

Assignment 9

Due: Wednesday, December 9, 2009 (before class)

FINAL EXAM: Thursday, December 17, 8-10am, BRNG 2280

Set your alarm so you arrive on time!

1) (18 pts.) Let $G = (V, E)$ be a directed, weighted, n -vertex, m -edge graph represented by adjacency lists. All weights are positive. The graph represents a transportation network and the weight on edge (u, v) , $w(u, v)$, represents the maximum size of an object that can be transported from u to v along this edge.

(i) Given two vertices s and t and an object of size p , determine whether a path for the transport. Your algorithm should have an $O(n + m)$ time performance.

(ii) Given two vertices s and t , describe an algorithm determine the maximum size object that can be transported from s to t and determine a path allowing such a transport. Your algorithm should adapt Dijkstra's algorithm and have an $O((n + m) \log n)$ running time. Make sure to address its correctness.

2) (22 pts.)

(i) Let Π_1 and Π_2 be two decision problems. Assume $\Pi_1 \leq_P \Pi_2$. If there exists an $O(n^2)$ time algorithm for Π_1 , does this imply that there exists an $O(n^2)$ time algorithm for Π_2 ? If not, what does it imply? Explain your answers.

(ii) Degree constrained minimum spanning tree problem (DCMST): Given is an undirected, weighted graph $G = (V, E)$, a quantity W , and an integer d , $d \geq 2$. Does there exist a spanning tree T of G such that every vertex of T has degree at most d and the sum of the weights of the edges in T is at most W ?

Show that DCMST is NP-complete. To show this, first show that $DCMST \in NP$. Then show that any algorithm for DCMST can be used to solve the Hamiltonian Path (HP) problem; i.e., $HP <_{poly} DCMST$.