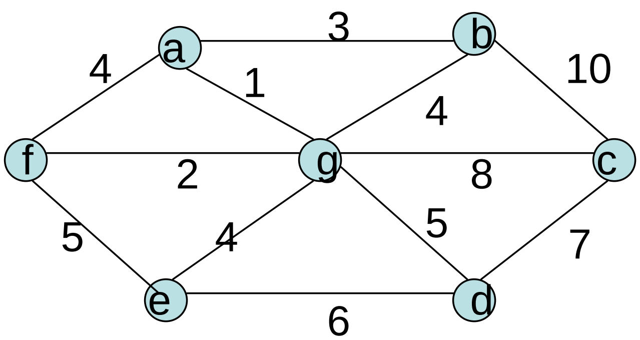
Purdue University | Gopal/Hambrusch
CS 381 | Spring 2008

Assignment 7

Due: Wednesday, April 2, 2008 (before class)

1) (10 points) Consider the following weighted, undirected graph $G = (V, E)$:



The goal is to find a minimum cost spanning tree (MST) of G. Kruskal's and Prim's algorithms start with an empty set and add edges in some order till a spanning tree results.

(a) List the edges of the MST in the *order* they are added by Kruskal's algorithm (i.e., list the first edge that is added by the algorithm, then second edge, and so on).
(b) Give the cut in G that validates the addition of edge (f, g) in your output of Kruskal's algorithm.
(c) List the edges of the MST in the *order* they are added by Prim's algorithm. Assume the algorithms starts at vertex c (i.e., the spanning tree will be rooted at c).
(d) Give the cut in G that validates the addition of edge (f, g) in your output of Prim's algorithm.

2) (15 points) Let T be a minimum cost spanning tree of a weighted, undirected graph $G = (V, E)$. Let T' be a minimum cost spanning tree of a new graph G' obtained from G by one of the following changes in the edge weights:

- all edge weights are increased by a constant number c
- all edge weights are decreased by a constant number c
- the weight of a single edge known **not** to be in T is increased by c
- the weight of a single edge known **not** to be in T is decreased by c

For each case describe and analyze an efficient algorithm to generate T' from T. Argue that your algorithm is correct.

3) (10 points) Let $G = (V, E, w)$ be a directed, weighted graph, where $w(e)$ denotes the weight of edge e. The weights can be negative, but G contains no negative cycles. Let s be a distinguished vertex in V. The goal is to compute the shortest paths from s to all vertices.

(i) Given G as input, show that Dijkstra's algorithm does not always generate the correct solution.

(ii) Consider the following algorithm that modifies the weights of G before running Dijkstra.

Dijkstra-new(G):

1. From G, generate a new graph $G' = (V, E, w')$ having the same node and edge set and weights are set as follows. Let $W = \max_{e \in E}\{|w(e)|\}$, i.e., W is the largest weight in G (in absolute value). Set $w'(e) = w(e) + W$ for every edge in E.
2. Run Dijkstra's algorithm on G'.
3. Output the shortest paths obtained from s to every other vertex as the corresponding shortest paths for G.

Does the above algorithm always generate the shortest paths for G? Note we want to generate the paths, not the value of the shortest paths. Justify your answer.

4) (15 points) The input is a weighted, undirected graph $G = (V, E, w)$ having n vertices and m edges. All weights are non-negative. The cost of a path is defined as the *maximum* of the weights of its edges (instead of the sum of the weights).

Modify Dijkstra's algorithm to solve the single-source shortest-paths problem in G where the path costs are defined as above. What is the running time of your algorithm?