CS 580 Examples of supplemental qual questions

A supplemental qual typically consists of 2 to 3 such questions.

Question A. Let T be an n-leaf tree rooted at some node r, and assume that T is k-ary, i.e., every internal node has at most k children. Assume that every vertex v stores $D(v) =$ the number of leaves in the subtree of v in T (counting the node as a descendent of itself, so that $D(v) = 1$ if v is a leaf, $D(r) = n$ where r is the root). In what follows, h denotes the height of T (i.e., the length of a longest root-to-leaf path in T). Keep in mind that the leaves of T can occur at different depths in T, and that the internal nodes of T can have different numbers of children (anywhere between 1 and k).

1. If $k = 2$, describe an $O(h)$ time algorithm for finding a vertex w such that

   $$n/3 \leq D(w) \leq 2n/3$$

   and prove that such a vertex w always exists.

2. What is, in terms of $k > 2$, the statement that corresponds to the above statement (1) ? That is, what are the bounds $\alpha(k)$ and $\beta(k)$ such that there is always a w for which $\alpha(k) \leq D(w) \leq \beta(k)$? Also sketch an $O(kh)$ time algorithm for finding such a w.

Question B. Let $G = (V', V'', E)$ be an undirected bipartite graph (that is, every edge has one endpoint in V' and the other endpoint in V''). Give a polynomial time algorithm for computing a minimum vertex cover of G. (*Hint:* Use maximum matching as a subroutine.)

Question C. Give an $O(n \log n)$ time *divide and conquer* solution to the pattern matching problem, one whose recurrence is $T(n) = 2T(n/2)+cn$ and $T(1) = d$ where c, d are constants. Assume that the text has length $2n$, the pattern length n, and that and that the symbols appearing in P and T are integers $\leq 3n$. For convenience, assume n is a power of two.
(*Comment:* Such a scheme is apparently worse that the KMP algorithm that we gave in class, but it is more "parallel" in the sense that both subproblems can be solved simultaneously by a parallel computer, whereas KMP appears less suitable to a parallel solution.)