

### Assignment 9

Due: Monday, Dec. 6, 1999, in class

**The final exam is Monday, Dec. 13, 3:20-5:20pm, in MATH 175.**

**You need to bring a picture id to the exam.**

1) (15 pts.) (i) Design an efficient and simple algorithm to check whether a given text of length  $n$  contains a string of 50 or more consecutive blanks. Analyze your algorithm. If you are not using KMP, state why not. If you are using KMP, state why you don't think there is a simpler solution.

(ii) Give a pattern of length 8 beginning with  $A$  and using only characters of the alphabet  $\Sigma = \{A, B, C\}$  that would have the following fail indexes for the KMP algorithm: 0 1 1 2 3 4 2 2.

2) (15 pts) Let  $T$  and  $P$  be two sequences  $t_1 t_2 \cdots t_n$  and  $p_1 p_2 \cdots p_k$  of characters,  $k \leq n$ . Sequence  $P$  is a subsequence of  $T$  if there exists a sequence of indices  $i_1 < i_2 < \cdots < i_k$  such that for all  $j$ ,  $1 \leq j \leq k$ , we have  $t_{i_j} = p_j$ . For example, for  $T = A B R A C A D A B R E$  and  $P = A R C A D E$ ,  $P$  is a subsequence of  $T$ . Design an  $O(n)$  time algorithm to determine whether  $P$  is a subsequence of  $T$ . Argue the correctness of your algorithm.

3) (20 pts) (i) Define the decision version of the Hamiltonian Cycle (HC) problem.

(ii) Define the optimization version of the HC problem.

(iii) The Hamiltonian Cycle problem is known to be NP-complete. Give a brief explanation what this means.

(iv) Show that the existence of a polynomial-time algorithm for the HC decision problem implies the existence of a polynomial time algorithm for the HC optimization problem.

(v) The degree-restricted spanning-tree problem is (DRST) defines as follows: Given is an undirected graph  $G$  and an integer  $k$ . Determine whether  $G$  contains a spanning tree  $T$  such that every vertex of  $T$  has degree at most  $k$ . Show that DRST is NP-complete (by making a reduction from HC to DRST).