Purdue University | Hambrusch
CS 381 | Fall 2008

Assignment 2

Due: Friday, September 12, 2008 (before class)

1) (20 pts.) Disk space gets fragmented and a good allocation algorithm assigns space so that a file is not split into too many pieces and large free spaces are preserved for future requests. This motivates the following simplified allocation problem:

Assume F is an array of size n and $F[i]$ represents the size of the i-th free space on the disk. $F[i]$ is an integer and the array entries are arranged by disk locations (not their sizes). Assume there is a request for space of size r.

(i) Describe and analyze an efficient algorithm for determining an index i such that $F[i] \geq r$ and $F[i] - r$ is a minimum. Make sure to state the running time.

(ii) Describe and analyze an efficient algorithm for determining two locations i and j such that $F[i] + F[j] \geq r$ and $F[i] + F[j] - r$ is a minimum, $i \neq j$. Make sure to state the running time (for full credit, it needs to be better than $O(n^2)$).

Note: If the request cannot be satisfied, report so. If there is more than one solution, an arbitrary one can be selected.

2) (18 pts.) Partition the following functions representing running times into equivalence classes so that $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$. Rank the classes from smallest to largest (in terms of growth rate with respect to n).

Include all work necessary to produce the ranking (relationships following from transitivity do not need to be shown). Logarithms are base 2.

$4 \log \log n$	$4n^2 \log n^2$	4^{n-4}	$4^{\log n}$	$14 + (\frac{1}{2})^n$
$n^{n/4}$	$\frac{n^2}{6}(\log n)^2$	$(n-4)!$	$n^{\log 8}$	$n!$
4^n	$4^{n/10}$	$(\log \sqrt{n})^2$	$16n^6$	$\frac{n^2}{\log n}$

3) (12 pts.) For each claim decide whether it is always true, never true, or sometimes true for asymptotically nonnegative functions f and g. If it is always true or never true, explain your answer. If it is sometimes true, give one example for which it is true and one for which it is false.

(i) $f(n) = O((f(n))^3)$

(ii) $f(n) + g(n) = \Theta(3f(n) + 5(g(n))$