Purdue University
CS 381

Gopal/Hambrusch
Spring 2008

Assignment 1

Due: Wednesday, January 16, 2008 (before class)

1) (10 pts.) Comparison-based sorting algorithms (i.e., merge sort, quicksort, heap sort) move one element at a time to a new location after comparing two elements. Considers now a different model of computation which views the input as elements in a string. The reversal operation available does the following: cut the entire string into two pieces, reverse one piece, and glue the two pieces back together. Scanning the string and comparing two elements is free, but the only operation which can move elements is the one described above (which is possible on DNA strings). For example, the string A C D H F A J X B I L A C can be cut into A C D H F A J X and B I L A C, the shorter string can be reversed into C A L I B, and the new string is either A C D H F A J X C A L I B or C A L I B A C D H F A J X.

Describe an algorithm that takes as input a string S of length n, outputs the string sorted, and makes $O(n)$ reversals.

2) (8 pts.) Assume you are given an unsorted array A of size n containing integers. Let $M = \max_{1 \le i,j \le n} |A[i] - A[j]|$, where $|A[i] - A[j]|$ stands for the absolute value of $A[i] - A[j]$. Describe an efficient algorithm to find M. Make sure to state and analyze the time bound.

3) (18 pts.) Partition the following functions representing running times into equivalence classes so that $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$. Rank the classes from smallest to largest (in terms of growth rate with respect to n).

Include complete work for the eight relations which were the least obvious to you and state why you found them more difficult. Use either the definition of big-O notations or take limits. Note: Logarithms are base 2 if not indicated otherwise.

$4 \log n$	$3n^2 + 4n \log n$	$\sqrt{4n} + 4 \log n$	$4^{\log n}$	$2^{\log \log n}$	$n^{\frac{1}{4}} + 4 \log n$	$\ln n$
$n^{n/2}$	$\frac{n}{6}(\log n)^2$	$n^2 \log n^3$	$n^{\log 6}$	$n!$	$3^{\log_3 n}$	3^{n-3}
3^n	$3^{n/3}$	$\sqrt{\log n}$	$\log \log n$	$\frac{n^2}{\log n}$	$(\frac{4}{5})^{n/2}$	

4) (14 pts.) Use induction to prove the following:
(a) $n! > 2^n$ for $n \ge 4$
(b) If $T(n) \le 3T(n/3) + 4n$, then $T(n) = O(n \log n)$. Assume $T(n) = c$, for $n \le 3$, where c is a constant. You can assume that n is a power of 3.