Purdue University | Hambrusch
CS 381 | Spring 2015

Assignment 1

Due: Tuesday, January 27, 2015 (hand in before class)

0) (2 pts.) Housekeeping tasks:

1. **Visit the course website at http://www.cs.purdue.edu/homes/seh/381webSp15/.** Follow the links and read the information provided. Set appropriate bookmarks needed during the semester.
2. **Sign up to Piazza at piazza.com/purdue/spring2015/cs381.** Material will be posted on Piazza. Piazza also serves as the discussion forum of topics and questions of interest to the entire class.
3. **Register your i-Clicker on Blackboard.** Use the same registered clicker during the semester.

1) (8 pts.) Partition the following functions representing running times into equivalence classes so that $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$. Rank the classes from smallest to largest (in terms of growth rate with respect to n). Logarithms are base 2 unless stated otherwise. You only need to give the final ordering from small to large with clearly indicating functions in the same equivalence class.

$3n \log\log n$	$10n^3 + 14n^2 \log^6 n$	$10n\sqrt{\log n}$	$4\log^3 n$	$(\sqrt{2})^{\log n}$	$\sqrt{n} + (\log n)^3$
$8\sqrt{n} + 6\log^6 n$	$(\frac{n}{\log n})^3$	$81 \log n^3$	$3n \log n$	$\sqrt{\log n^3}$	$3n + 5\sqrt{n}$

2) (8 pts.) Review proofs by inductions. In your proofs, be precise and show all needed steps.

1. Use induction to show that $n! > 2^n$ for $n > 3$.
2. Use induction to show that $\sum_{i=1}^{n}(2i-1) = n^2$, $n \geq 1$.

3) (22 pts.) Given is an array A containing n distinct integers in arbitrary order. Describe and analyze an efficient algorithm for each of the following problems. Clearly state the achieved running time in big-O and Θ-notation. Do not give code, but explain your solution in a clear and precise manner. In addition to the running time, argue the correctness of your algorithm.

1. Determine the maximum difference between any two elements in A. For example, for A = [2, 10, 12, 4, -9, 0, -5, 8, 1, -7] the answer is 21 (difference between -9 and 12).
2. Determine the number of elements between two given values x and y with $x < y$ (i.e., the number of elements z such that $x < z < y$).
3. Determine the 5-th largest element in array A.
4. Given is also an integer r, $r > 0$. Determine two elements $A[k]$ and $A[p]$, $k < p$, such that $p - k \leq r$ and $|A[p] - A[k]|$ is a maximum. For example, for $r = 2$ and A = [-8, 3, 2, -5, 6, 10, 14] the answer is $A[k] = -5$ and $A[p] = 10$ achieving a difference of 15.

4) (10 pts.) How many times is function F called in each code segment given below when $n = 2^r$? Clearly explain your answer and express bounds in terms of n in big-O and Θ-notation. Review the pseudocde convention in Section 2.1 (if needed).

Code Segment 1

for $i = 1$ **to** n **do**
  $j = 1$
  while $j \leq n$ **do**
    j = 2*j
    for $k = 1$ **to** j **do**
      F(i,j,k)

Code Segment 2

while $n > 1$ **do**
  for $i = 1$ **to** n **do**
    F(i,n)
  $n = n/4$