

Assignment 3

Due: Friday, Sept. 24, 1999, in class

1) (18 pts.) For each of the following recurrence equations determine whether the Master Theorem (3.17 on page 139) can be applied. If it can, give the asymptotic bound and show which one of the three cases holds and why. If it cannot, show why all cases fail. Assume that $T(1) = 1$.

(i) $T(n) = 4T(n/4) + 6$

(ii) $T(n) = 4T(n/3) + n^2$

(iii) $T(n) = 2T(n/2) + \frac{n}{\log n}$

(iv) $T(n) = 6T(n/6) + 3n - 5$

(v) $T(n) = T(\sqrt{n}) + \log n$.

2) (15 pts.) Consider the recurrence relation $T(n) = 2T(n/2) + 5n \log n$ with $T(1) = 1$ and $n = 2^k$, $k \geq 0$. Show by induction that $T(n) = O(n \log^2 n)$.

3) (17 pts.)

(i) Assume you are given an array A of size n containing real numbers. Describe and analyze an efficient algorithm to determine $n/4$ numbers **not** in array A .

(ii) Assume you are given an array A of size n and an array B of size m , $n \geq m$. The elements in each array are in arbitrary order. Describe and analyze an efficient algorithm to determine whether the elements in the two arrays are disjoint. State your running time in terms of n and m . Make sure to consider the case when m is substantially smaller than n .