CS590R: Randomized Algorithms and Probabilistic Techniques in CS

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"It is remarkable that this science, which originated in the consideration of games and chances, should have become the most important object of human knowledge... The most important questions of life are, for the most part, really only problems of probability"

— Pierre Simons, Marquis de Laplace (1749–1827).

Randomization and Computing

"If somebody would ask me, what in the last 10 years, what was the most important change in the study of algorithms I would have to say that people getting really familiar with **randomized algorithms** *had to be the* **winner**.*"*

— Donald Knuth ("Randomization and Religion", 1999).

Probability and Computing

- Randomized algorithms - random steps help!
- Probabilistic analysis of algorithms - Why hard problems are sometimes easy to solve in practice?
- Probabilistic deduction, statistical inference, machine learning...

Applications: Communication networks; Cryptography; Search engines; Fast data structures; Scheduling; Optimization algorithms; Simulation and Modeling, AI reasoning; Learning; Bioinformatics, Quantum Computing, Complexity Theory ...

Topics

Theme: Emphasis on **Probabilistic techniques** — illustrated by **various applications**.

1. Introduction to probability theory

— Expectation, Linearity of Expectation. — Moments, Variance. — Deviation bounds: Markov, Chebyshev, and Chernoff Bounds.

2. Introduction to Randomized Algorithms and Probabilistic Analysis

— Las Vegas and Monte Carlo Algorithms. — Fingerprinting technique, Pattern Matching. — Randomized Quicksort. — Randomized Selection. — Stable Marriage Problem. — Packet Routing.

3. Balls and Bins Paradigm

— Birthday Paradox. — Coupon Collector's Problem. — Poisson Approximation. — Hashing, applications. — Two choices paradigm.

4. Randomized Graph Algorithms

— Karger's Min Cut Algorithm. — Linear time Minimum Spanning Tree Algorithm. — Shortest Paths.

5. Random walks and Markov chains

— Markov Chain Basics. — 2-SAT and 3-SAT algorithms. — Random Walks on Graphs and Undirected Connectivity. — Variation Distance and Mixing Time. — Expander Graphs. — Coupling and Convergence. — Algorithmic Applications.

6. The Probabilistic Method.

— Counting and Expectation techniques, Applications. — Second Moment Method. — Lovasz Local Lemma. — Algorithmic application of the Lovasz Local Lemma.

7. Martingales.

— Introduction. — Azuma's Inequality — Applications of Azuma's Inequality.

8. Random Graphs.

— Models of Random Graphs. — Threshold Phenomena. — Connectivity and Giant Component. — Random Graph Algorithms.

9. Online Algorithms.

— Competitive Analysis. — Caching problem. — k-server problem.

10. Parallel and Distributed Algorithms.

— Sorting on a PRAM. — Parallel Connectivity. — Maximal Independent Set (Luby's algorithm). — Distributed Dominating Set Problem. — Contention Resolution Protocols.

References

- Probability and Computing by M. Mitzenmacher and E. Upfal.
- Randomized Algorithms by R. Motwani and P. Raghavan.
- Probabilistic Method by Alon and Spencer.
- A First Course in Probability by S. Ross (useful reference for probability).
- Probability and Random Processes by G. Grimmett and D. Stirzaker (reference for more advanced concepts in probability).

All books are on reserve in the Math Sciences library.

Grading

Homeworks:

4 or 5 assignments.

Individually written (in Latex).

Concise and correct proofs.

Work **must** be submitted on time.

Research Project.

Class Participation.

Academic Dishonesty policy: All submitted work should be on your own. Copying or using other people's work (including from the Web) or using unauthorized material (the reference books listed above are the only authorized material allowed) will result in $-MAX$ points, where MAX is the maximum possible number of points for that assignment/problems/quiz. Repeat offense will result in getting a failure grade in the course and reporting to the Dean of students.