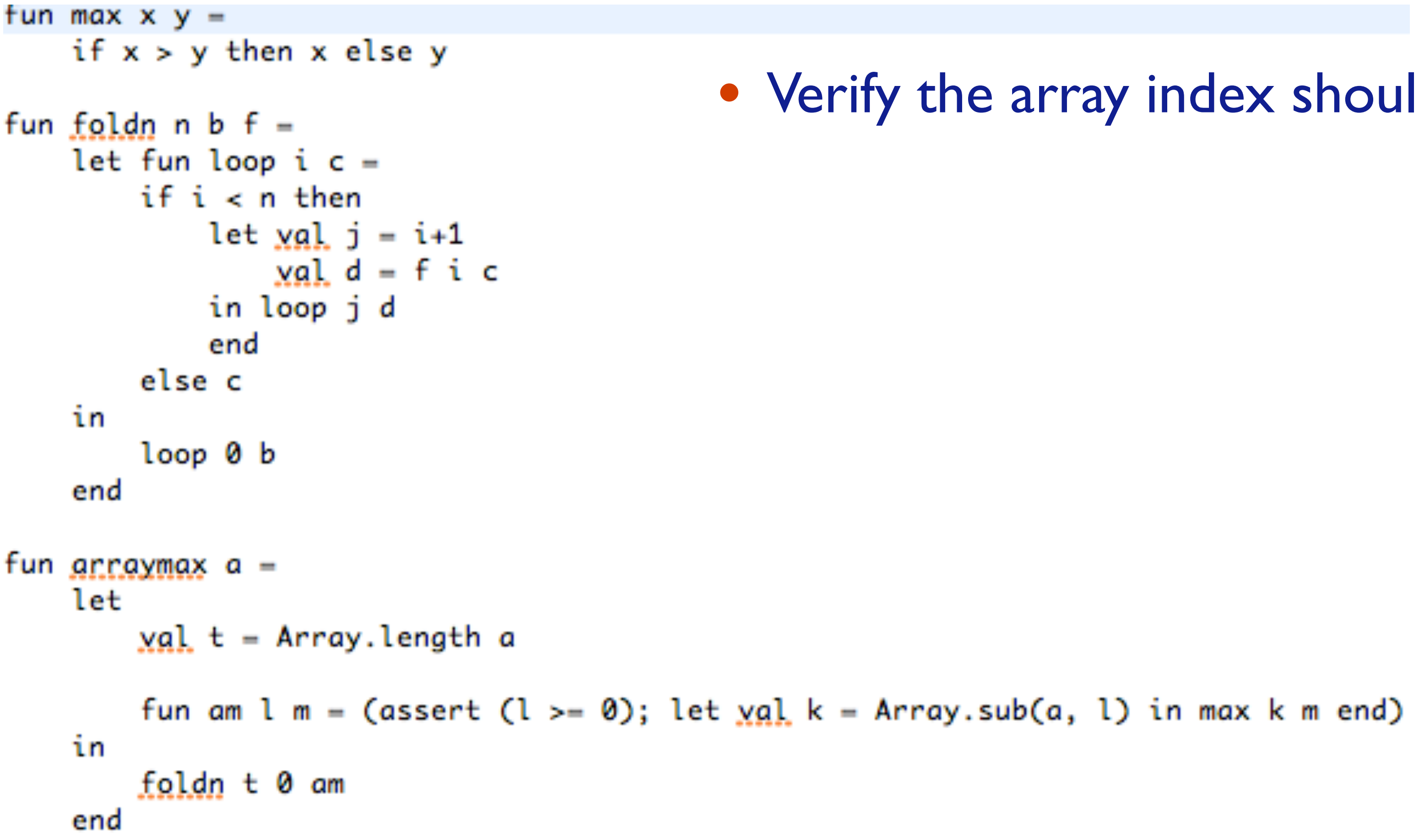

- Verify the array index should always be less than the length of the array

Preliminary Results



```
Verifying reult of function arraymax is true
Verification completes. Result is true

Program is safe. The final depenedent types for functions inferred are listed below:

foldn : {x_3 : {int32 | true} -> {{x_4 : Var (a114) -> {{x_5 : {{{int32 | (V580<t)} -> {Var (a114)
 -> Var (a114)}}} -> Var (a114)}}}}}
arraymax : {x_0 : {array{int32 | true} | true} -> {int32 | true}}
am : {x_1 : {int32 | (V580<t)} -> {{x_2 : {int32 | true} -> {int32 | true}}}}
max : {x_8 : {int32 | true} -> {{x_9 : {int32 | true} -> {int32 | true}}}}
loop : {x_6 : {int32 | true} -> {{x_7 : Var (a241) -> Var (a241)}}}
```



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fun max x y =
    if x > y then x else y

fun foldn n b f =
    let fun loop i c =
        if i < n then
            let val j = i+1
                val d = f i c
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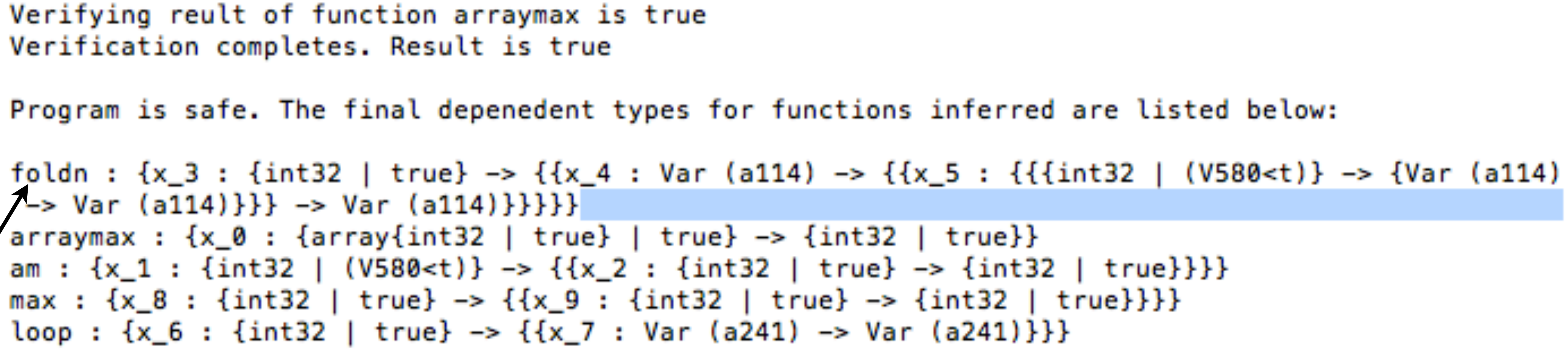
            end
        else c
    in
        loop 0 b
    end

fun arraymax a =
    let
        val t = Array.length a

        fun am l m = (assert (l >= 0); let val k = Array.sub(a, l) in max k m end)
    in
        foldn t 0 am
    end
```

- Verify the array index should always be less than the length of the array

Preliminary Results



```
Verifying reult of function arraymax is true
Verification completes. Result is true

Program is safe. The final depenedent types for functions inferred are listed below:

foldn : {x_3 : {int32 | true} -> {{x_4 : Var (a114) -> {{x_5 : {{{int32 | (V580<t)} -> {Var (a114)
-> Var (a114)}}} -> Var (a114)}}}}}
arraymax : {x_0 : {array{int32 | true} | true} -> {int32 | true}}
am : {x_1 : {int32 | (V580<t)} -> {{x_2 : {int32 | true} -> {int32 | true}}}}
max : {x_8 : {int32 | true} -> {{x_9 : {int32 | true} -> {int32 | true}}}}
loop : {x_6 : {int32 | true} -> {{x_7 : Var (a241) -> Var (a241)}}}
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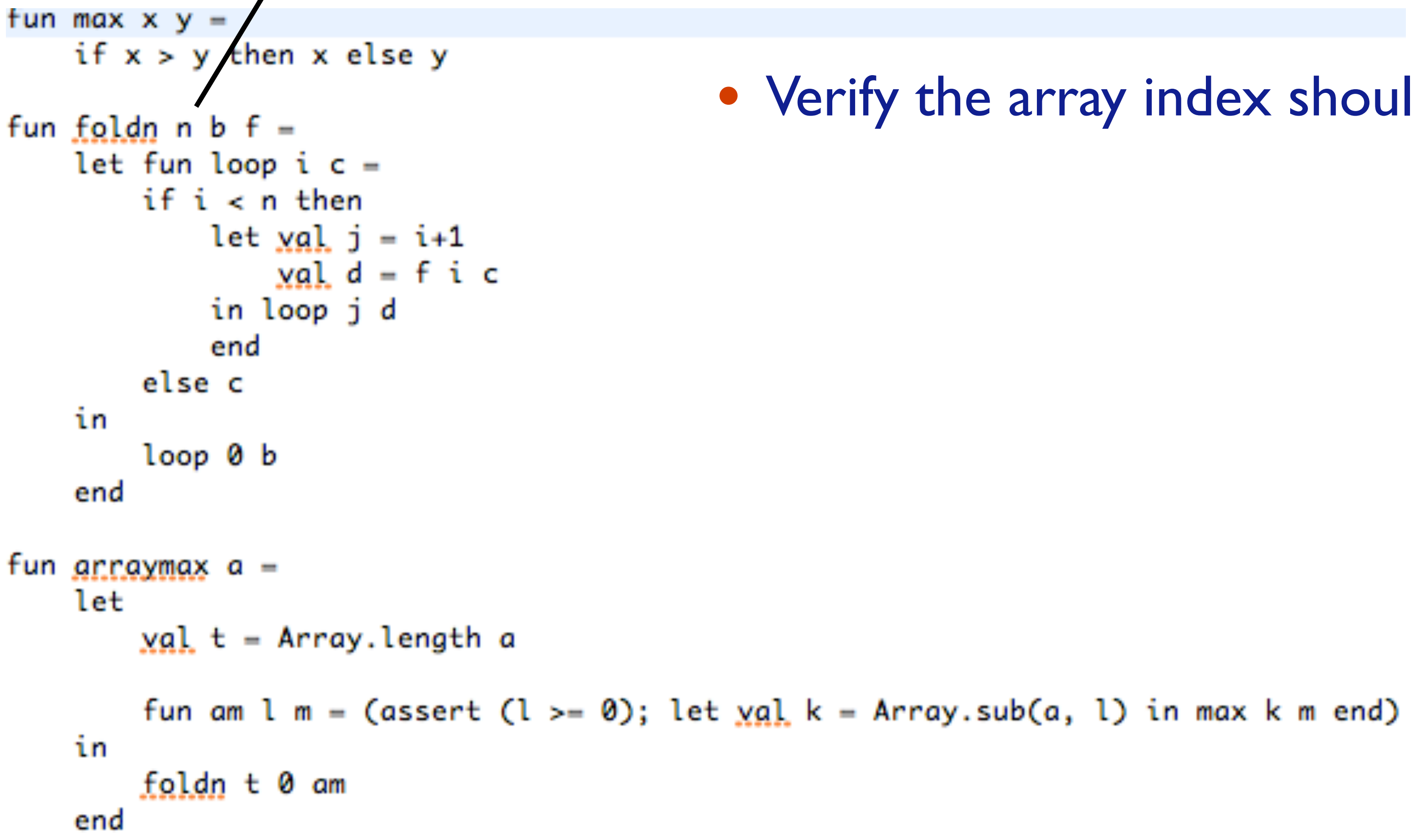

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fun max x y =
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    let fun loop i c =
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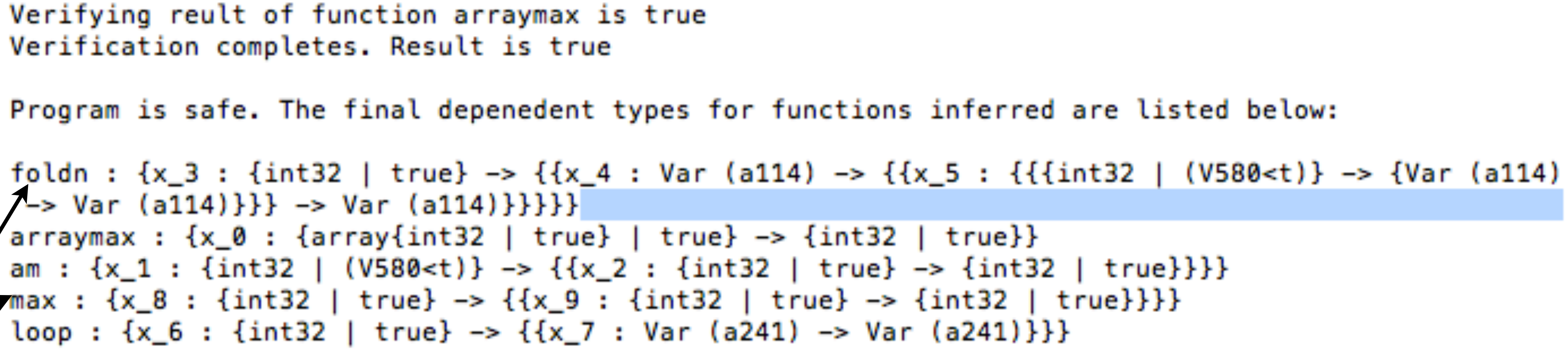
- Verify the array index should always be less than the length of the array

Preliminary Results



```
Verifying reult of function arraymax is true
Verification completes. Result is true

Program is safe. The final depenedent types for functions inferred are listed below:

foldn : {x_3 : {int32 | true} -> {{x_4 : Var (a114) -> {{x_5 : {{{int32 | (V580<t)} -> {Var (a114)
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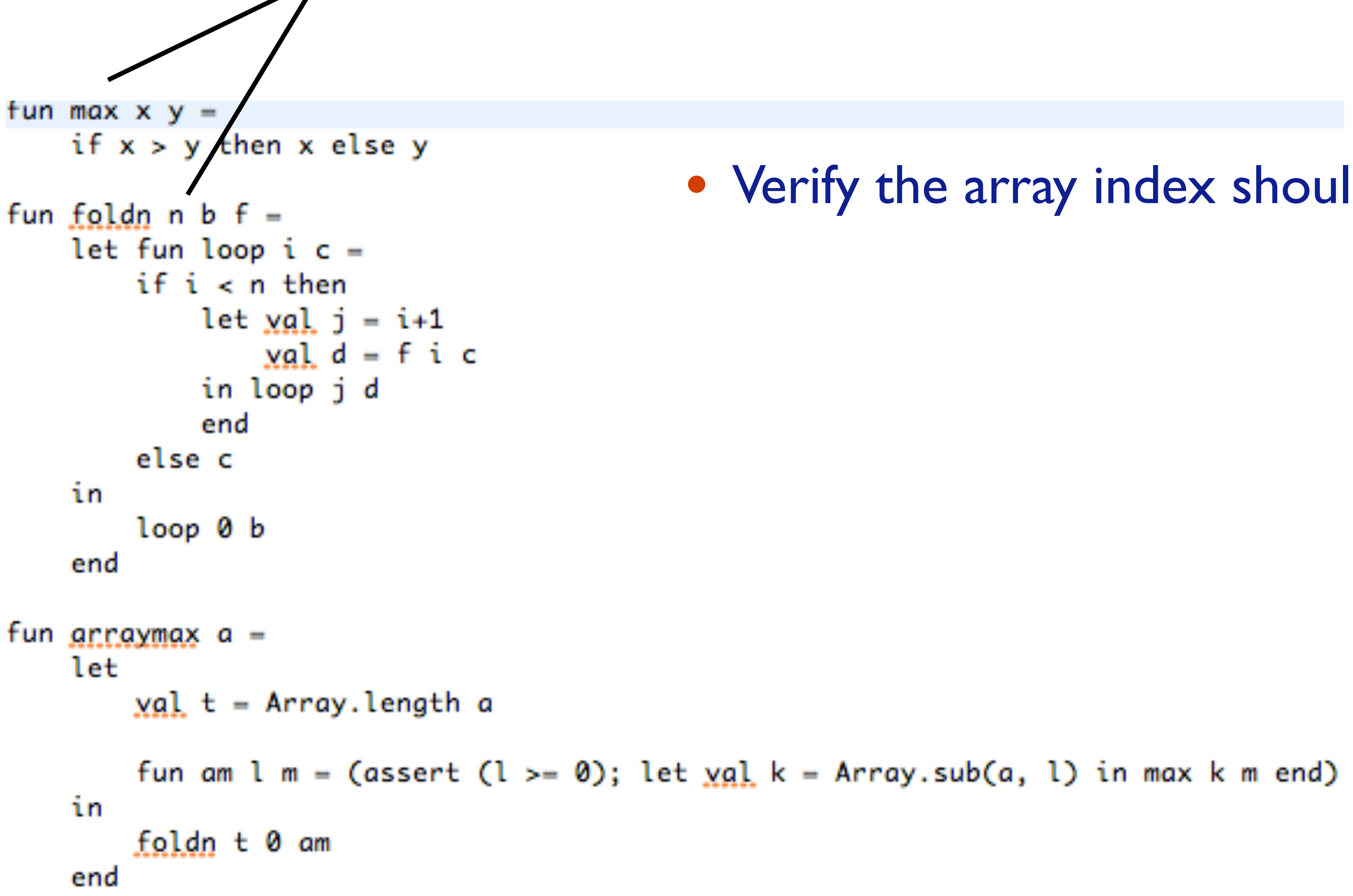

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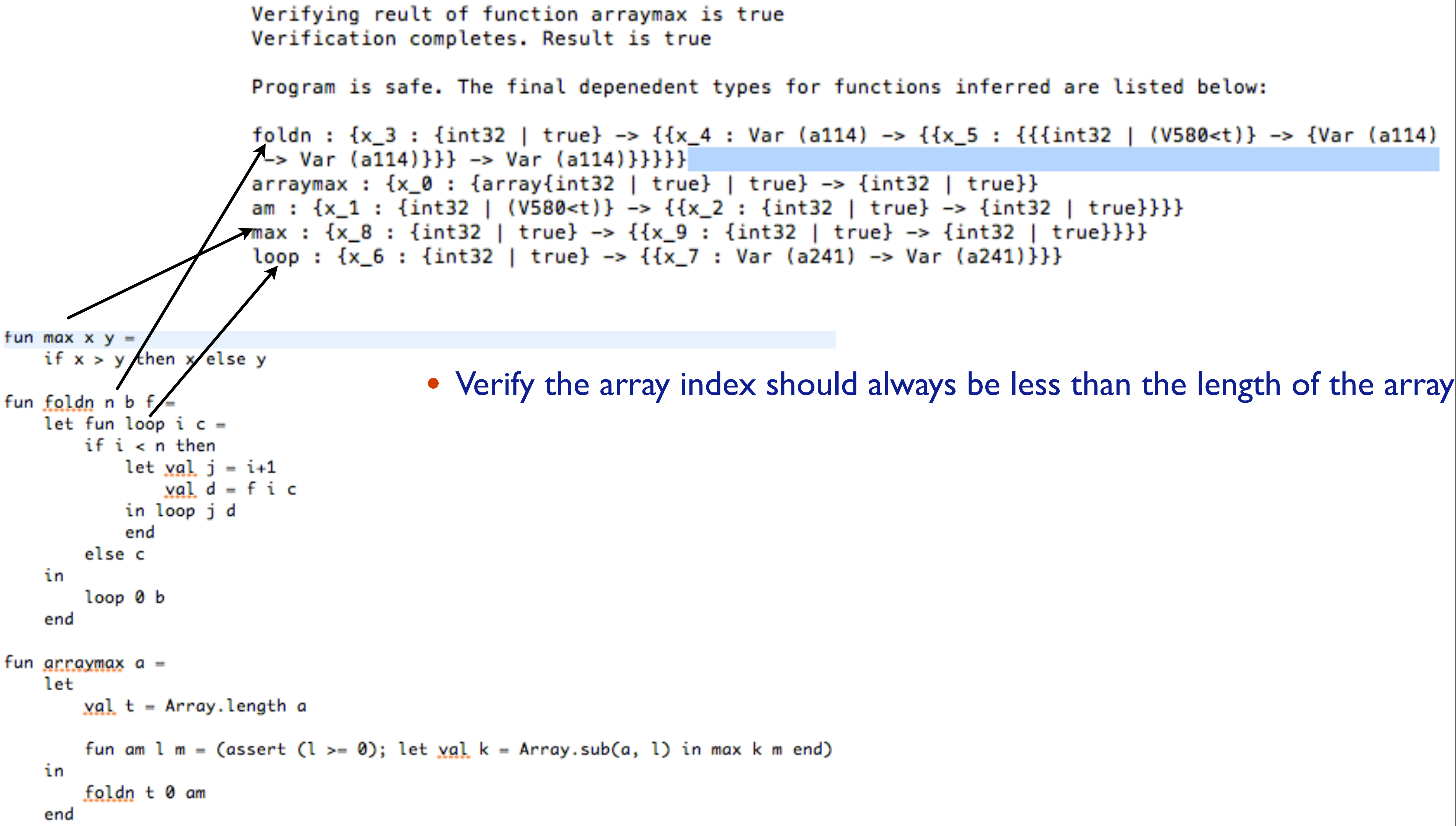
Preliminary Results



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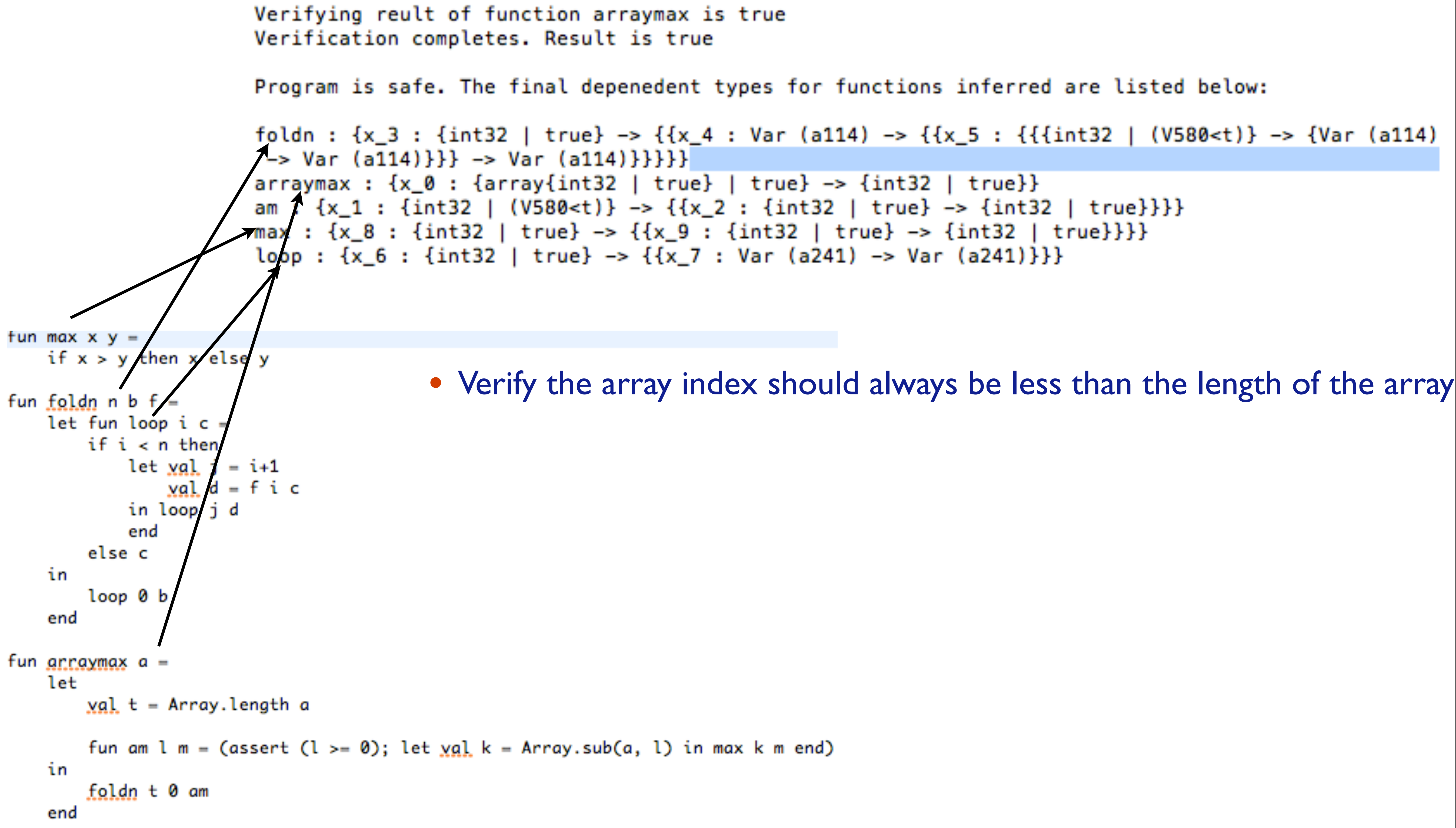

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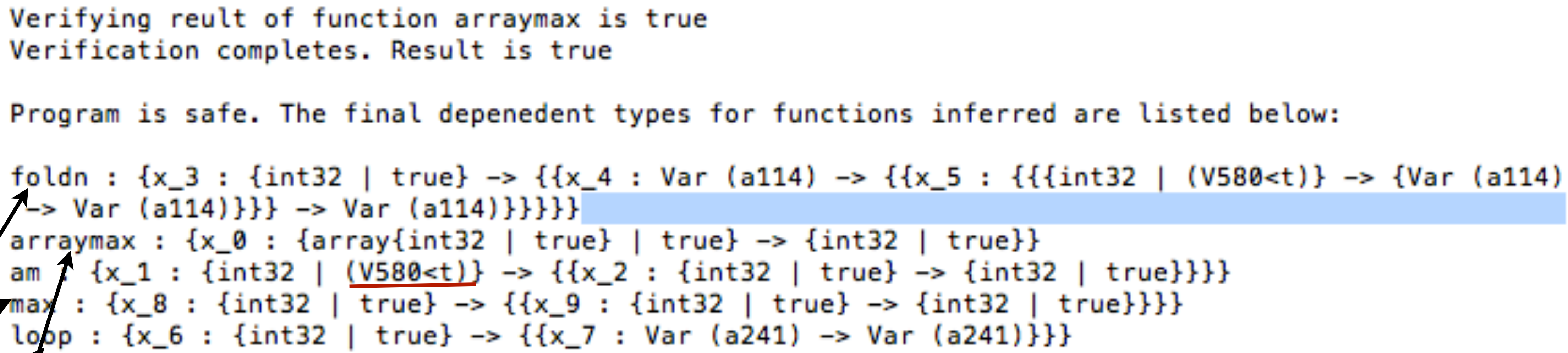
Preliminary Results



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loop : {x_6 : {int32 | true} -> {{x_7 : Var (a241) -> Var (a241)}}}
```
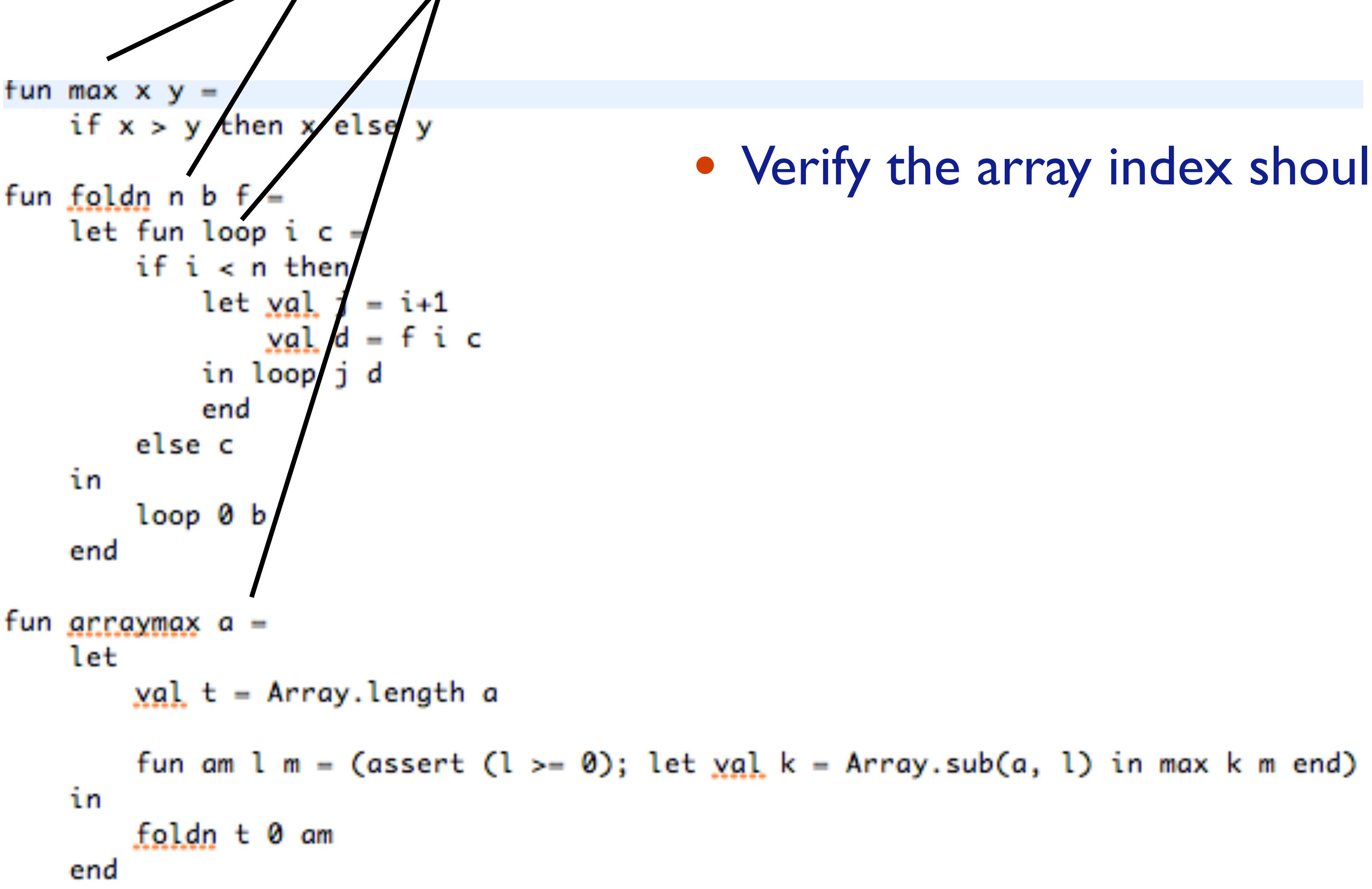
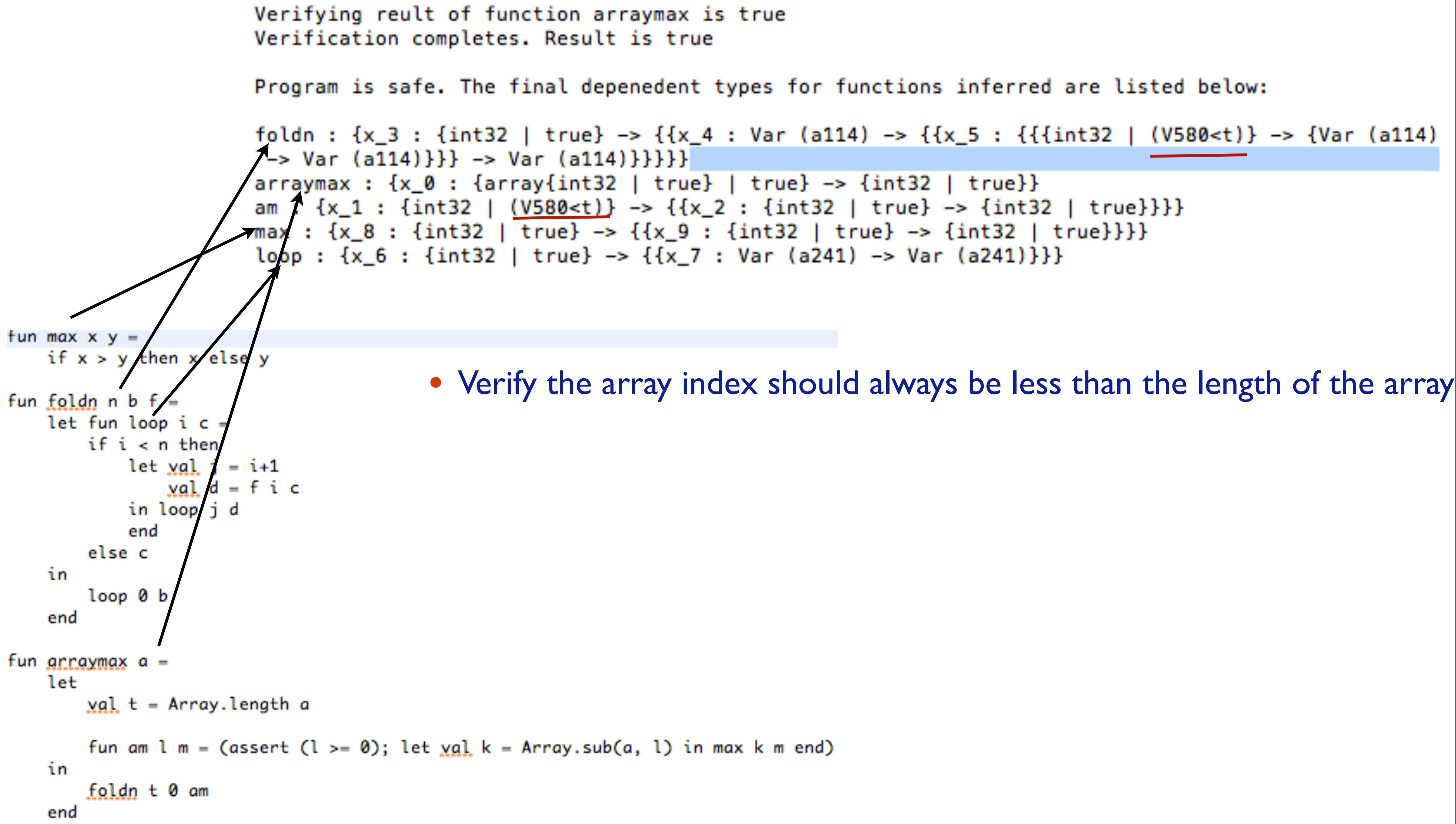



```
fun max x y =
    if x > y then x else y

fun foldn n b f =
    let fun loop i c =
        if i < n then
            let val j = i+1
                val d = f i c
            in loop j d
            end
        else c
    in
        loop 0 b
    end

fun arraymax a =
    let
        val t = Array.length a

        fun am l m = (assert (l >= 0); let val k = Array.sub(a, l) in max k m end)
    in
        foldn t 0 am
    end
```

- Verify the array index should always be less than the length of the array

Preliminary Results



```
Verifying reult of function arraymax is true
Verification completes. Result is true

Program is safe. The final depenedent types for functions inferred are listed below:

foldn : {x_3 : {int32 | true} -> {{x_4 : Var (a114) -> {{x_5 : {{{int32 | (V580<t)} -> {Var (a114)
-> Var (a114)}}} -> Var (a114)}}}}}
arraymax : {x_0 : {array{int32 | true} | true} -> {int32 | true}}
am : {x_1 : {int32 | (V580<t)} -> {{x_2 : {int32 | true} -> {int32 | true}}}}
max : {x_8 : {int32 | true} -> {{x_9 : {int32 | true} -> {int32 | true}}}}
loop : {x_6 : {int32 | true} -> {{x_7 : Var (a241) -> Var (a241)}}}
```

```
fun max x y =
    if x > y then x else y

fun foldn n b f =
    let fun loop i c =
        if i < n then
            let val j = i+1
                val d = f i c
            in loop j d
            end
        else c
    in
        loop 0 b
    end

fun arraymax a =
    let
        val t = Array.length a

        fun am l m = (assert (l >= 0); let val k = Array.sub(a, l) in max k m end)
    in
        foldn t 0 am
    end
```

- Verify the array index should always be less than the length of the array

Related Work

- Liquid Types (Rondon et. al [PLDI'08], Kawaguchi et. al [PLDI'09])
  - ★ Qualifier discovery vs. selection
- Higher-Order Program Model Checking (Kobayashi et. al [PLDI'11]),
  - ★ First-order vs. higher-order verification engine
- Dependent Types from Counterexamples (Terauchi [POPL'10])
  - ★ Concrete tests vs. abstract counterexamples
- Verifying Functional Programs using Abstract Interpreters (Jhala et. al, [CAV'11])
  - ★ Program analysis vs. program transformation

Conclusions

Preliminary evidence that incorporating modular verification techniques into an optimizing compiler is feasible

- ★ A first step towards devising optimizations that leverage automatically derived "rich" specifications