Networks: how Information theory met the space and time

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Plan of the talk

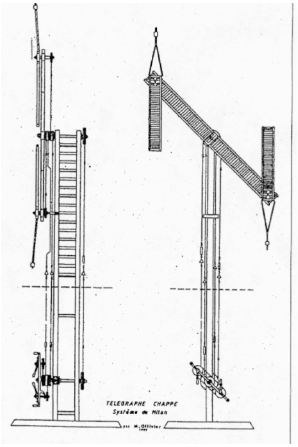
- History of networking and telecommunication
- Physics, mathematics, computer science
- Internet Routing complexity
- Mobile ad hoc networking complexity
- Wireless networking: Shannon’s law.
- Information and space-time
- Wireless capacity and space-time

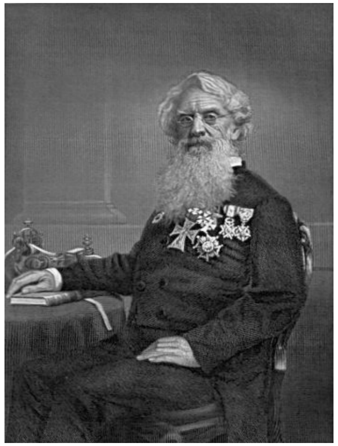
History of networking and telecommunication

- 1900: Marconi on Eiffel tower: first wireless telecommunication
- 1948: Shannon and Information Theory, transistor: first telecommunication massive optimization
- 1986: Birth of Internet, World Wide Web: First massive data, multimedia, multi-user network.
- 2000: the internet got wireless

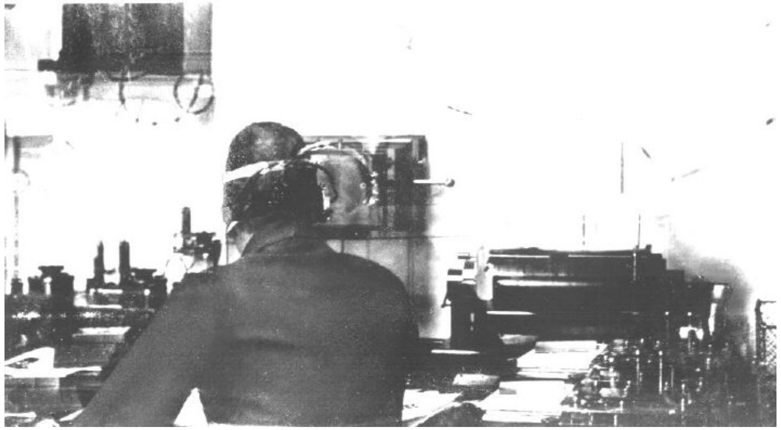
Rise of Telecommunications (1800-1900)

C. Chappe 1793



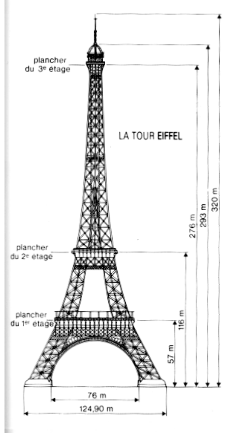


S. Morse 1837

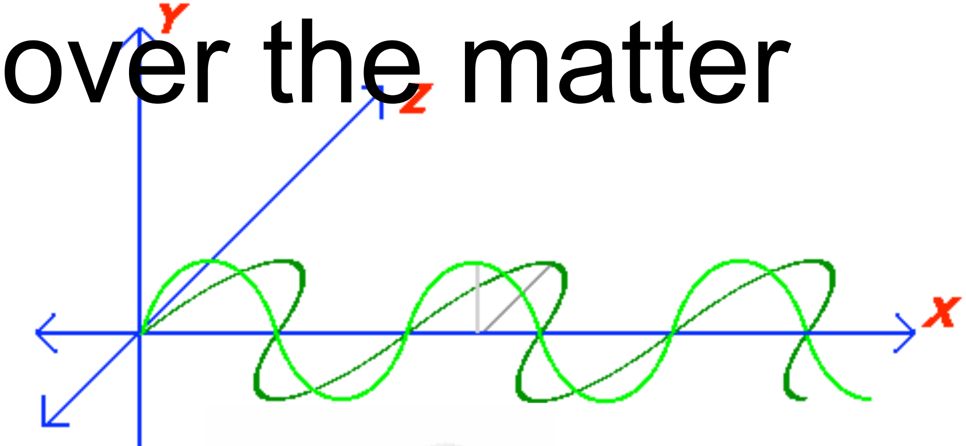


T. Edison 1878

G. Marconi 1899



From pigeon to light speed: the triumph over the matter



- Electromagnetic

- Journal paper

- Pigeon post

From lamp to transistor: code and signal processing revolution. The triumph over the numbers

- Telecommunications cross the border from physics to mathematics

enigma 1941

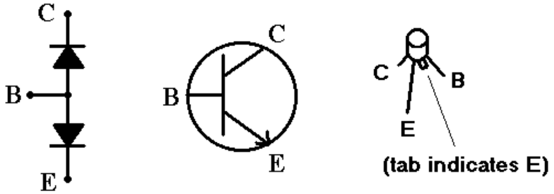
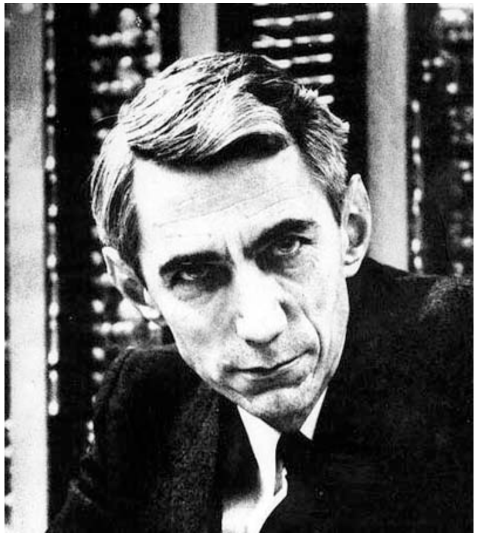


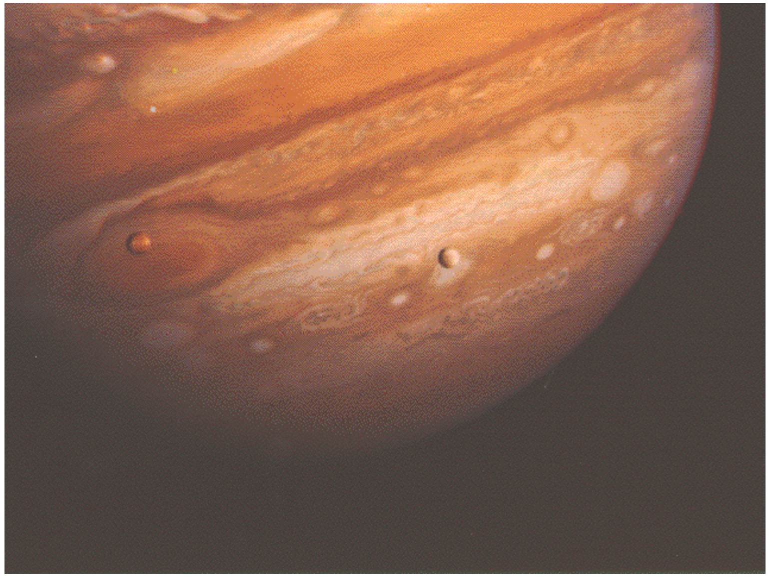
Fig. 1 The NPN Transistor

transistor 1947

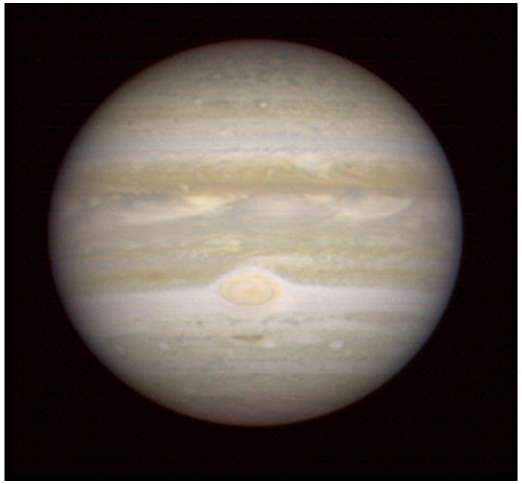


C. Shannon 1948

The wonders of the digital revolution



C. Berrou 2000

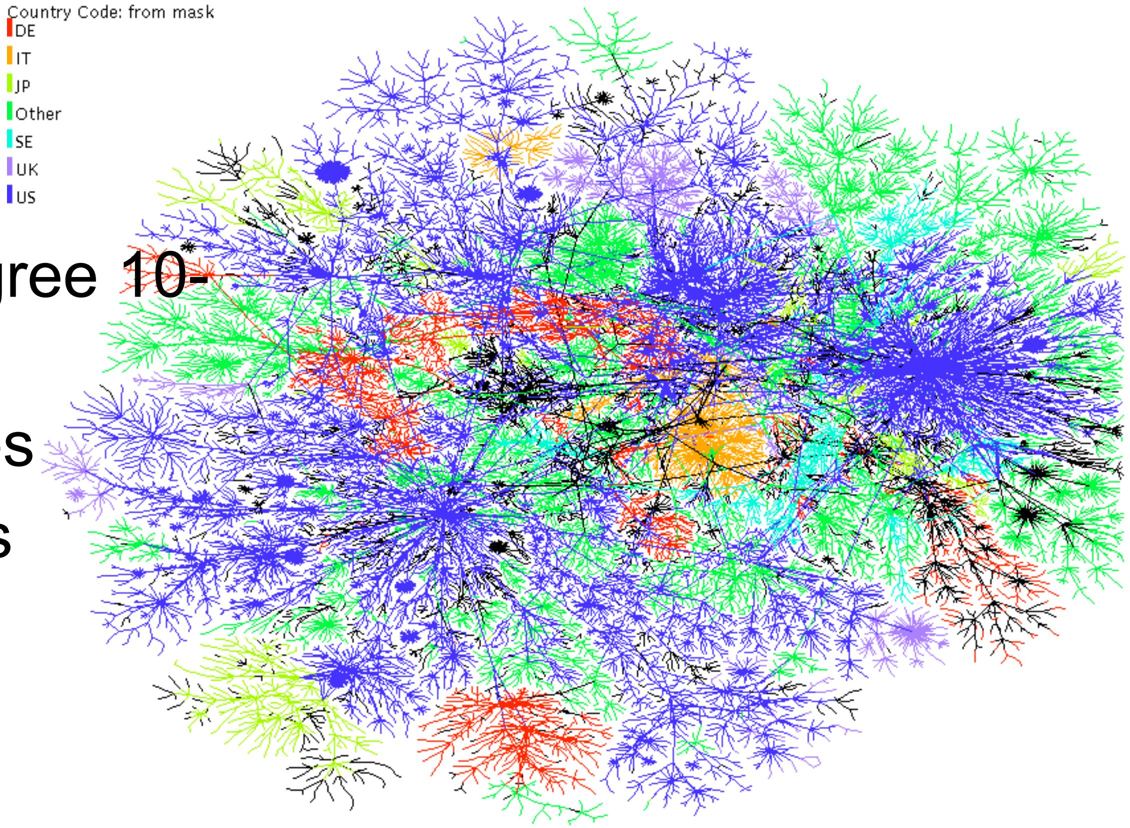


Internet: The triumph over the complexity

- Telecommunications cross the border of computer science
  - Cold war 1970: the strategic network identified as the weak point
- Answer: ARPANET then INTERNET
  - First, connect missiles sites
  - Then, universities
  - Every user
- The protocol (IP) binds heterogeneous networks (fiber, telephone, *etc*)
- Detects and avoids damaged parts in real time.

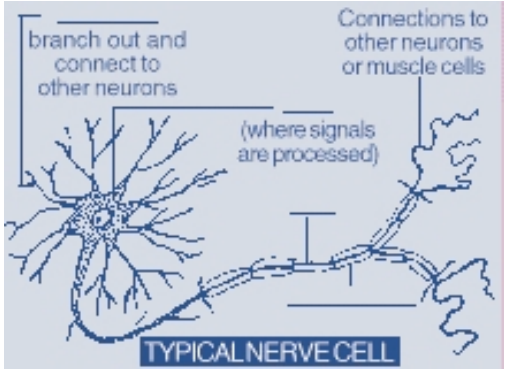
Facts and figure about internet

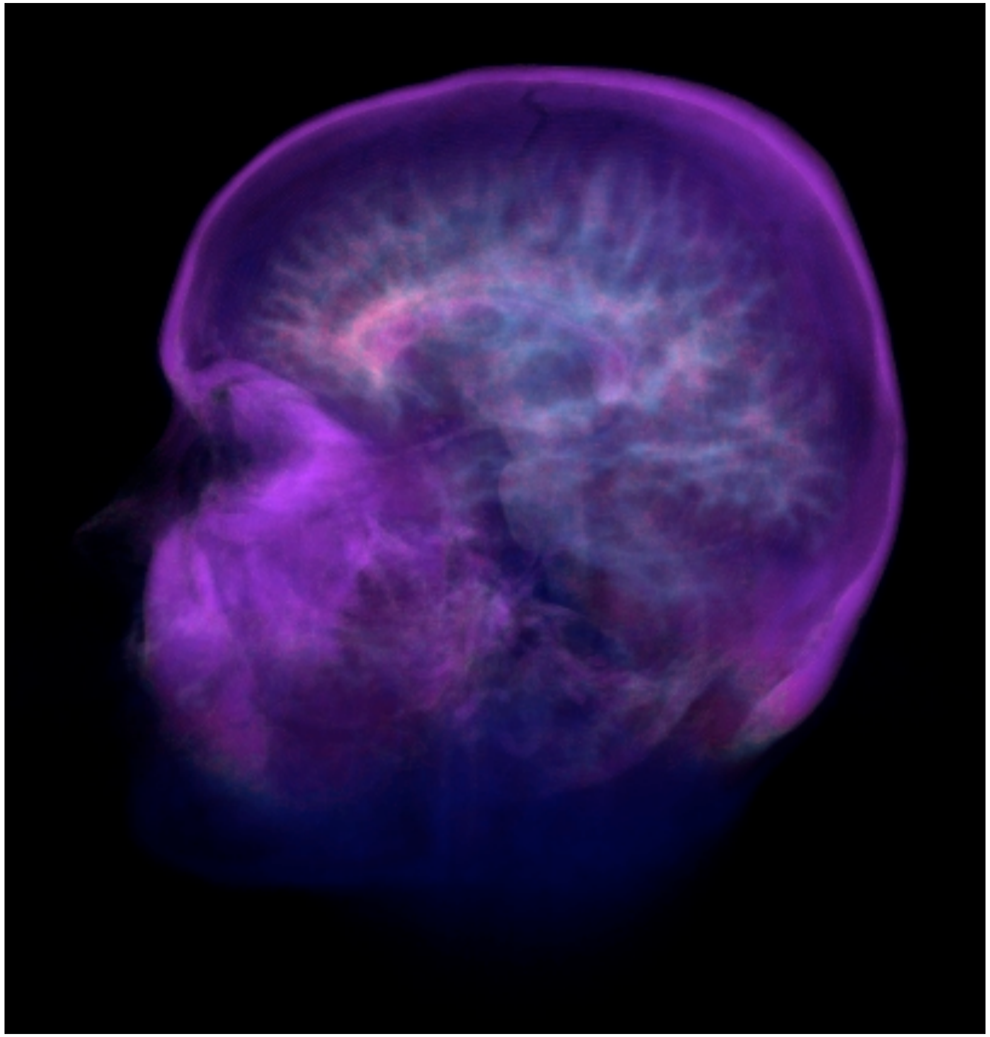
- internet:
  - 6.10^8 connected machines
  - Connectivity degree 10-100
  - 3.10^{10} web pages
  - 2.10^{15} online bits
  - speed sub c



Internet is the most complicated artefact

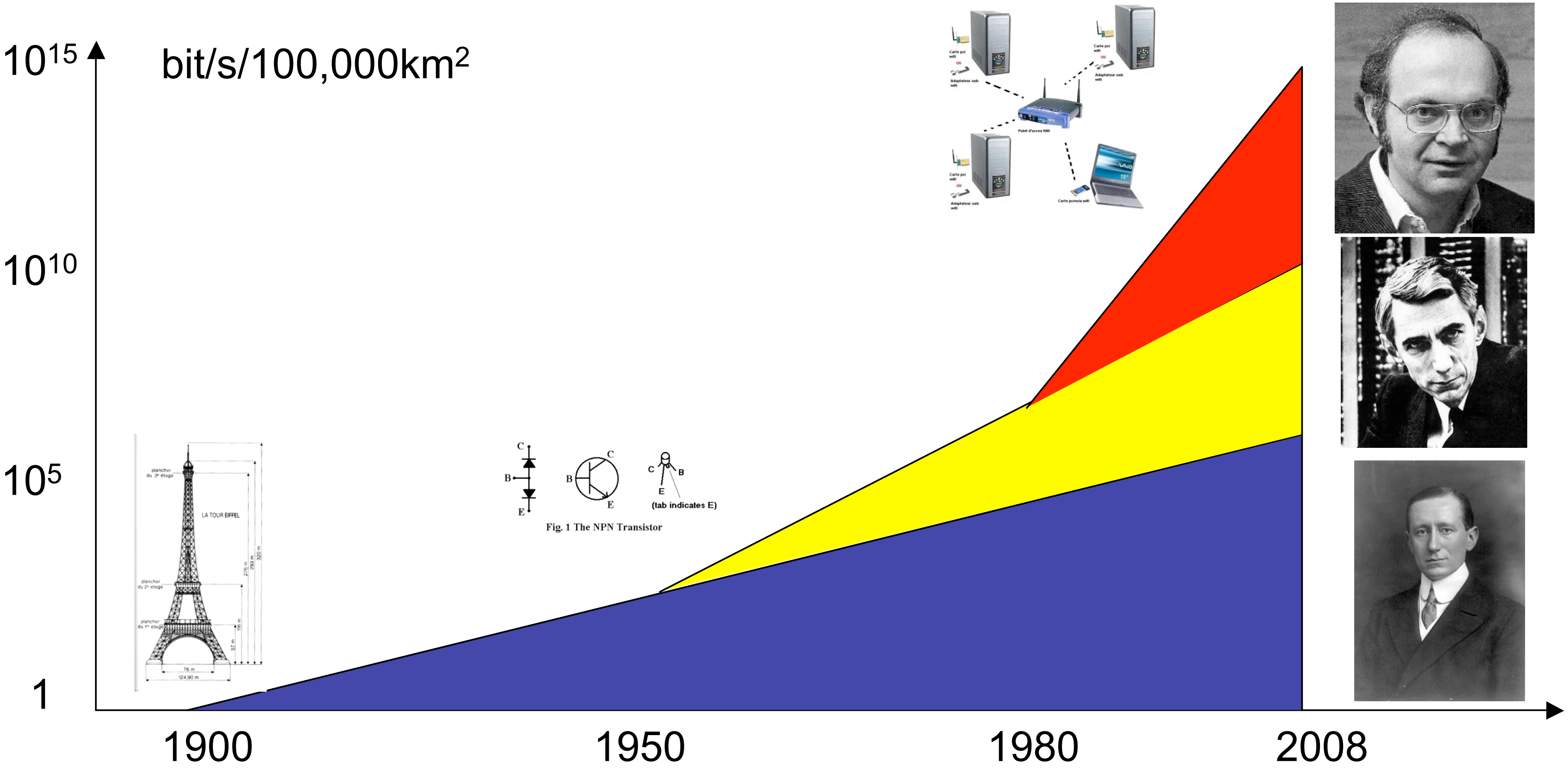
- Far from human brain:
    - 10^{11} neurons
    - Connectivity degree : 10,000 -100,000
    - Speed: 100-400 m/s (scale with world internet)





Histogram

- Physics, math and computer sciences in telecommunication



wireless performance from Marconi to Wifi

- Traffic density
  - 1900:
    - 10 bit/s/100.000 km^2 ,
    - 1000 watt
  - 2008:
    - 10,000,000 bit/s/ha,
    - 0.01 watt

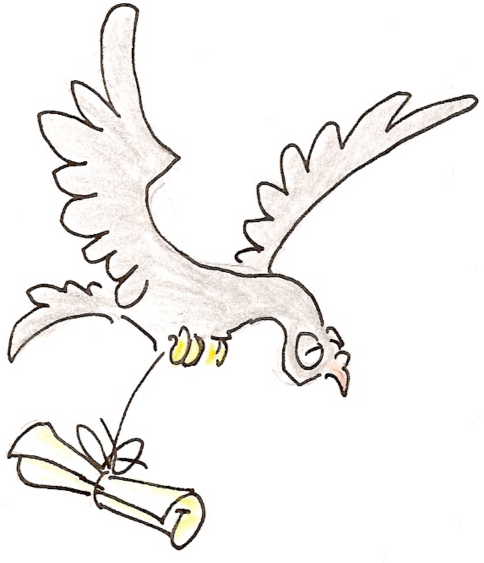
A factor 10^{14}
100,000,000,
000,000

Comparison: road traffic

Road traffic increase 1900-2008: $< 10^5 = 100{,}000$?

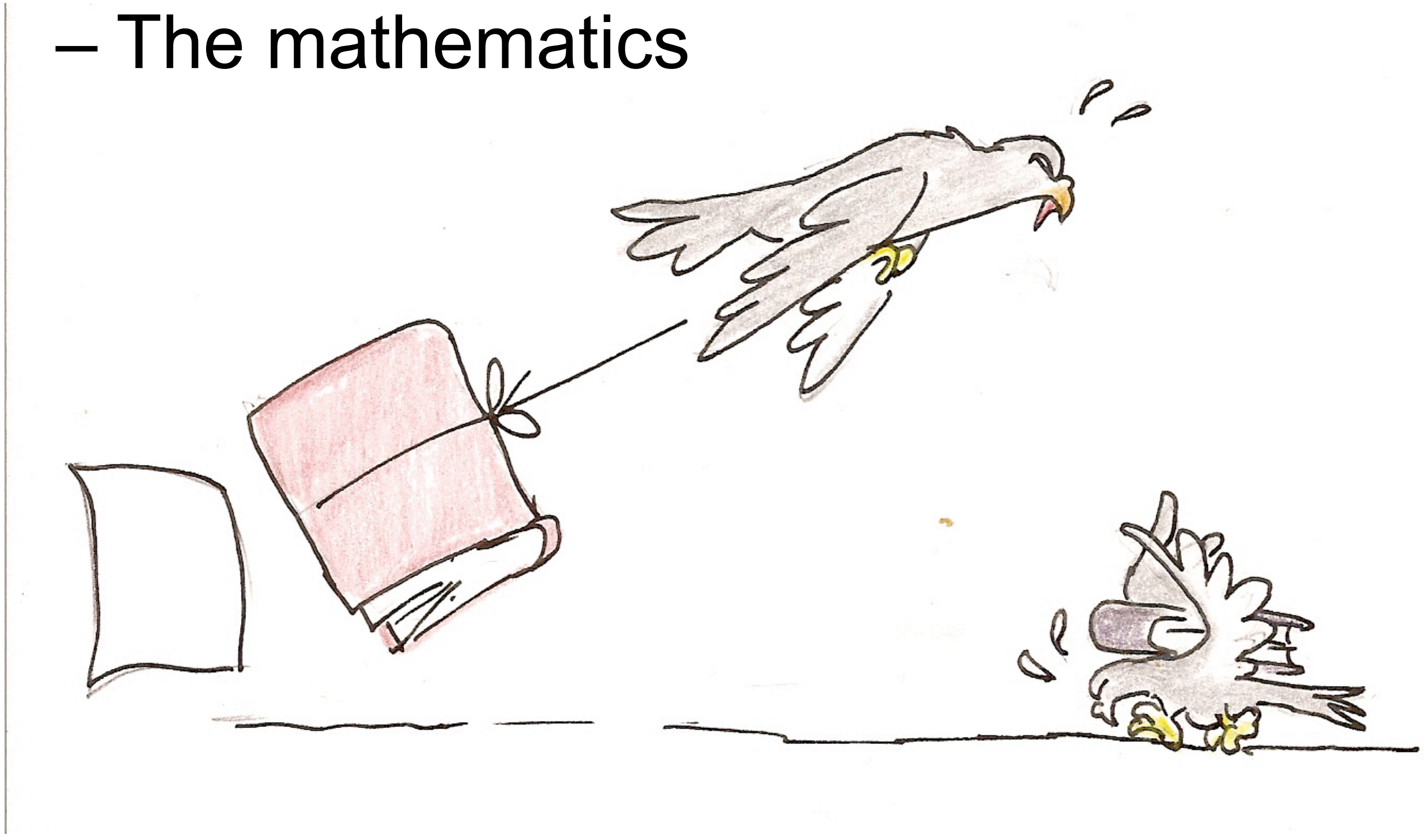
Physics,mathematics,computer science

- The telecommunications without
  - – The physics



Physics,mathematics,computer science

- The telecommunications without
  - The mathematics



Physics,mathematics,computer science

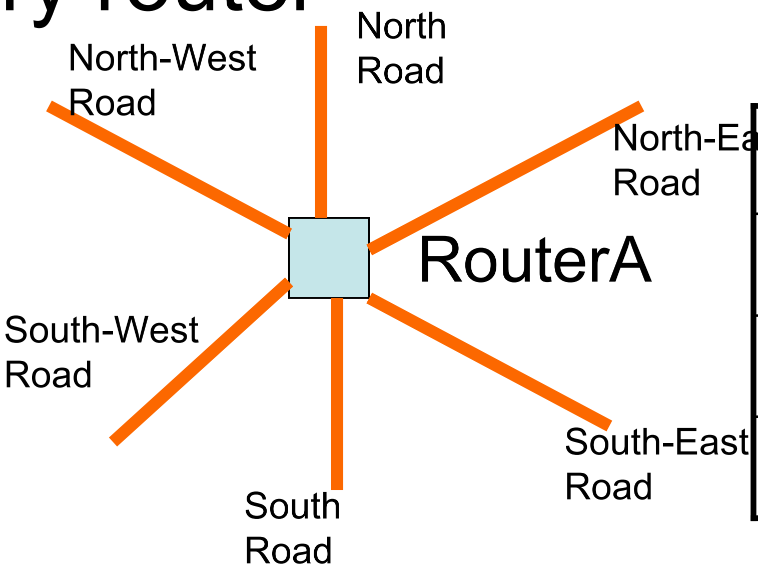
- The telecommunications without
  - The computer science

Physics-math-computer science

- Networks are together physics, math and computer science
- Computer science is the science which makes a complex system a simple object
  - Gee, how hard sometimes to reach this simplicity…

Example: routing protocol

- Routing tables is like orientation maps in every router



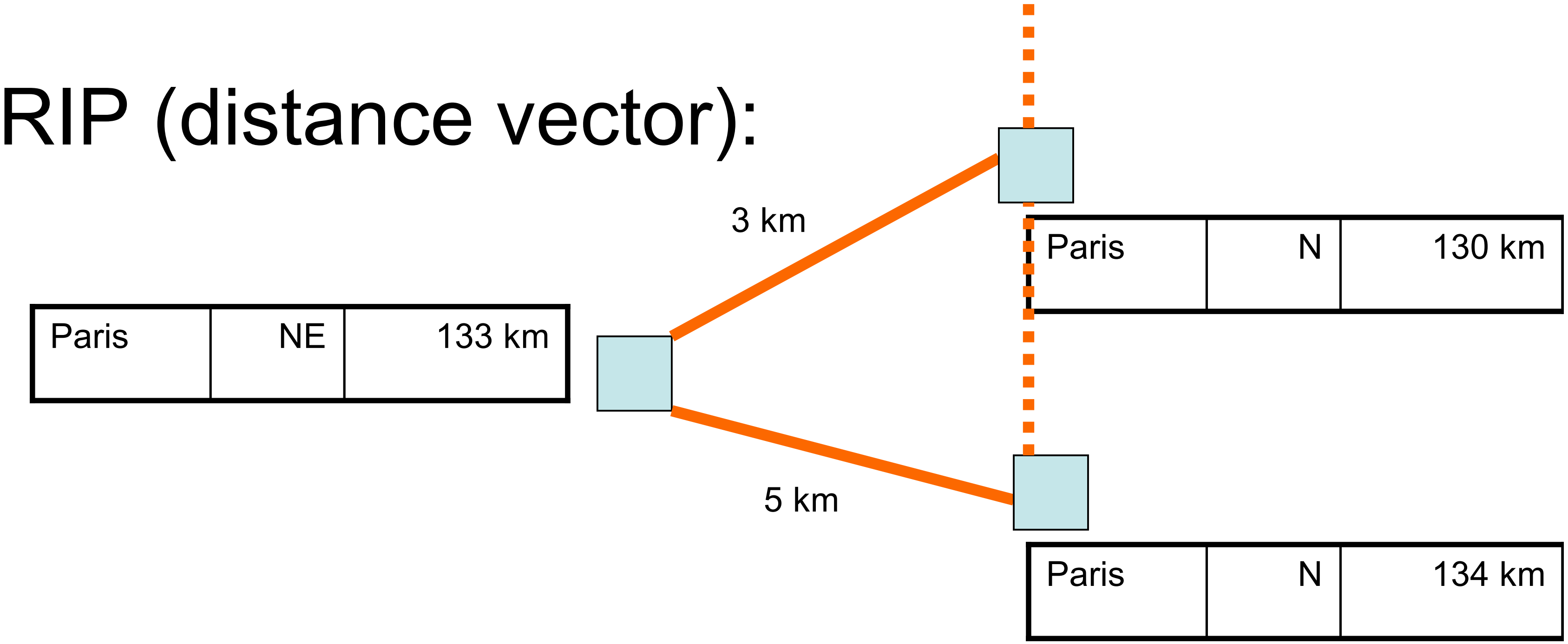
destination	exit	distance
routerB	NE	62 km
Paris	N	133 km
Beijing	NW	12880 km

Example: routing protocol

- Two protocols;
  - RIP: run to neighbor protocol
    - Deliver the routing table to local neighbor
  - BGP: run theTour de France protocol
    - Deliver the local routing table to whole network

Example: routing protocol

- RIP (distance vector):

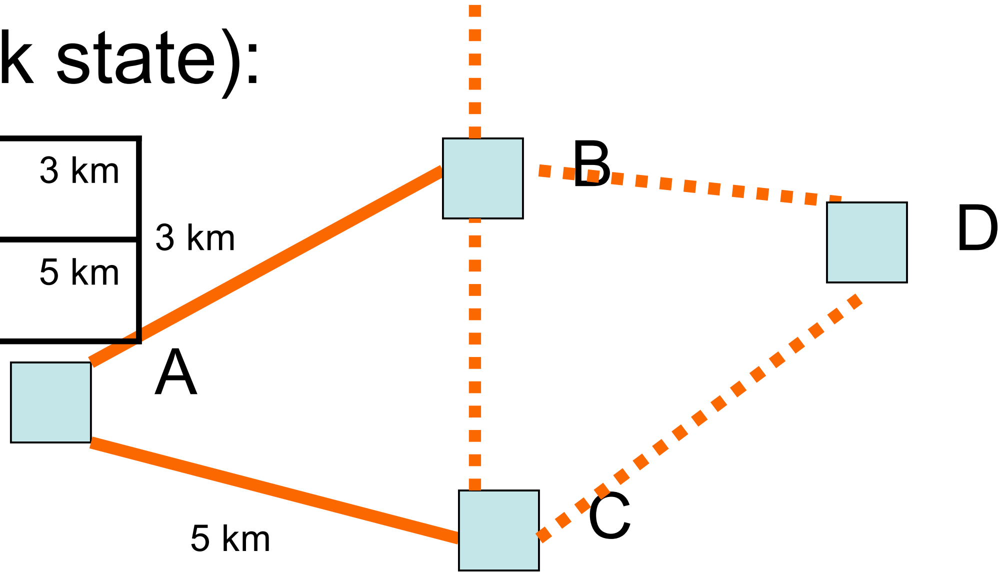


  - Complexity: NL (per refresh period)
  - Convergence time: network diameter

Example: routing protocol

- BGP (link state):

B	NE	3 km
C	SE	5 km



  - Routing table computed on local link database
  - Complexity L^2 (per refresh period)
  - Convergence: network diameter

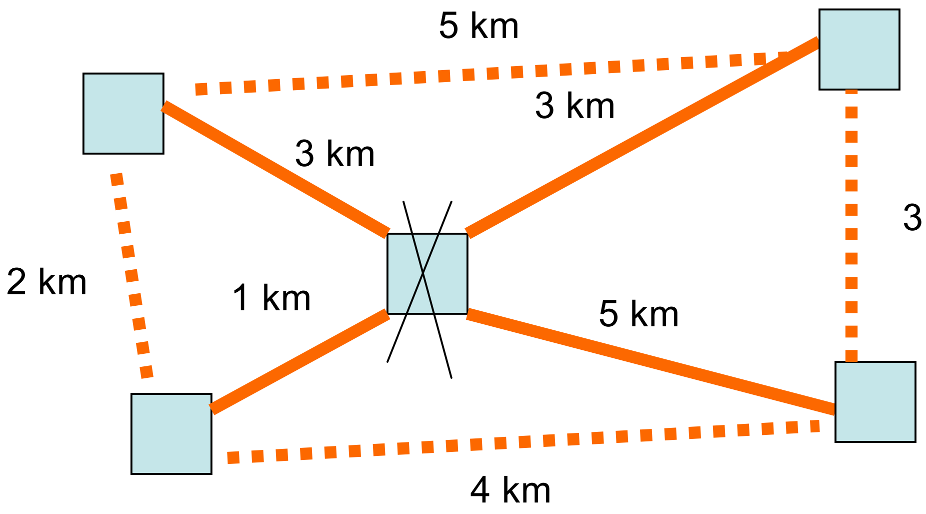
Example: routing protocol

- Which is best:
  - Deliver whole table to local?
  - Deliver local table to all? ✗

- Divergence time makes the difference!
  - Network Diameter with BGP (symmetric)
  - At least Diameter × L with RIP (asymmetric)

RIP failure

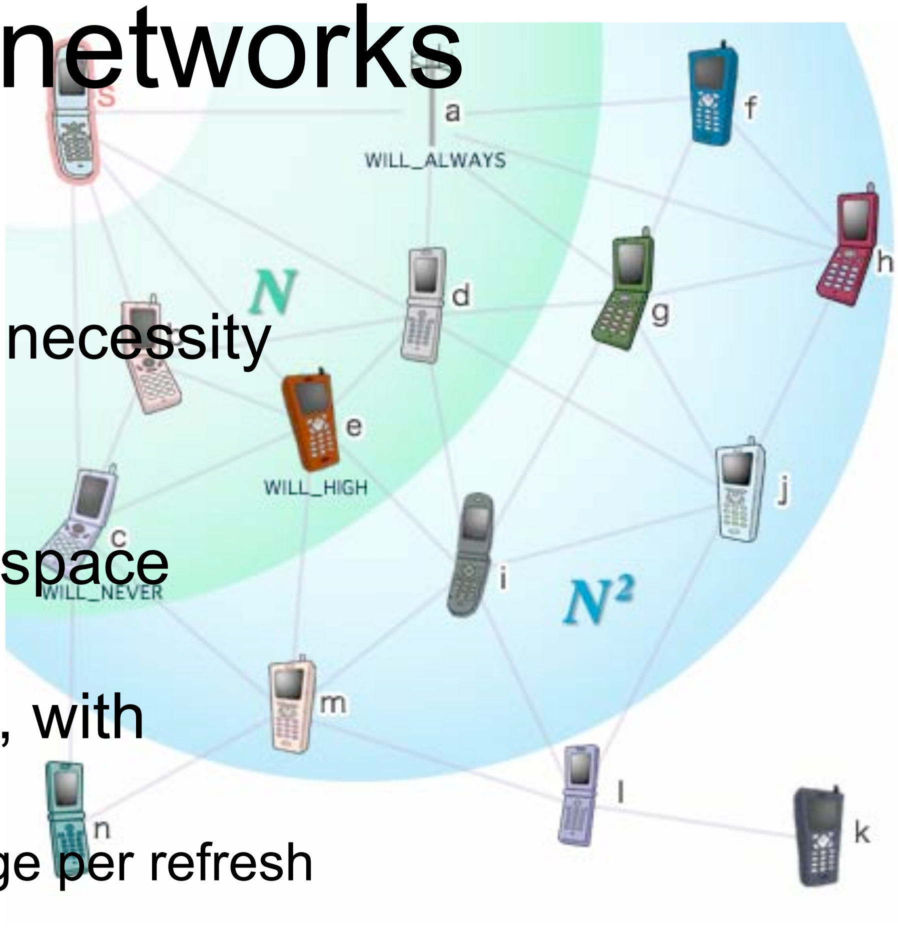
- Count to infinity (1983 ARPANET incident)



Diameter max limited to 15

$$\infty \times L < 15 \times L$$

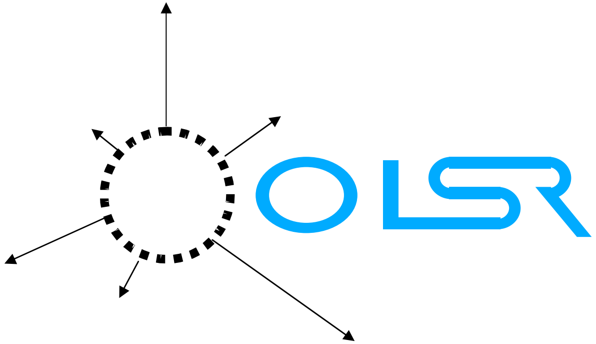
Wireless networks



Mobile *ad hoc* networks

- Mobility makes link failure a necessity
  - Refresh period 1 second
  - Automatic self-healing
- Local neighborhood is local space
  - Unlimited neighborhood size
- Stadium network: N=10,000, with average degree 1,000
  - BGP needs 10^{14} links exchange per refresh time
  - BGP fails on Wifi networks with 20 users at walking speed.
  - Heavy density kills link state management.

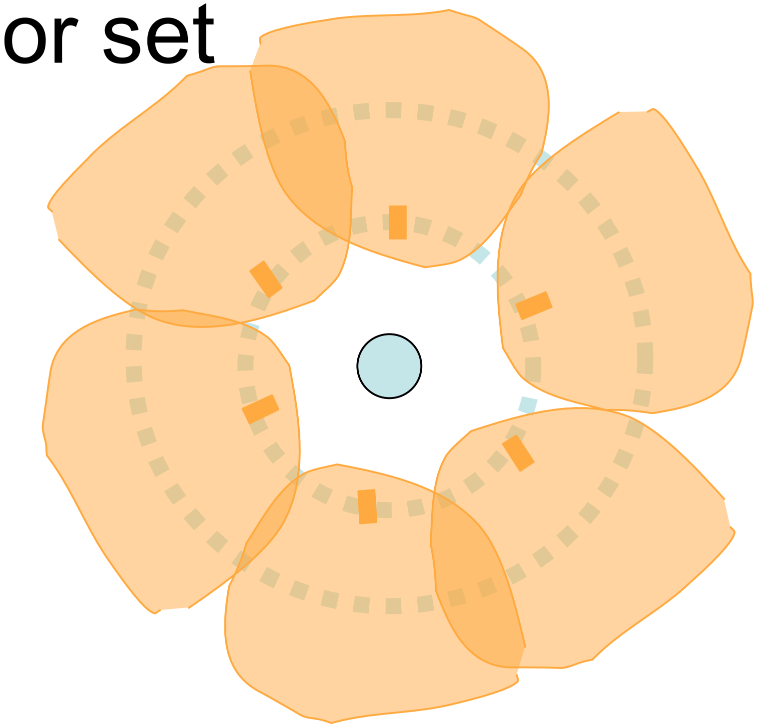
Wireless topology compression



- Optimized Link State Routing protocol
  - Advertize local table subset to whole network
  - Local neighborhood subset is the MultiPoint Relay (MPR) set of the node.
  - Every node receives a compressed topology information.

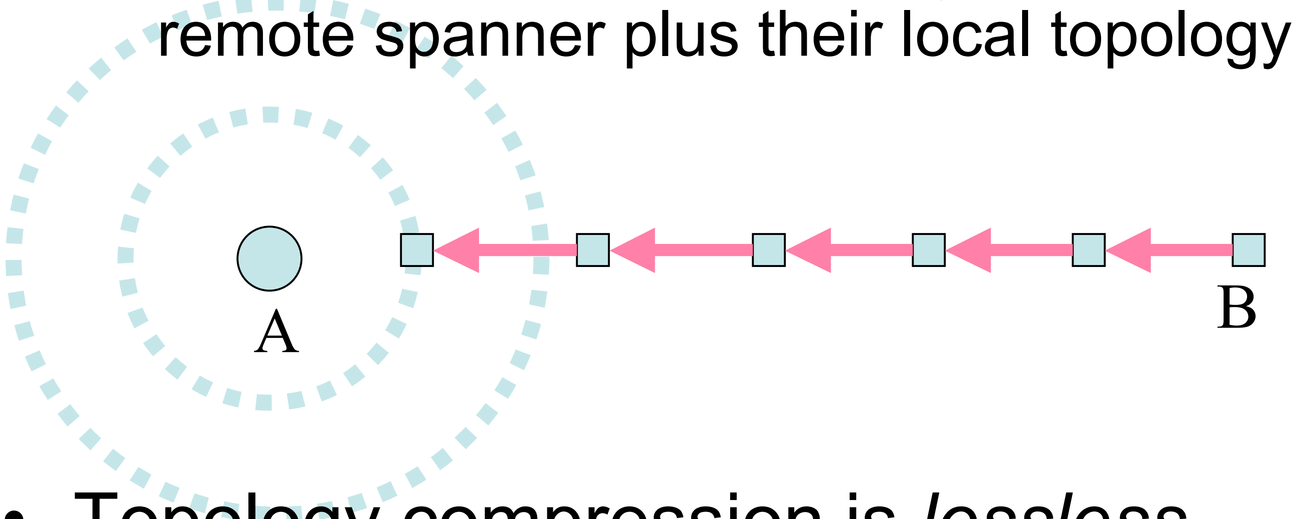
Wireless topology compression

- the MPR set covers the two-hop neighbor set



Wireless topology compression

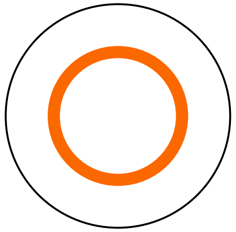
- MPR sets forms a *remote spanner*
  - Nodes compute their routing table on remote spanner plus their local topology.



- Topology compression is *lossless*
  - Optimal routes on remote spanner also optimal without compression.

Wireless topology compression

- In Erdoss-Renyi random graphs
  - Random selection gives compression rate $\tau_r = \frac{N^3}{L^2}\log N$
- In unit disk graph model
  - Greedy selection gives compression rate $\tau_r = 3\left(\frac{\pi N}{L}\right)^{\frac{2}{3}}$
- Stadium network
  - Compression rate 10^{-2}



Dissemination compression

- Only MPR retransmit local routing table
  - Another compression of rate $\approx \tau_r \frac{N}{L}$
- Stadium network
  - Overall compression 10^{-7}

Wireless protocol compression

- 10^7 ratio is like
  - your car traveling at light speed

Information theory in networks

- Originally for point to point coding

A

B

X: Emitted code

Y: Received code

Channel capacity: I(X,Y)=entropy of Y - channel entropy

$$I(X,Y) = h(Y) - h(Y \mid X)$$

Example: wireless transmission

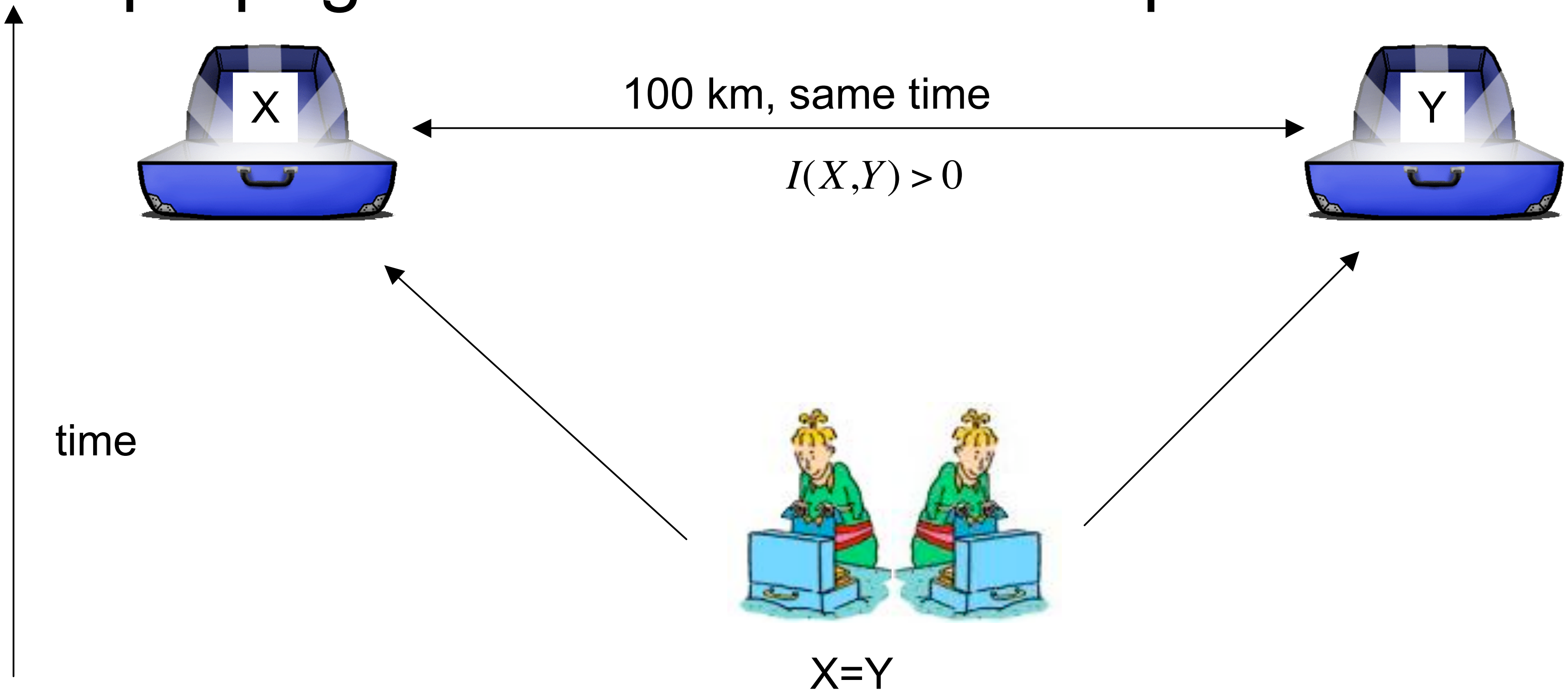
- Emitted Symbol: X an integer in $[1,S]$
- Received Symbol: $Y = X + \beta$
  - Noise β an integer in $[1,\mathrm{B}]$
  - Capacity per symbol $I(X,Y) = \log(1+\frac{S}{B})$

$$h(Y) = \log(S+B)$$

$$h(Y \mid X) = \log B$$

Limitation of information theory within space and time

- Fake superluminal information propagation: the twin traveler paradox



100 km, same time

$I(X,Y) > 0$

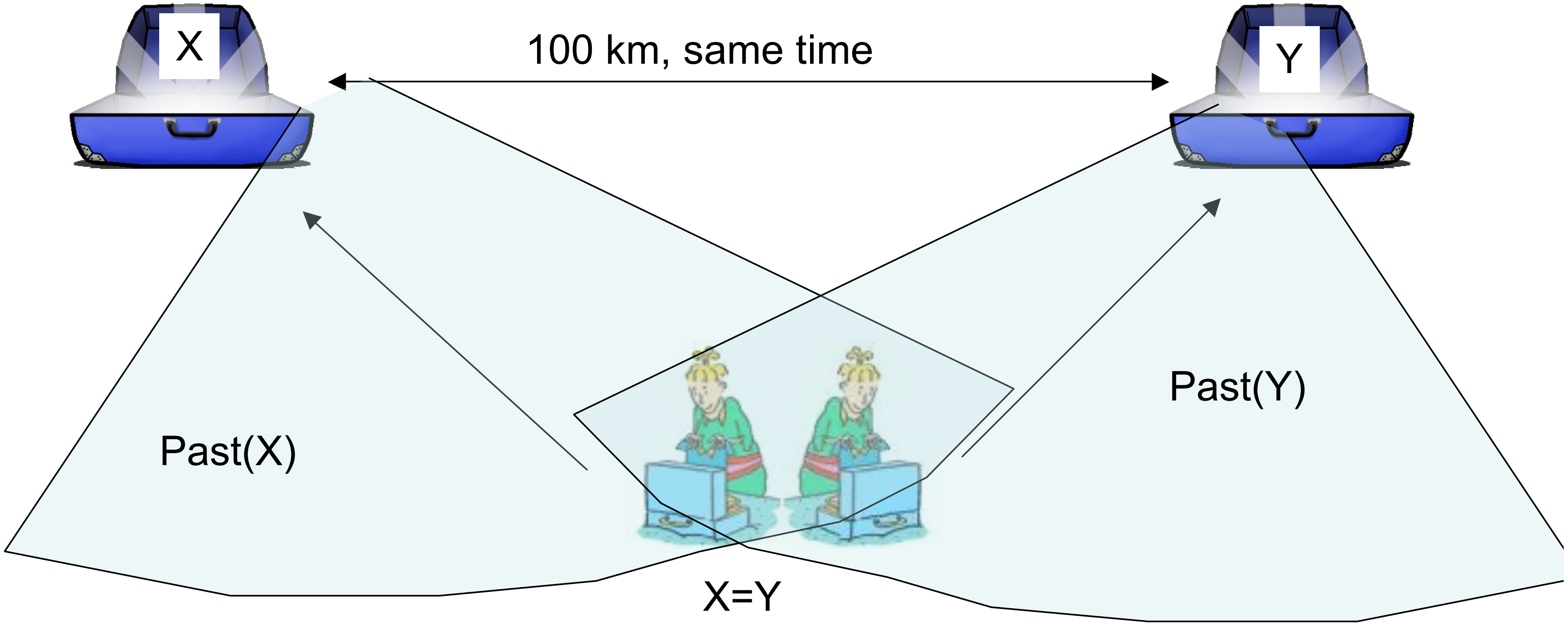
time

X=Y

Causality in information theory

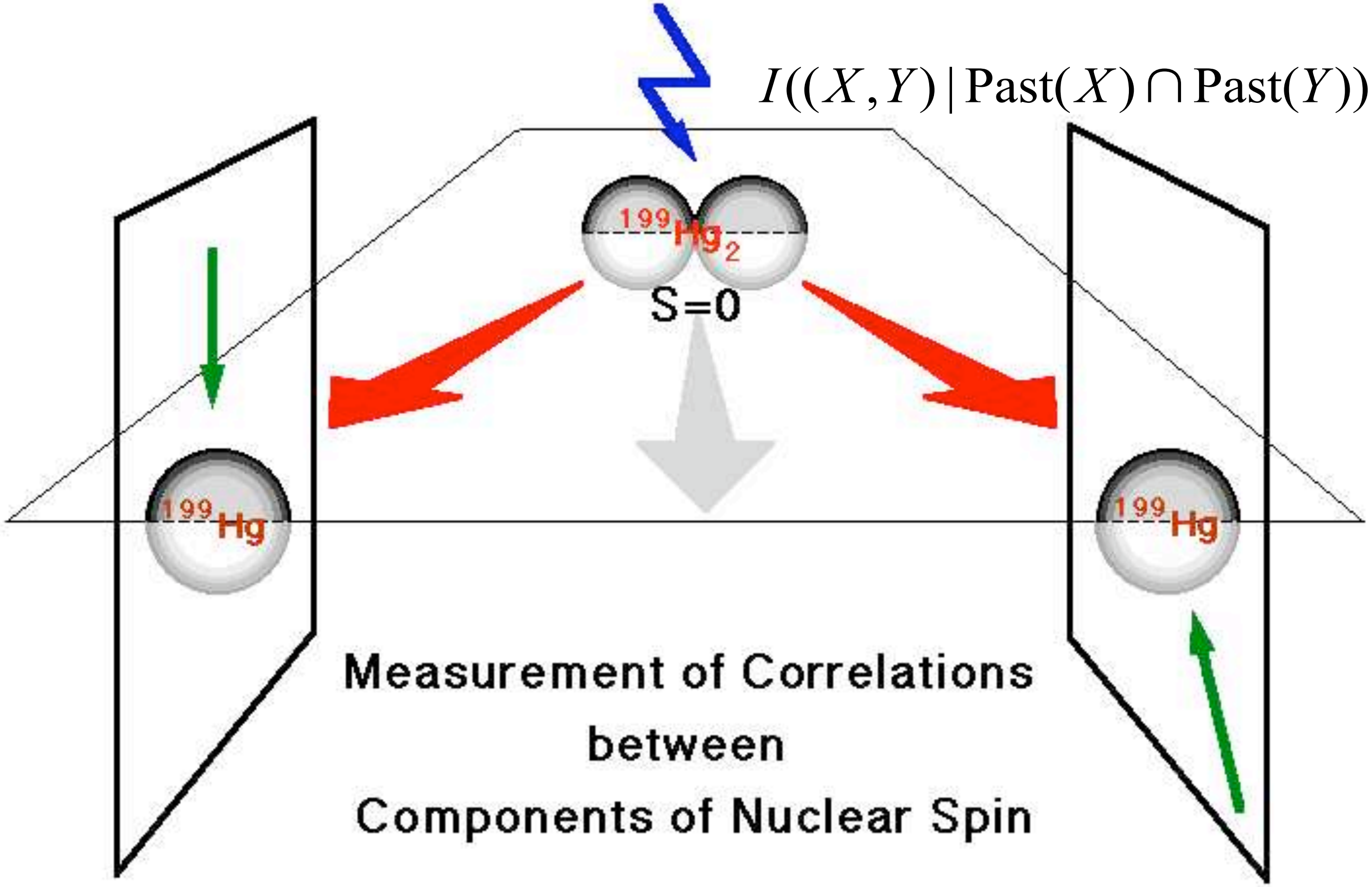
- Twin traveler paradox resolution?.
  - For a common fixed past X et Y should be independent.

$$I((X,Y)\,|\,\mathrm{Past}(X)\cap\mathrm{Past}(Y))=0$$



Bell theorem

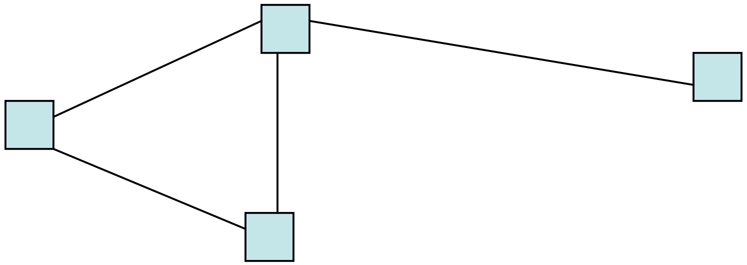
- Information is non causal:



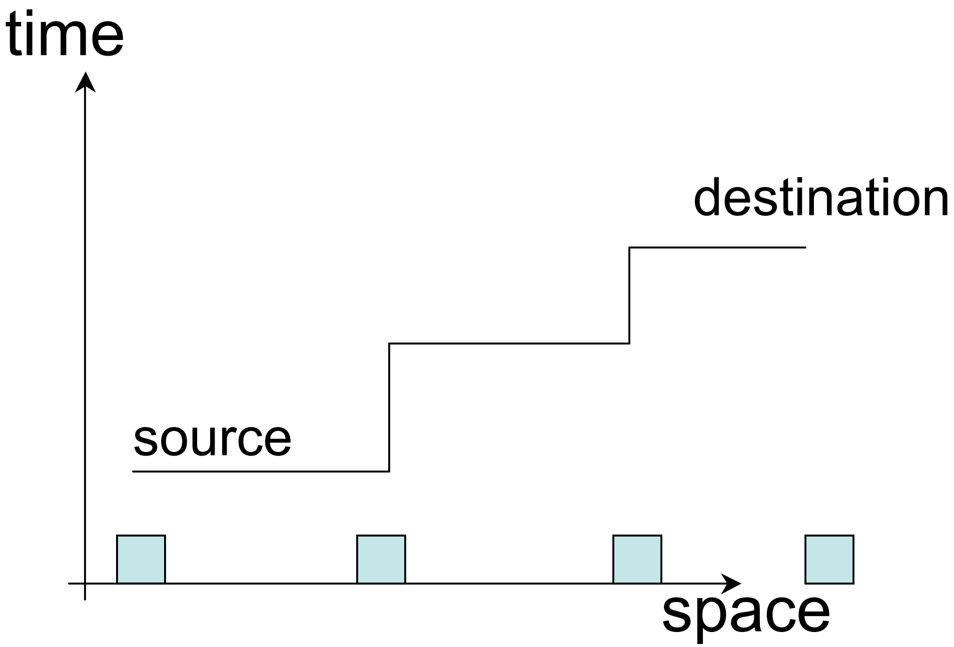
$$I((X,Y)\,|\,\mathrm{Past}(X)\cap\mathrm{Past}(Y)) > 0$$

Network: information and space-time

- A Network is
  - Set of objects in physical space
  - relay information
    - from arbitrary source
    - to arbitrary destinations.

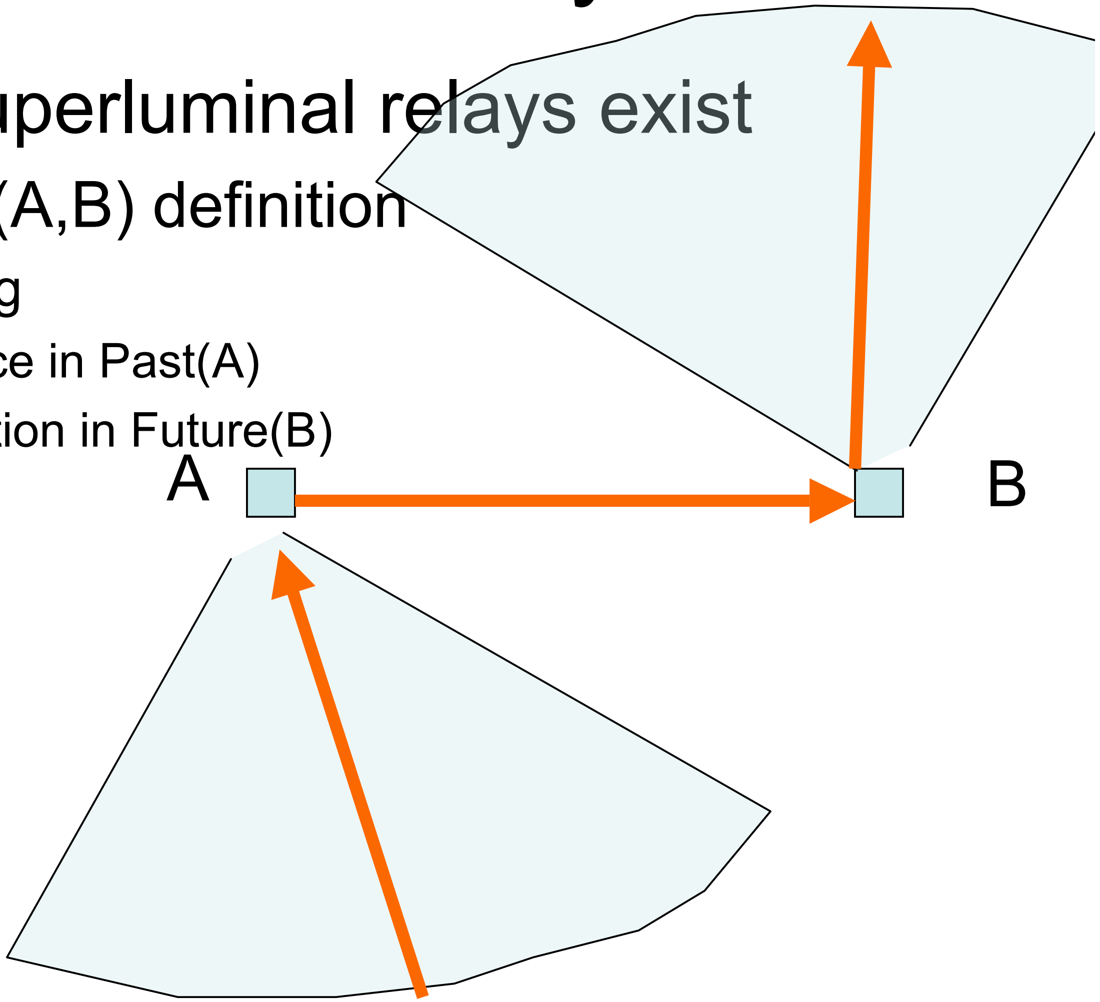


  - Concept of router
  - Concept of information propagation path

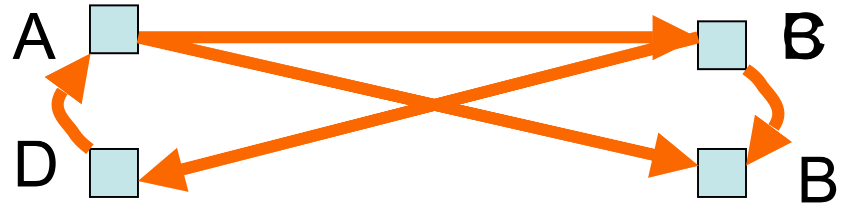


Space time relay versus information causality

- Can network of superluminal relays exist
  - Space time relay (A,B) definition
    - Can relay anything
      - From any source in Past(A)
      - To any destination in Future(B)



Space time relay versus information causality



time

temporal loop: quantum unitarity violation

$$\text{capacity } I = \frac{|\nu|}{\log 2}$$

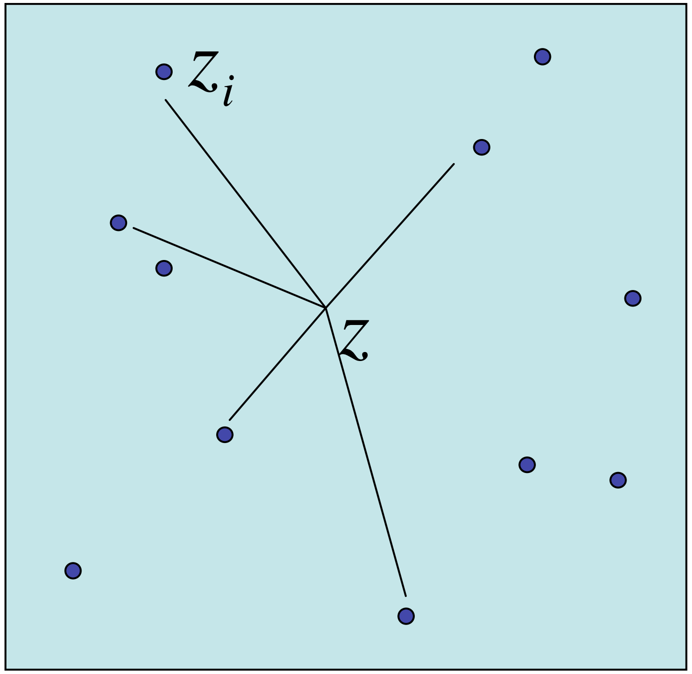
A wireless model

- Emitters are distributed as Poisson process in the plan

  density λ

  - Signals sum

$$S = \sum_i |z - z_i|^{-\alpha}$$

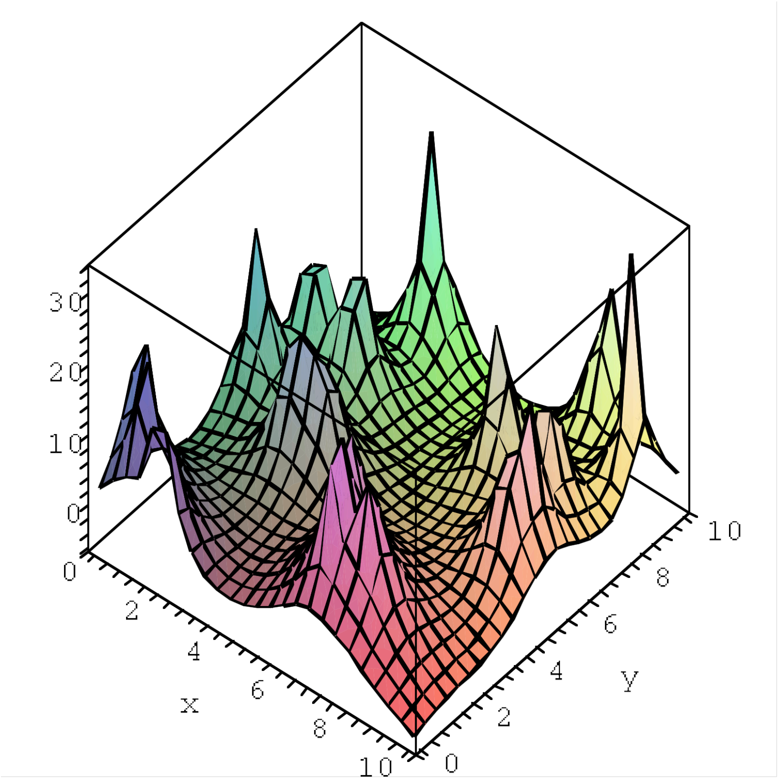


A wireless model

- Signal distribution

$$E(e^{-\theta S}) = \exp(-\lambda\pi\Gamma(1-\gamma)\theta^{\gamma})$$

$$\gamma = \frac{2}{\alpha}$$



Wireless information theory

- Wireless capacity,
  - emiters send independent information

$$I_0 = E\left(\sum_i \log_2(1+\frac{|z-z_i|^{-\alpha}}{\sum_{j\neq i}|z-z_j|^{-\alpha}})\right)$$

  - Computable and invariant even with random fading

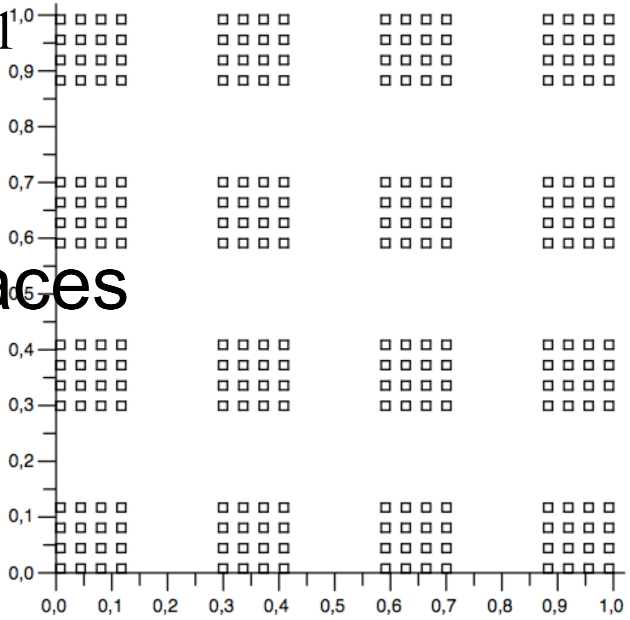
$$I_0 = \frac{\gamma}{\log 2}$$

Wireless Shannon

- Invariant with dimensions

$$I_0 = \frac{\alpha}{D}(\log 2)^{-1}$$

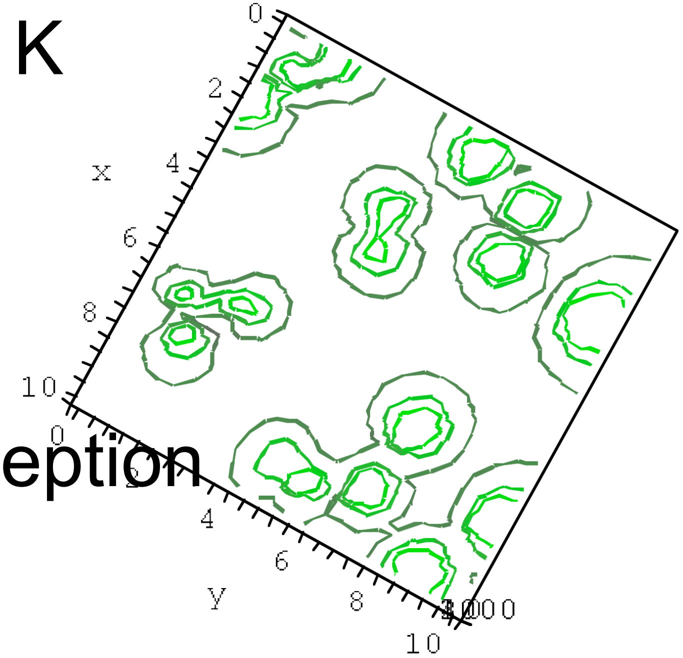
  - – Works also with fractal spaces
    - D=4/3



Wireless Space capacity

- With signal over noise ratio K requirement

$$I(K) = \frac{\sin(\gamma\pi)}{\gamma\pi} K^{-\gamma}$$



- Average area of correct reception

$$\sigma(K) = \frac{I(K)}{\lambda}$$

$$\sigma(10) \approx 0.037066$$

Wireless Space capacity

- Reception probability vs distance

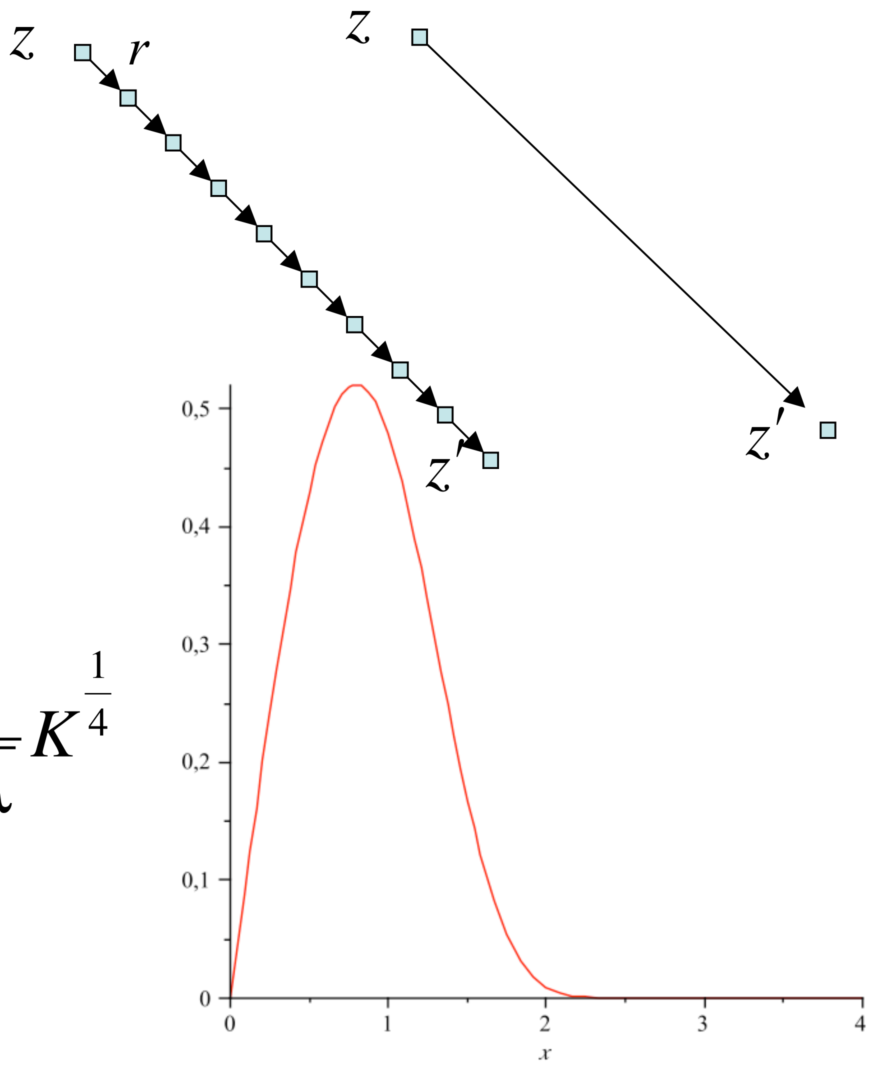
$$p(r,\lambda,K) = p(r\sqrt{\lambda}K^{-\frac{1}{4}},1,1)$$

$$p(r,1,1) = \sum_n (-1)^n \frac{\sin(\pi n\gamma)}{\pi} \frac{\Gamma(n\gamma)}{n!} r^{2n}$$

$$p(r,1,1) = 1 - \mathrm{erf}(\frac{r^2}{2}) \text{ when } \alpha = 4$$

- Optimal routing radius

$$r_m = \arg\max_{r>0}\{rp(r,\lambda,K)\} = \frac{r_1}{\sqrt{\lambda}}K^{\frac{1}{4}}$$



Wireless Space capacity

- Average number of retransmissions

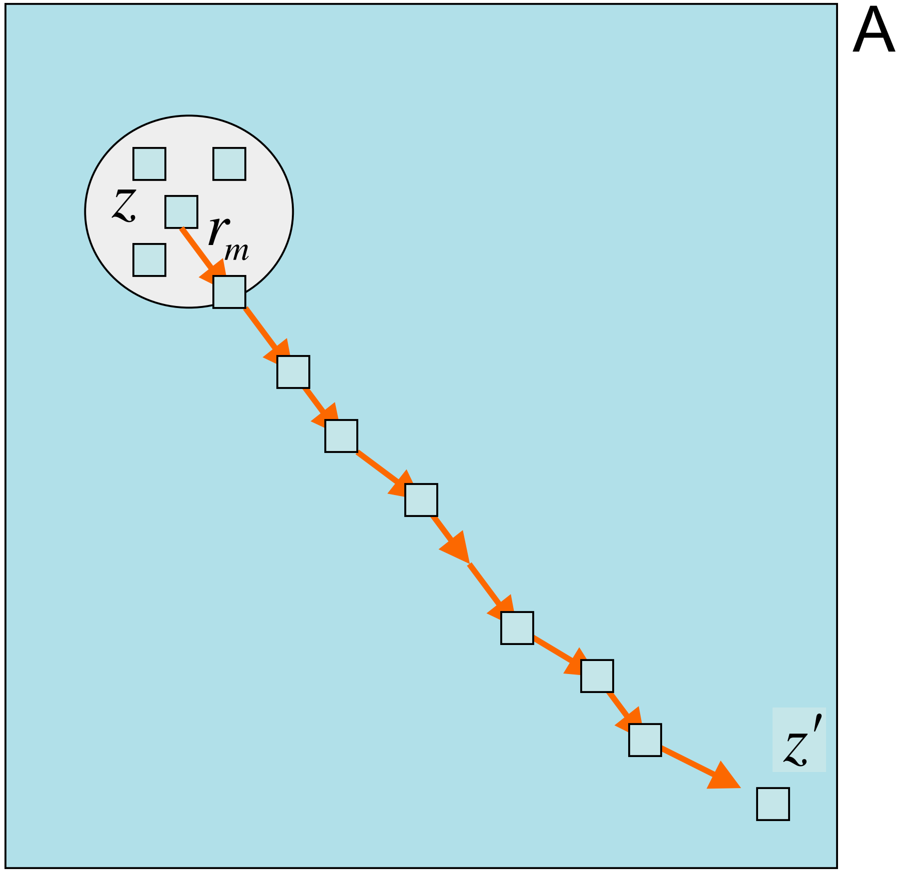
$$\frac{|z - z'|}{r_m p(r_m)}$$

- Net traffic density

$$\rho = \lambda \frac{r_m p(r_m)}{E(|z - z'|)}$$

- True if neighborhood is dense enough

$$\pi r_m^2 \frac{N}{\mathrm{A}} > \log N$$

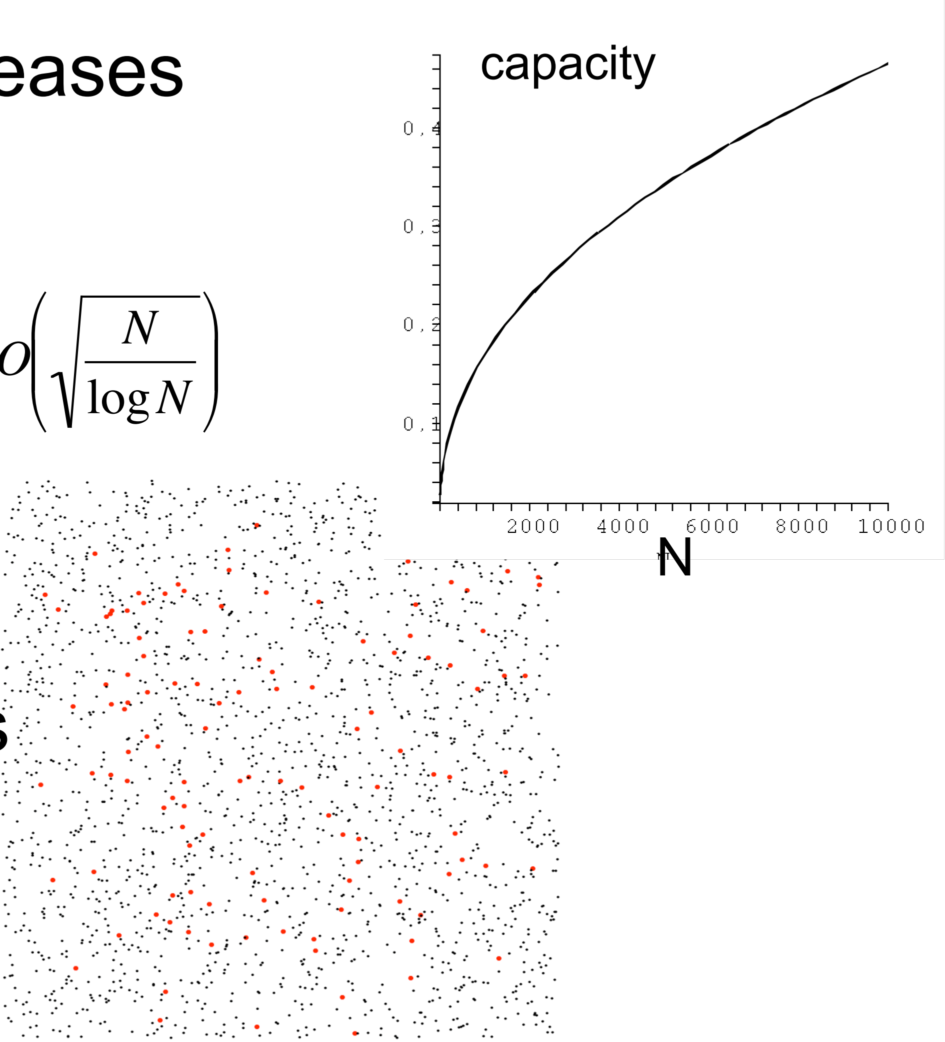


Space capacity result (Gupta-Kumar 2000)

- The capacity increases with the density

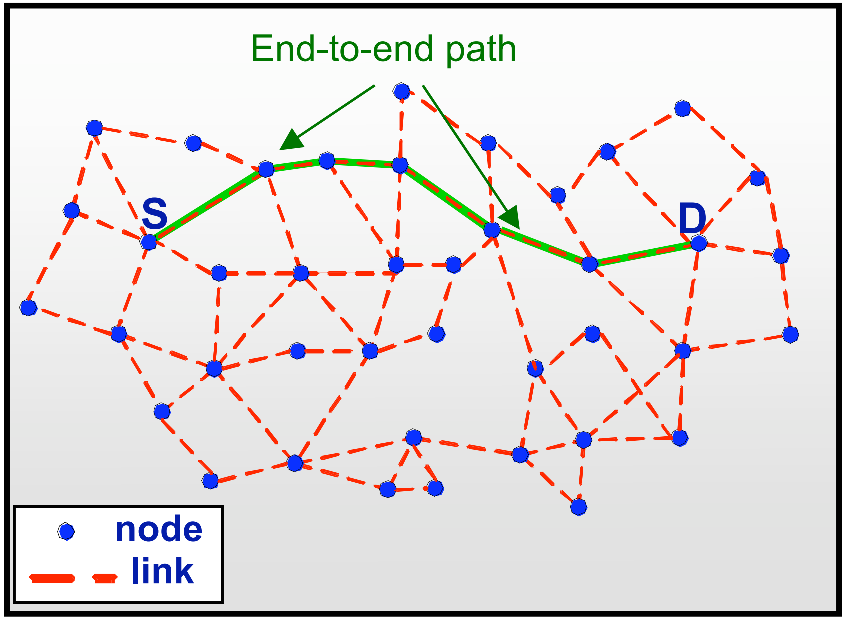
$$A\rho = \sqrt{\pi} r_1^2 p_1 \frac{\sqrt{A}}{E(|z - z'|)} \sqrt{\frac{N}{\log N}} = O\left(\sqrt{\frac{N}{\log N}}\right)$$

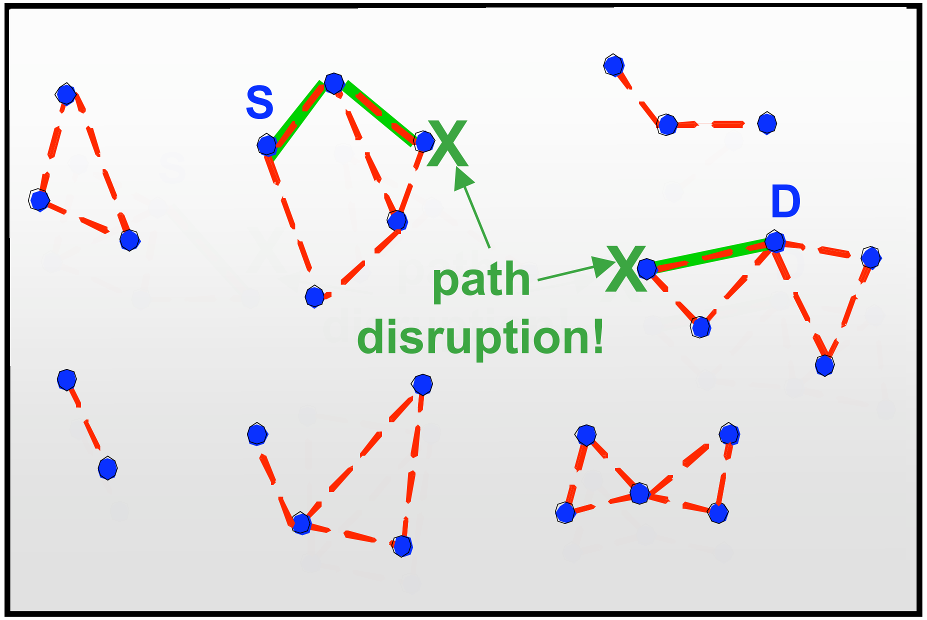
- Massively dense wireless networks



Time capacity paradox

- Mobility can create capacity in sparse networks

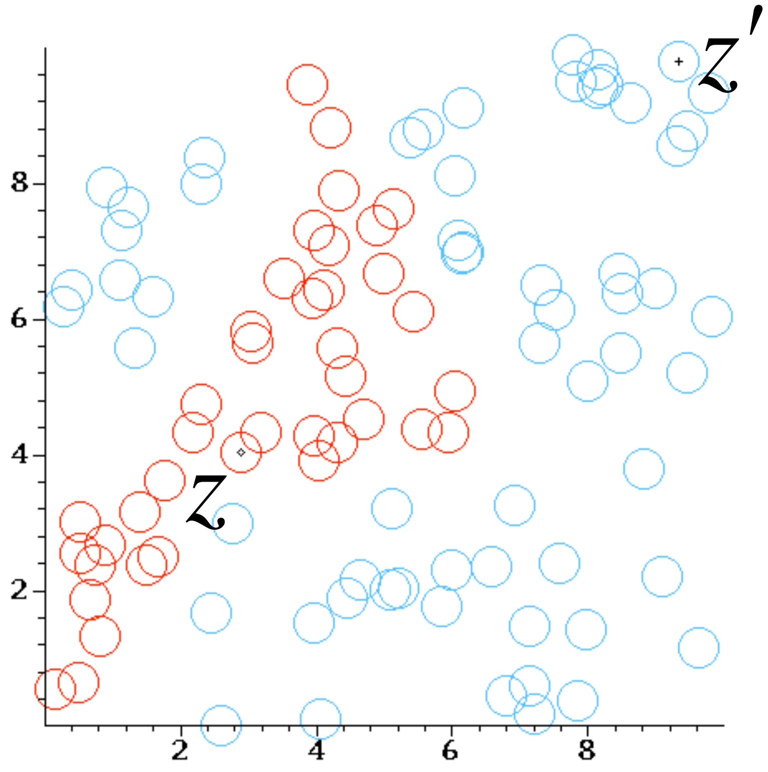




- Delay Tolerant Networks

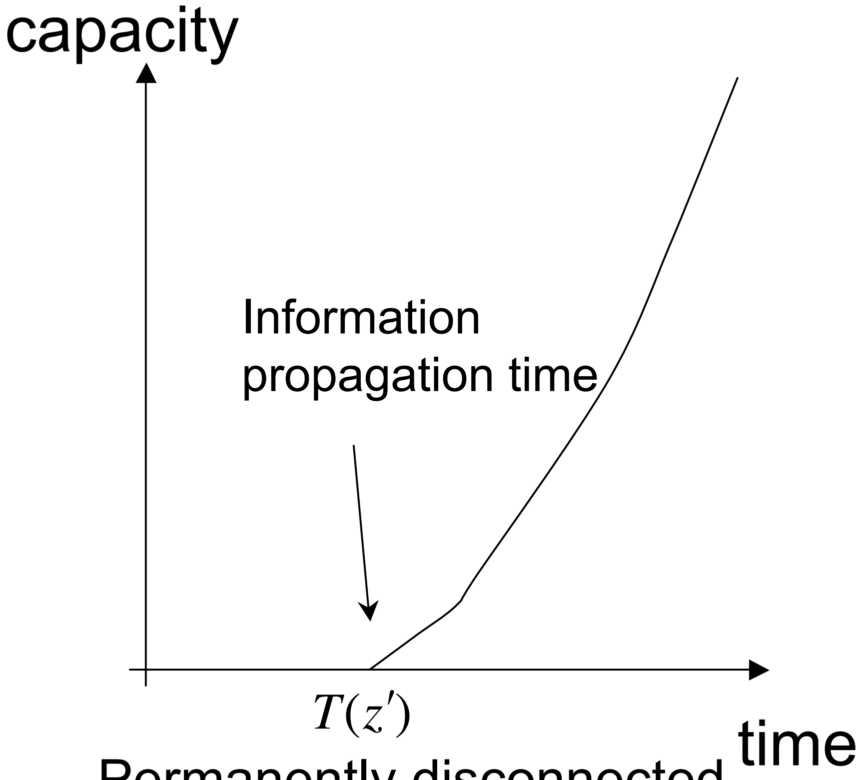
Information propagation speed

- Unit disk graph model
- Random walk mobility model

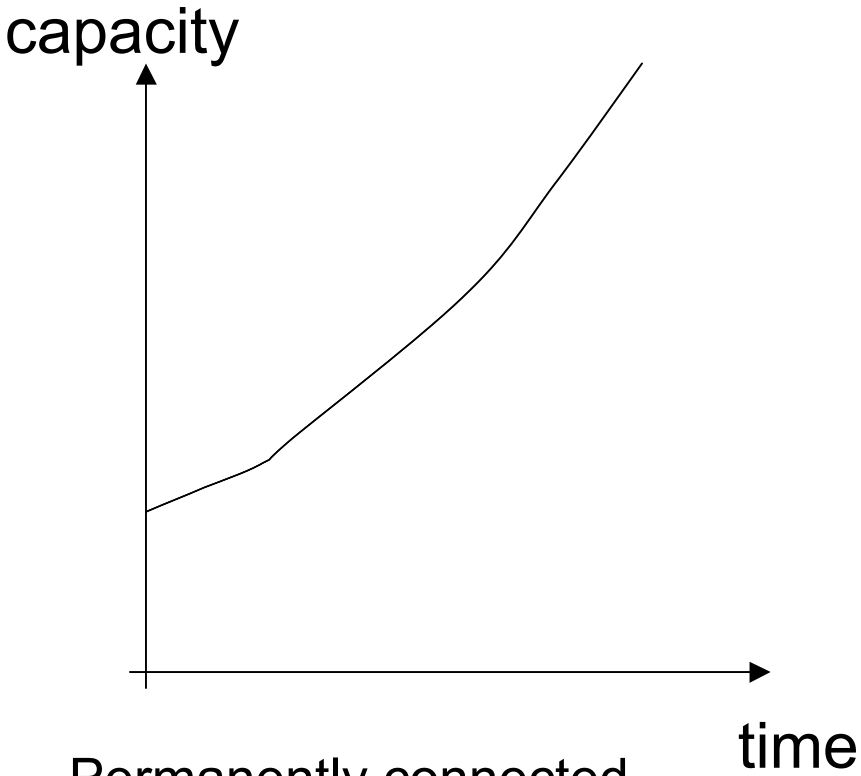


Time capacity paradox

- Mobility creates capacity



Permanently disconnected



Permanently connected

Information propagation speed

- Upper bound of information propagation speed
  - Any quantity c such that

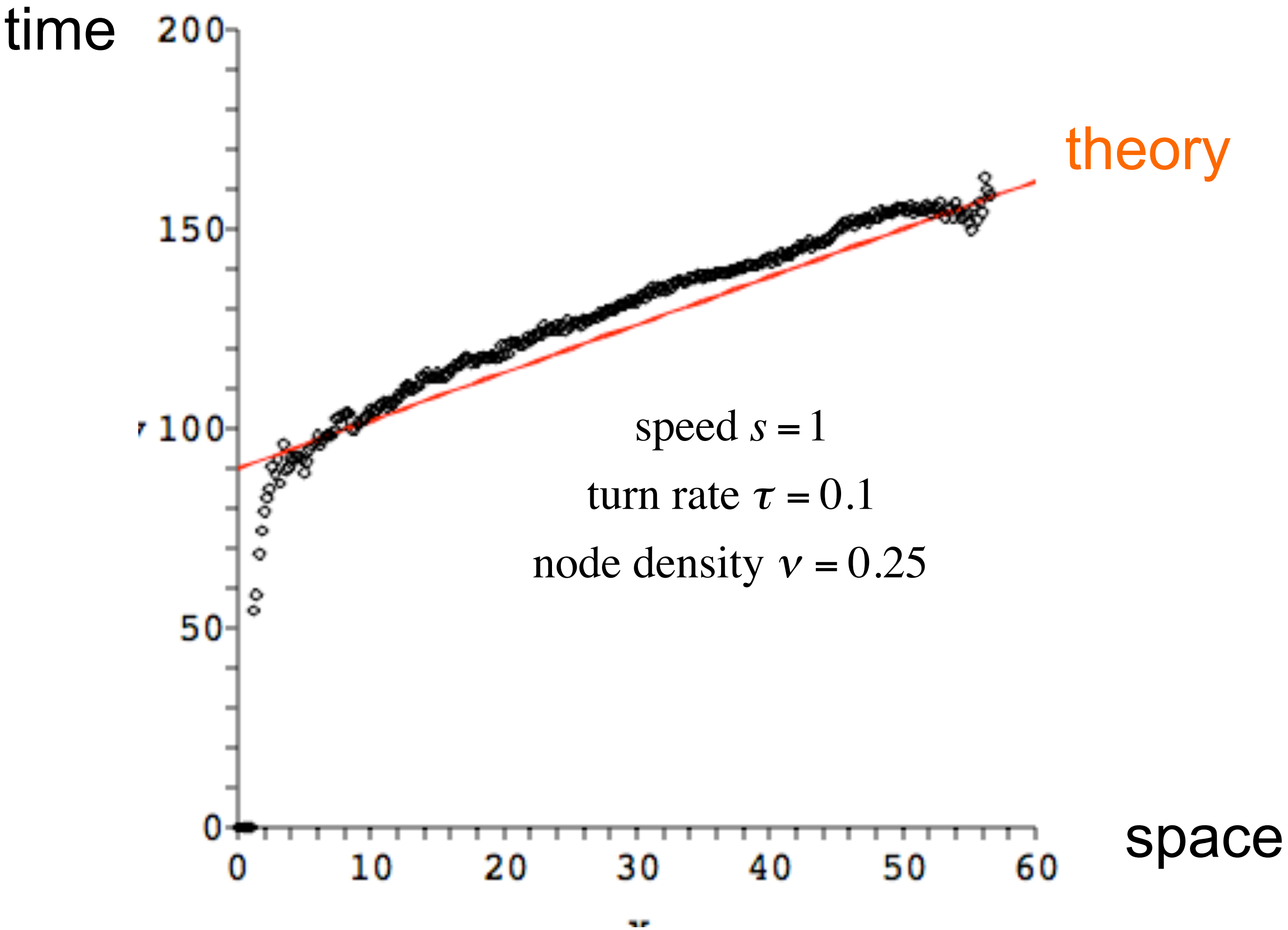
$$\lim_{z' \to \infty} P(T(z') < \frac{|z - z'|}{c}) = 0$$

  - Is the smallest ratio $\frac{\theta}{\rho}$ in the kernel of

$$D(\rho,\theta) = \sqrt{(\tau + \theta)^2 - \rho^2 v^2} - \tau - \frac{2\pi v s I_0(\rho)}{1 - \pi v \frac{2}{\rho} I_1(\rho)}$$

$I_k()$ are modified Bessel functions

Information propagation speed



speed $s = 1$

turn rate $\tau = 0.1$

node density $\nu = 0.25$