Purdue University | Hambrusch
CS 580 | Spring 2010

Assignment 2

Due: Tuesday, February 2, 2010 (before class)

1) (18 pts.) (a) For each of the following recurrence relations give asymptotically tight bounds. If the Master Theorem applies, explain which case holds and why (no proof of the bound resulting from the theorem is needed). If the Master Theorem does not apply, give a proof by induction. You can assume that n is of a convenient form. $T(1)$ is constant for all cases.

(i) $T(n) = 2T(n/2) + n^{1/3}$
(ii) $T(n) = 8T(n/3) + (\frac{n}{\log n})^2$
(iii) $T(n) = T(\sqrt{n}) + \log n$
(iv) $T(n) = 2T(n/2) + \frac{n}{\log n}$
(v) $T(n) = 2T(n/5) + n^{\log_5 2}$

(b) Proof or disprove: If $f(n) = O(g(n))$, then $2^{f(n)} = O(2^{g(n)})$.

2) (16 pts.) A bank has confiscated n credit cards suspected to have been involved in fraud. Every card is associated with a customer and a customer can have many cards. The bank needs to determine if there exists one customer who owns at least $n/2$ of the cards. For security reasons, the only test that can be done involves an expensive equivalence tester. The tester is a little black box into which two cards are inserted and the tester indicates whether the cards belong to the same customer or not.

(i) Design a divide-and-conquer algorithm determining whether there exists a customer owning at least $n/2$ cards. Your algorithm should make $O(n \log n)$ uses of the tester and, if such a user exists, it should identify the associated cards.

(ii) Assume two testers are available and can be used simultaneously. Adapt your answer for (i) for two testers. Can the testing time be reduced by a factor of 2? Justify your answer.

(iii) If there is only one tester available, do you believe $O(n \log n)$ time is an asymptotically optimal? Justify your answer.

3) (16 pts.) Let $G = (V, E)$ be an n-vertex, undirected graph representing a social network. The vertices represent people; an edge between A and B means that A and B are friends. The **clustering coefficient of a person** is the fraction of pairs of friends of A that are friends with each other. It is 0 if when none of A's friends are friends with each other or when A has only one friend. It is 1 when all of A's friends are friends with each other. The **global clustering coefficient** is defined as the average of the n clustering coefficients. See also `http://en.wikipedia.org/wiki/Clustering_coefficient` where C_i for undirected graphs corresponds to the clustering coefficient of person i.

(i) Describe an efficient algorithm determining the n clustering coefficients and the global clustering coefficient when G is represented as an $n \times n$ adjacency matrix. Give the running time in terms of n.

(ii) Describe an efficient algorithm determining the n clustering coefficients and the global clustering coefficient when G is represented by adjacency lists. Give the running time in terms of n and m, where m is the number of edges in G.