Solution of Homework 1: *Basic Logic*

Q.1 Make truth tables for the following statement:

- $p \vee (\overline{r \vee q})$;

Answer

p	q	r	$r \vee q$	$\overline{r \vee q}$	$p \vee (\overline{r \vee q})$
T	T	T	T	F	T
T	T	F	T	F	T
T	F	T	T	F	T
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	F	T	T

- $(p \wedge \neg q) \rightarrow r$.

Answer

p	q	r	$\neg q$	$p \wedge \neg q$	$(p \wedge \neg q) \rightarrow r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	F	T
F	F	F	T	F	T

Q.2 Using *logical equivalences* discussed in class prove that

$$(p \wedge q) \rightarrow (p \vee q)$$

is a tautology, that is, prove that

$$(p \wedge q) \rightarrow (p \vee q) \equiv T.$$

Answer

$$\begin{aligned}(p \wedge q) \rightarrow (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q)\\ &\equiv (\neg p \vee \neg q) \vee (p \vee q)\\ &\equiv (\neg p \vee p) \vee (\neg q \vee q)\\ &\equiv T \vee T\\ &\equiv T\end{aligned}$$

Note: Another way to solve this question is by constructing the truth table for the given logical expression and showing that it always yields T for all values of p and q.

Q.3 Let

$$P(x,y): \ x+y \geq 5 \text{ where } x, \ y \text{ are positive integers.}$$

Tell whether the following statements are true or false:

- $\forall_x \forall_y \ P(x,y)$
- $\forall_x \exists_y \ P(x,y)$.

Answer

- $\forall_x \forall_y \ P(x,y)$: **"False"**
  Counterexample: $P(1,2): 1+2 \not\geq 5$.
- $\forall_x \exists_y \ P(x,y)$: **"True"**
  If we pick an arbitrary value for x, say a, then there always exists a value for y (for example, $a+5$) such that $x+y = 2a+5 \geq 5$.

Q.4 Which of the following is equivalent to $\overline{\forall_x \exists_y P(x,y)} \equiv \neg \forall_x \exists_y P(x,y)$:

(a) $\exists_x \overline{\forall_y P(x,y)}$;

(b) $\forall_x \overline{\exists_y P(x,y)}$;

(c) $\exists_x \forall_y \overline{P(x,y)}$;

(d) $\exists_x \exists_y \overline{P(x,y)}$.

Answer

(c) $\exists_x \forall_y \overline{P(x,y)}$.