

Assignment 2

Due: Wednesday, Sept. 15, 1999, in class

1) (20 pts.)

a) Solve the following recurrence equation exactly by expanding it. Assume $n = 2^k$. Show your work.

$$T(1) = 1$$

$$T(n) = 4T(n/2) + n \text{ for } n \geq 2.$$

b) Prove by induction that your answer is correct.

2) (15 pts.) The *internal path length* of a binary tree is defined as the sum of the levels of all internal nodes of the tree. The *external path length* of a binary tree is defined as the sum over all the leaves of the level of each leaf. (Recall that the level of a node x is 0 if x is the root of the tree and is 1 + the level of the parent node of x , otherwise.) Your text also defines the external path length (page 116, definition 3.3). Realize that the two definitions are equivalent.

A *2-tree* is a binary tree in which a node has either two or no children. Consider a 2-tree with n internal nodes, internal path length ipl and external path length epl . Prove by induction that $epl = ipl + 2n$.

3) (15 pts.) Do exercise 1.26 pages 63-64 of the textbook.