

CS381 Homework 7

Out: Dec. 01, Friday

Due: Dec. 8, Friday before 4 pm.

Submit to Paula Perkins (or the receptionist) at LWSN 1123.

Submissions will not be accepted afterwards.

Instructions:

Read the course policy (including academic dishonesty) in the course webpage at <http://www.cs.purdue.edu/homes/gopal/cs381>.

Text refers to the Introduction to Algorithms (second edition) book.

Justify your answers. Show appropriate work.

Reading: Chapter 24 and 25 of Text (Dijkstra, Bellman-Ford, Floyd-Warshall).

Problem 1

A communications provider has a number of computers that are connected using a network. Assume that the network is an undirected graph $G = (V, E)$ where the vertices represent the computers and the edges represent the links between them. Each edge $(u, v) \in E$ has an associated bandwidth $b(u, v)$ (a positive number). The **bottleneck bandwidth** of a path is the *minimum* of the bandwidths of its edges. Develop an algorithm that given two computers a and b computes the **maximum** bottleneck bandwidth path from a to b . What is the running time of your algorithm. (Hint: Modify Dijkstra's algorithm to solve this problem.) (25 points)

Problem 2

Let $G = (V, E)$ be a weighted directed graph. (Weights may be positive or negative.) Give an $O(|V|^3)$ algorithm to find a non-trivial cycle (i.e., a cycle consisting of at least two vertices) in G of minimum weight. Note that

your algorithm should not only output the minimum weight of the cycle but also the vertices of the cycle. (Hint: If the graph has a negative weight cycle then you should output one such cycle. Otherwise you should output a positive weight cycle of minimum weight. You might want to modify Floyd-Warshall's algorithm.) (25 points)