Purdue University
CS 381

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Assignment 6

Due: Monday, Nov. 1, 1999, in class

1) (22 pts.) (i) Consider the undirected graph shown in Figure 7.10 on page 326. Assume DFS starts at vertex 3. Show the graph partitioned into tree and back edges and give for every vertex its depth-first number.

(ii) Consider the directed graph shown in Figure 7.29 on page 377. Assume the vertices are listed in alphabetical order in the adjacency lists and DFS starts at vertex A. If necessary, it restarts on the alphabetically smallest vertex remaining. Show the graph partitioning into tree, back, cross, and forward edges. Give the depth-first number for each vertex.

(iii) Give an example of a graph in which DFS backs up from a vertex before all the vertices that can be reached from it via one or more edges are discovered. Explain how this can happen.

2) (18 pts.) Let $G = (V, E)$ be a directed, acyclic graph (dag) represented by adjacency lists.
(i) Can a dag have $\Theta(n^2)$ edges?

(ii) A dag G is a lattice if G contains two vertices s and t such that every vertex in G can be reached from s and every vertex in G can reach vertex t. Describe and analyze an efficient algorithm to determine whether a given dag is a lattice. If it is, s and t should be output.

3) (10 pts.) Let G be a connected graph, and let s be a vertex in G. Let T_D be the depth-first search tree formed by doing a depth-first search of G starting at s. Let T_B be the breadth-first spanning tree formed by doing a breadth-first search of G starting at s. Is it always true that $height(T_D) \geq height(T_B)$? Does it matter whether the graph is directed or undirected? Give a clear argument or a counterexample.