Purdue University
CS 381

S. Hambrusch/C. Guerra
Fall 1999

Assignment 1

Due: Friday, September 3, 1999, in class

1) (15 pts.) Find a closed form for the expression

$$1 \star 2 + 2 \star 3 + 3 \star 4 + \ldots + n \star (n+1)$$

and prove it correct by induction.

2) (20 pts.) Partition the following functions representing running times into equivalence classes so that $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$. Rank the classes from smallest to largest (in terms of growth rate with respect to n). Include work for the five relations which were the least obvious to you and state why they were not obvious. Use either the definition of big-O or take limits.
Note: Logarithms are base 2 if not indicated otherwise.

$4 \log n, 3n^2 + 4n \log n, \frac{n}{6}(\log n)^2, \sqrt{n} + 4n \log n, n^{\log 6},$

$3^n, 3^{n/3}, 3^{n-3}, 3^{\log_3 n}, 3^{\log n},$

$n^{10} - n^6 + 15n^3, 4 \log(n^3), n^{n/2}, 2^n, \sqrt{4n} + 4 \log n$

3) (15 pts.) The health department needs to test n water samples for pesticide X. For this purpose water is collected and the samples are labeled. There exists an expensive test to determine whether water contains pesticide X. One may mix portions of water samples to conduct tests on several water samples simultaneously. (A positive outcome implies that at least one original sample contains the pesticide.)
(i) Describe an efficient method to determine p, the number of water samples which contain pesticide X. The amount of water available for each sample is not a constraint. Your method should be efficient when p is considerably smaller than n. State the number of tests necessary in terms of n and p.

(ii) Bonus question. Your solution for (i) only needs to determine the number of water samples containing pesticide X. Can your algorithm also be used to identify the samples with pesticide X? If yes, state how. If no, describe a solution which can.