

Assignment 7

Due: Thursday, April 13, 2010 (before class)

FINAL EXAM: Tuesday, May 4, 8-10 am, KRAN G018

1) (20 pts.) A local municipal office of a city prone to spring flooding provides the following web-based query to residents: a home owner can enter the distance of a house from the river and the system returns the estimated damage to houses at that distance. The office receives updates on the damage in the forms of "all houses within distance d experience damage a ." Design a data structure that dynamically maintains damage information and responds to home owner queries. More specifically,

- The data structure is empty at the time flood season starts.
- $Update(d, a)$: decreases the value of every house within distance d of the river by amount a .
- $Assess(d)$: returns the current damage done to a house distance d from the river.

Describe and analyze a data structure that implements each operation in $O(\log n)$ time, where n is the number of updates (i.e., decreases) that have already taken place.

2) (20 pts.) After graduation you move to Seattle and you become a serious jogger. You run every morning, starting at your house, choosing one of a number of routes. You always return home by bus. Unfortunately, you have injured your knee. While you can continue to jog, you are told to choose a route where you first run uphill and then downhill.

To help you determine which routes you can take under this restriction, you model your environment by an undirected graph, $G = (V, E)$, where vertex s represents your house. Selected vertices are labeled as B-vertices, indicating locations from which you can take the bus home. Every vertex u records the elevation of location u in $h(u)$. For every edge $e = (u, v)$, the weight, $w(e)$, represents the distance from u to v and either $h(u) < h(v)$ or $h(u) > h(v)$ holds. A valid jogging route is a path from vertex s to a B-vertex such that the h -values on the path first increase and then decrease.

(i) Given the n -vertex and m -edge graph G , design and analyze an efficient algorithm for determining the shortest valid jogging route you can take from your house (the trip back on the bus does not count).

(ii) Given the n -vertex and m -edge graph G , design and analyze an efficient algorithm for determining the longest valid jogging route you can take from your house (the trip back on the bus does not count).

3) (10 pts.) Given is a directed, unweighed, n -vertex, m -edge graph $G = (V, E)$ and two vertices s and t . Describe and analyze an $O(n + m)$ time algorithm to determine the number of different shortest paths from s to t . The paths do not have to be vertex disjoint.