

## CS 580 Examples of supplemental qual questions

A supplemental qual typically consists of 2 to 3 such questions.

**Question A.** Let  $T$  be an  $n$ -leaf tree rooted at some node  $r$ , and assume that  $T$  is  $k$ -ary, i.e., every internal node has at most  $k$  children. Assume that every vertex  $v$  stores  $D(v)$  = the number of leaves in the subtree of  $v$  in  $T$  (counting the node as a descendent of itself, so that  $D(v) = 1$  if  $v$  is a leaf,  $D(r) = n$  where  $r$  is the root). In what follows,  $h$  denotes the height of  $T$  (i.e., the length of a longest root-to-leaf path in  $T$ ). Keep in mind that the leaves of  $T$  can occur at different depths in  $T$ , and that the internal nodes of  $T$  can have different numbers of children (anywhere between 1 and  $k$ ).

1. If  $k = 2$ , describe an  $O(h)$  time algorithm for finding a vertex  $w$  such that

$$n/3 \leq D(w) \leq 2n/3$$

and prove that such a vertex  $w$  always exists.

2. What is, in terms of  $k > 2$ , the statement that corresponds to the above statement (1)? That is, what are the bounds  $\alpha(k)$  and  $\beta(k)$  such that there is always a  $w$  for which  $\alpha(k) \leq D(w) \leq \beta(k)$ ? Also sketch an  $O(kh)$  time algorithm for finding such a  $w$ .

**Question B.** Let  $G = (V', V'', E)$  be an undirected bipartite graph (that is, every edge has one endpoint in  $V'$  and the other endpoint in  $V''$ ). Give a polynomial time algorithm for computing a minimum vertex cover of  $G$ . (*Hint:* Use maximum matching as a subroutine.)

**Question C.** Give an  $O(n \log n)$  time *divide and conquer* solution to the pattern matching problem, one whose recurrence is  $T(n) = 2T(n/2) + cn$  and  $T(1) = d$  where  $c, d$  are constants. Assume that the text has length  $2n$ , the pattern length  $n$ , and that and that the symbols appearing in  $P$  and  $T$  are integers  $\leq 3n$ . For convenience, assume  $n$  is a power of two.

(*Comment:* Such a scheme is apparently worse than the KMP algorithm that we gave in class, but it is more “parallel” in the sense that both subproblems can be solved simultaneously by a parallel computer, whereas KMP appears less suitable to a parallel solution.)