Three Proofs Without Words for the Sine of the Sum

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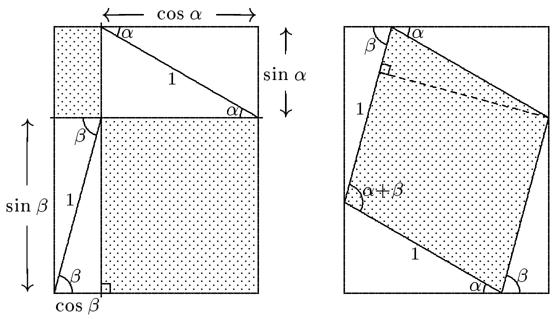
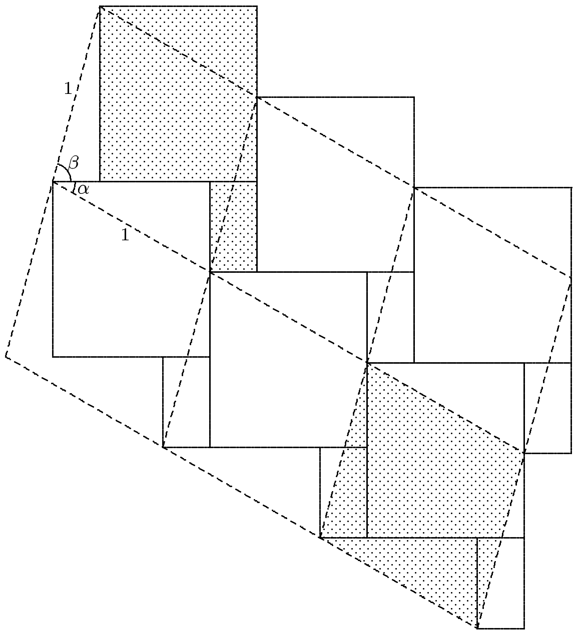
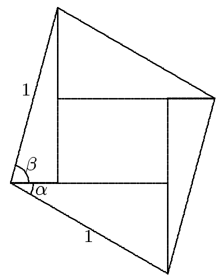
Three of the most ancient (and most elegant) proofs of the Pythagorean Theorem (see, for example, [1, 2]) generalize to proofs of the addition theorem of sines. We assume $\alpha < \beta < \pi/2$ throughout.



$$\sin\alpha \cdot \cos\beta + \sin\beta \cdot \cos\alpha = \text{ shaded area on the left } = \text{ shaded area on the right } = \sin(\alpha+\beta)$$



$$\sin\alpha \cdot \cos\beta + \sin\beta \cdot \cos\alpha = \text{ area of shaded rectangles } = \text{ area of shaded parallelogram } = \sin(\alpha+\beta)$$



$$\begin{aligned}
\sin(\alpha+\beta) &= \text{area of (outer) parallelogram}\\
&= 2\cdot\left(\tfrac{1}{2}\cdot\cos\alpha\cdot\sin\alpha+\tfrac{1}{2}\cdot\cos\beta\cdot\sin\beta\right)+(\cos\alpha-\cos\beta)\cdot(\sin\beta-\sin\alpha)\\
&= \sin\alpha\cdot\cos\beta+\sin\beta\cdot\cos\alpha
\end{aligned}$$

References

[1] Roger B. Nelsen, *Proofs Without Words*, Classroom Resource Materials #1, Math. Assoc. of America, Washington DC, 1993

[2] John Stillwell, *Numbers and Geometry*, Springer-Verlag, New York, 1998