Question 1. (50 points) Let A be an $n \times n$ matrix with the following property: $A[i,j] = A[i-1, j-1]$ for all $1 < i \leq n$, $1 < j \leq n$.

1. (15 points) Give an $O(n)$ space representation of A, and explain how any $A[i,j]$ value can be obtained from it in constant time.

2. (35 points) Recall that the product AU of an $n \times n$ matrix A with an n-vector U is the n-vector V such that
$$V[i] = \sum_{k=1}^{n} A[i,k] * U[k]$$
Give an algorithm that, given vector U and the $O(n)$ space representation of matrix A, computes the product $V = AU$ in $O(n \log n)$ time.

Hint. Use convolution.

Question 2. (50 points) Let $T = t_0 \dots t_{2n-1}$ be a text string and $P = p_0 \dots p_{n-1}$ be a pattern string, where the text and pattern symbols are (possibly large) integers. We define the vector C of length $n+1$ as follows:

$$C[i] = \sum_{k=0}^{n-1} (t_{i+k} - p_k)^2$$

for $i = 0, \dots, n$. Give an $O(n \log n)$ time algorithm for computing the vector C.

Hint. Use convolution.

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