

Digital vs. Analog Data

Digital data: bits.

- discrete signal
- both in time and amplitude

Analog “data”: audio/voice, video/image

- continuous signal
- both in time and amplitude

Both forms used in today’s network environment.

- burning CDs
- audio/video playback

In broadband networks:

- use analog signals to carry digital data

Important task: analog data is often digitized

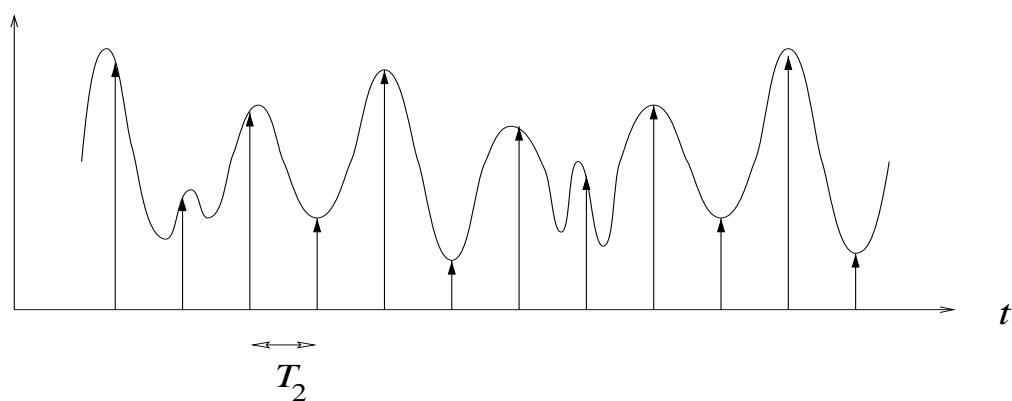
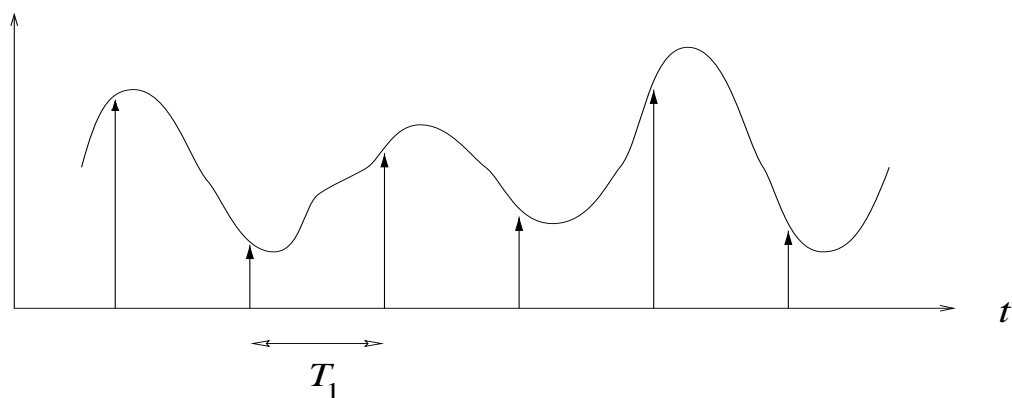
- useful: why?
- it's convenient
- use full power of digital computers
- simple form: digital signal processing
- analog computers are not as versatile/programmable
- cf. “Computer and the Brain,” von Neumann (1958)

How to digitize such that digital representation is faithful?

- sampling
- interface between analog & digital world

Intuition behind sampling:

→ slowly vs. rapidly varying signal



If a signal varies quickly, need more samples to not miss details/changes.

$$\nu_1 = 1/T_1 < \nu_2 = 1/T_2$$

Sampling criterion for guaranteed faithfulness:

Sampling Theorem (Nyquist): Given continuous bandlimited signal $s(t)$ with $S(\omega) = 0$ for $|\omega| > W$, $s(t)$ can be reconstructed from its samples if

$$\nu > 2W$$

where ν is the sampling rate.

→ ν : samples per second

Remember simple rule: sample twice the bandwidth

Issue of digitizing amplitude/magnitude ignored

→ problem of quantization

→ possible source of information loss

→ exploit limitations of human perception

→ logarithmic scale

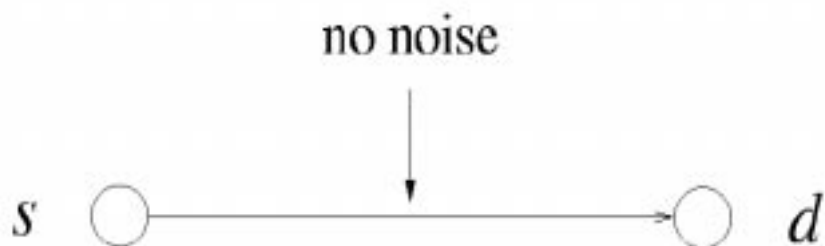
Compression

Information transmission over noiseless medium

- medium or “channel”
- fancy name for copper wire, fiber, air/space

Sender wants to communicate information to receiver over noiseless channel.

- can receive exactly what is sent
- idealized scenario



Set-up:

- take a system perspective
- e.g., modem manufacturer

Need to specify two parts: property of data source—what are we supposed to send?—and how compression is done.

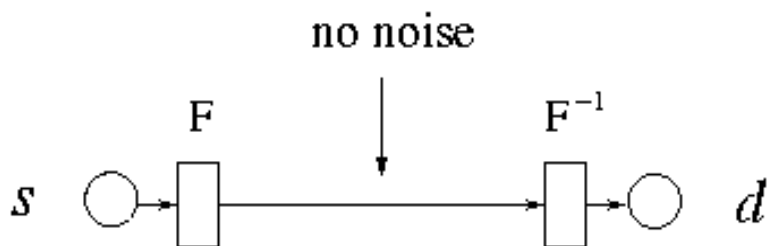
- need to know what we're dealing with
- if we want to do a good job compressing

Part I. What does the (data) source look like:

- source s emits symbols from finite alphabet set Σ
 - e.g., $\Sigma = \{0, 1\}$; $\Sigma =$ ASCII character set
- symbol $a \in \Sigma$ is generated with probability $p_a > 0$
 - e.g., books have known distribution for 'e', 'x' ...
 - let's play "Wheel of Fortune"

Part II. Compression machinery:

- code book F assigns code word $w_a = F(a)$ for each symbol $a \in \Sigma$
 - w_a is a binary string of length $|w_a|$
 - F could be just a table
- F is invertible
 - receiver d can recover a from w_a
 - F^{-1} is the same table, different look-up



Ex.: $\Sigma = \{A, C, G, T\}$; need at least two bits

- F^1 : $w_A = 00$, $w_C = 01$, $w_G = 10$, $w_T = 11$
- F^2 : $w_A = 0$, $w_C = 10$, $w_G = 110$, $w_T = 1110$

→ pros & cons?

Note: code book F is not unique

→ find a “good” code book

→ when is a code book good?

Performance (i.e., “goodness”) measure: average code length L

$$L = \sum_{a \in \Sigma} p_a |w_a|$$

→ average number of bits consumed by given F

Ex.: If DNA sequence is 10000 letters long, then require on average $10000 \cdot L$ bits to be transmitted.

→ good to have code book with small L

Optimization problem: Given source $\langle \Sigma, \mathbf{p} \rangle$ where \mathbf{p} is a probability vector, find a code book F with least L .

→ practically super-important

→ shrink-and-send

→ lossless shrinkage

A fundamental result on what is achievable to attain small L .

→ kind of like speed-of-light

First, define entropy H of source $\langle \Sigma, \mathbf{p} \rangle$

$$H = \sum_{a \in \Sigma} p_a \log \frac{1}{p_a}$$

Ex.: $\Sigma = \{A, C, G, T\}$; H is maximum if $p_A = p_C = p_G = p_T = 1/4$.

→ when is it minimum?

Source Coding Theorem (Shannon): For all code books F ,

$$H \leq L_F$$

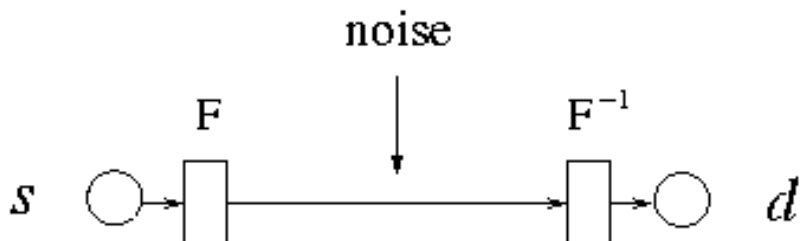
where L_F is the average code length under F .

Furthermore, L_F can be made to approach H by selecting better and better F .

Remark:

- to approach minimum H use blocks of k symbols
 - e.g., treat “THE” as one unit (not 3 separate letters)
 - called extension code
- entropy is innate property of data source s
- limitation of ensemble viewpoint
 - e.g., sending number $\pi = 3.1415927\dots$
 - better way?

Information Transmission under Noise



Uncertainty introduced by noise:

- encoding/decoding: $a \mapsto w_a \mapsto w \mapsto [?]$
- w_a gets corrupted, i.e., becomes w
- if $w = w_b$, incorrectly conclude b as symbol
- detect w is corrupted: error detection
- correct w to w_a : error correction

Would like: if received code word $w = w_c$ for some symbol $c \in \Sigma$, then probability that actual symbol sent is indeed c is high

→ $\Pr\{\text{actual symbol sent} = c \mid w = w_c\} \approx 1$

→ noiseless channel: special case (prob = 1)

In practice, w may not match any legal code word:

→ for all $c \in \Sigma$, $w \neq w_c$

→ good or bad?

→ what's next?

Shannon showed that there is a fundamental limitation to reliable data transmission.

→ the noisier the channel, the smaller the reliable throughput

→ overhead spent dealing with bit flips

Definition of channel capacity C : maximum achievable reliable data transmission rate (bps) over a noisy channel (dB) with bandwidth W (Hz).

Channel Coding Theorem (Shannon): Given bandwidth W , signal power P_S , noise power P_N , channel subject to white noise,

$$C = W \log \left(1 + \frac{P_S}{P_N} \right) \text{ bps.}$$

P_S/P_N : signal-to-noise ratio (SNR)

→ upper bound achieved by using longer codes

→ detailed set-up/conditions omitted

Increasingly important for modern day networking:

- Power control (e.g., pocket PCs)
 - trade-off w.r.t. battery power
 - trade-off w.r.t. multi-user interference
 - signal-to-interference ratio (SIR)
- Recent trend: software radio
 - hardware-to-software migration
 - kind of like cordless phones (e.g., 2.4 GHz)
 - configurable: make it programmable

Signal-to-noise ratio (SNR) is expressed as

$$\text{dB} = 10 \log_{10}(P_S/P_N).$$

Example: Assuming a decibel level of 30, what is the channel capacity of a telephone line?

Answer: First, $W = 3000$ Hz, $P_S/P_N = 1000$. Using Channel Coding Theorem,

$$C = 3000 \log 1001 \approx 30 \text{ kbps.}$$

- compare against 28.8 kbps modems
- what about 56 kbps modems?
- DSL lines?

Digital vs. Analog Transmission

Two forms of *transmission*:

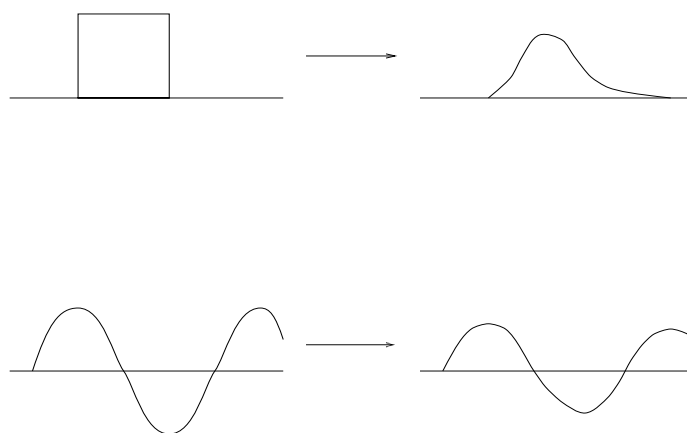
- digital transmission: data transmission using square waves
- analog transmission: data transmission using all other waves

Four possibilities to consider:

- analog data via analog transmission
→ “as is” (e.g., radio)
- analog data via digital transmission
→ sampling (e.g., voice, audio, video)
- digital data via analog transmission
→ broadband & wireless (“high-speed networks”)
- digital data via digital transmission
→ baseband (e.g., Ethernet)

Why consider digital transmission?

Common to both: problem of *attenuation*.



- decrease in signal strength as a function of distance
- increase in attenuation as a function of frequency

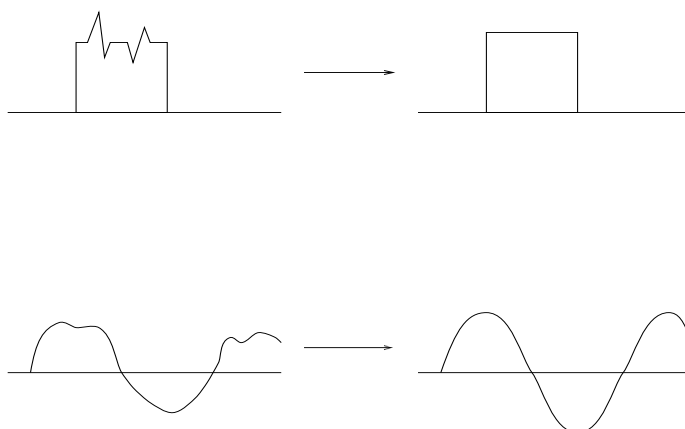
Rejuvenation of signal via amplifiers (analog) and repeaters (digital).

Delay distortion: different frequency components travel at different speeds.

Most problematic: effect of noise

→ thermal, interference, ...

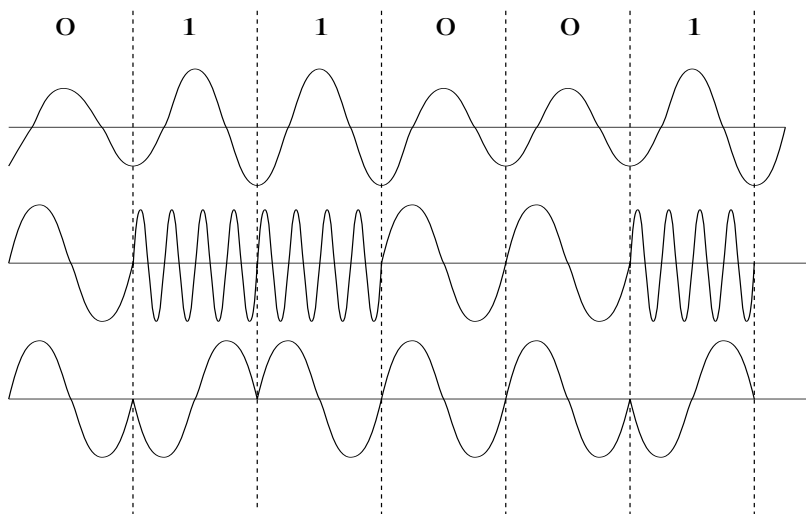
- Analog: Amplification also amplifies noise—filtering out just noise, in general, is a complex problem.
- Digital: Repeater just generates a new square wave; more resilient against ambiguity.



Analog Transmission of Digital Data

Three pieces of information to manipulate: amplitude, frequency, phase.

- Amplitude modulation (AM): encode bits using amplitude levels.
- Frequency modulation (FM): encode bits using frequency differences.
- Phase modulation (PM): encode bits using phase shifts.



FM radio uses . . . FM!

AM radio uses . . . AM!

iPod & radio experiment uses . . . ?

Why is FM radio clearer (“high fidelity”) than AM radio?

Broadband uses . . . ?

Baud Rate vs. Bit Rate

Baud rate: Unit of time within which carrier wave can be altered for AM, FM, or PM.

→ signalling rate

→ e.g., clock

Not synonymous with bit rate: e.g., AM with 8 levels, PM with 8 phases

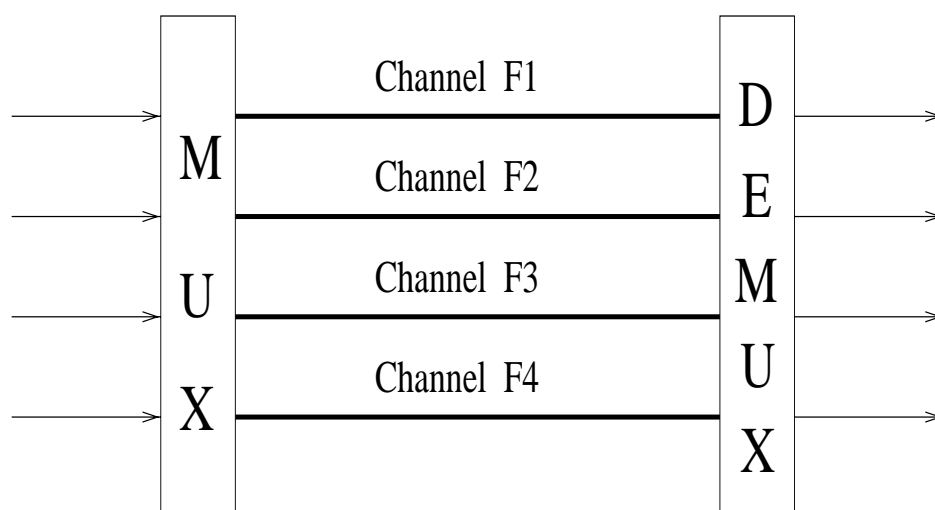
→ bit rate (bps) = 3 × baud rate

... less than one bit per baud?

Broadband vs. Baseband

Presence or absence of carrier wave: allows many channels to co-exist at the same time

→ frequency division multiplexing (FDM)



Ex.: AM radio (535 kHz–1705 kHz)

→ tuning to specific frequency: Fourier transform

→ coefficient (magnitude) carries bit information

Ex.: FM radio

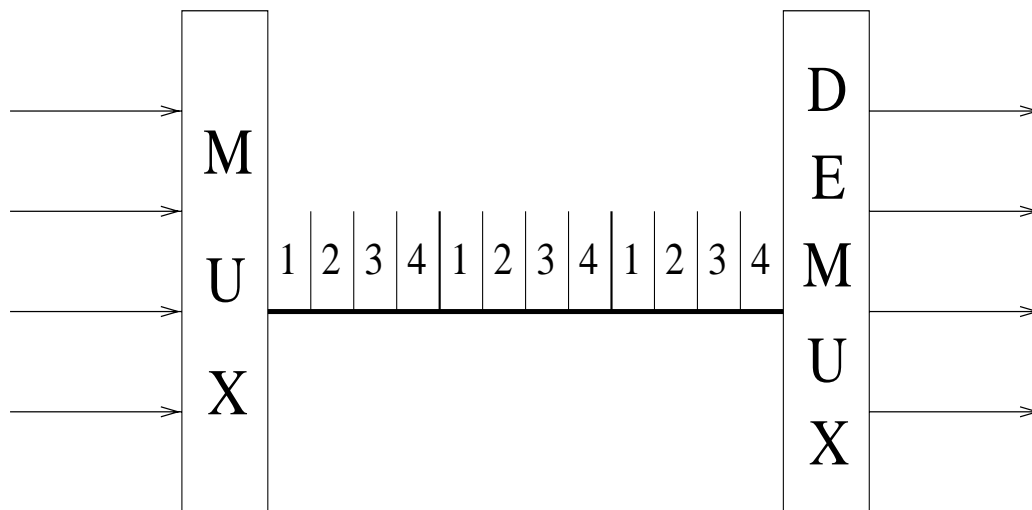
- 88 MHz–108 MHz
- 200 kHz slices
- how does it work?
- better or worse than AM?

Ex.: Digital radio

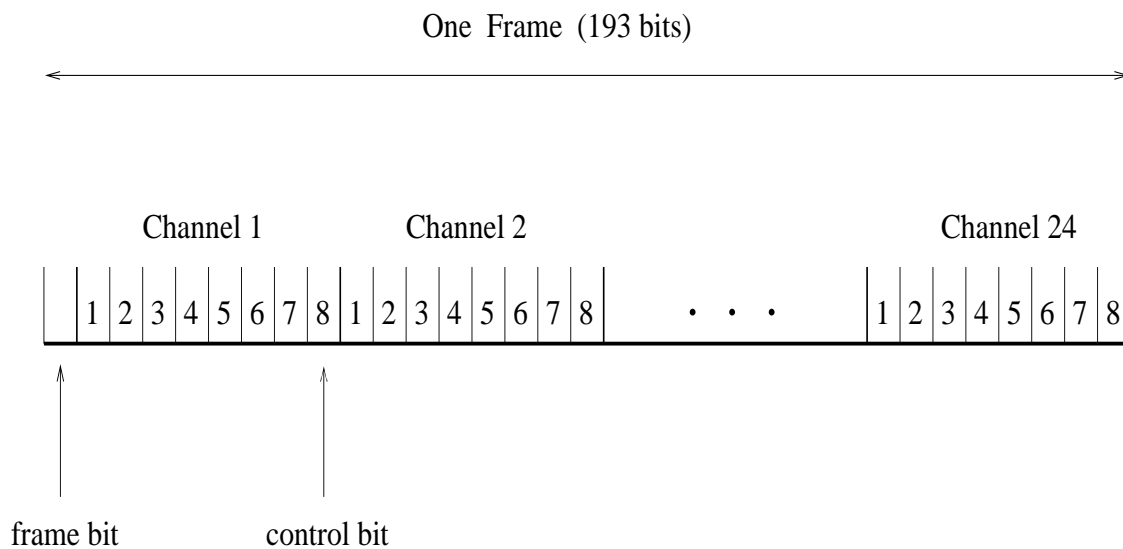
- digital audio radio service
- GEO satellites (a.k.a. satellite radio)
- uses 2.3 GHz spectrum (a.k.a. S-band)
- e.g., XM, Sirius

In the absence of carrier wave, can still use multiplexing:

→ time-division multiplexing (TDM)



- digital transmission of analog data
 - first digitize
 - PCM (e.g., PC sound cards), modem
- digital transmission of digital data
 - e.g., telephony backbone network

Example: T1 carrier (1.544 Mbps)

- 24 simultaneous users
- 7 bit quantization

Assuming 4 kHz telephone channel bandwidth, Sampling Theorem dictates 8000 samples per second.

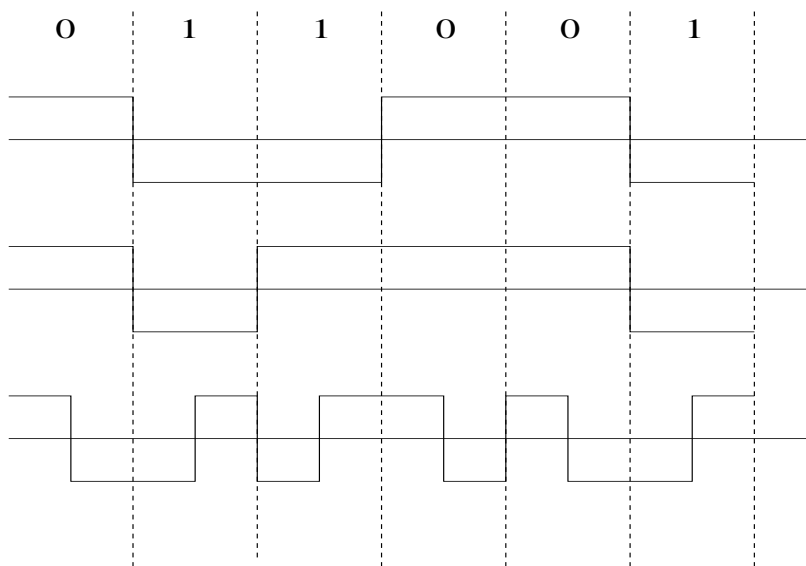
→ 125 μ sec inter-sample interval

Bandwidth = $8000 \times 193 = 1.544$ Mbps

Digital Transmission of Digital Data

Direct encoding of square waves using voltage differentials; e.g., -15V – $+15\text{V}$ for RS-232-C.

- NRZ-L (non-return to zero, level)
- NRZI (NRZ invert on ones)
- Manchester (biphase or self-clocking codes)



→ baud rate vs. bit rate of Manchester?

Trade-offs:

- NRZ codes—long sequences of 0's (or 1's) causes synchronization problem; need extra control line (clock) or sensitive signalling equipment.
- Manchester codes—synchronization achieved through self-clocking; however, achieves only 50% efficiency vis-à-vis NRZ codes.

4B/5B code

Encode 4 bits of data using 5 bit code where the code word has at most one leading 0 and two trailing 0's.

0000 \leftrightarrow 11110, 0001 \leftrightarrow 01001, etc.

→ at most three consecutive 0's

→ efficiency: 80%

Multiplexing techniques:

- TDM
- FDM
- mixture (FDM + TDM); e.g., TDMA
- CDMA (code division multiple access) or spread spectrum
 - wireless communication
 - competing scheme with TDMA