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FOCUS

Volume 24 Issue 6

Former MAA President Thomas F. Banchoff Receives NSF's 2004 Distinguished Teaching Scholars Award

By Harry Waldman

Former MAA president Thomas F. Banchoff has received the National Science Foundation's Director's Award for Distinguished Teaching Scholars. He was named one of eight winners of the foundation's highest honor for teaching and research excellence. Banchoff, who was MAA president from l999-2000, teaches at Brown University. The fourth-annual awards were presented at a gala reception in Washington, D.C. on June 2, 2004.

"These scholars have a special distinction in that they influence entire academic cultures. They make students major participants in the process of discovery. They also promote activities that expand the education process beyond the boundaries of the university into local schools and communities," said NSF's acting director, Arden L. Bement, Jr. "They are true leaders in both the scientific and academic realms. Their pioneering research, already well recognized, is equaled, and sometimes surpassed, by a rare talent and commitment to communicate and teach knowledge."

Banchoff, a mathematician and professor at Brown since 1967, has been influential because of his pioneering geometry work on smooth and polyhedral surfaces beyond the third dimension. His seminal paper — in which he was the first to show the importance of tautness on smooth surfaces — was the seed of many papers by many mathematicians. He has also written more than a dozen articles about the use of computer graphics in teaching and research. His use of visualization techniques has drawn many students into mathematics, teaching, and computer graphics professions.

Tom Banchoff, (right), receiving Director's Award for Distinguished Teaching Scholars from Rosemary R. Haggett, Director, Division of Undergraduate Education, and Arden L. Bement, Jr., Acting Director, NSF.

Worth about $300,000 to each scholar over the next four years, the awards recognize accomplishments by scientists and engineers whose roles as educators and mentors are as important as the ground-breaking research results they achieve. The grant will allow Banchoff and the other scholars to work on new projects, or continue present work in new ways that benefit their individual fields and the students they support.

Whether it's showing undergraduate engineering students how to take apart photocopiers to learn about design, teaching calculus through visualization techniques, or providing virtual research and data gathering opportunities on climate change, the fourth-annual winners of the teaching scholars the NSF named are wide-ranging in their approaches.

The others winners of this year's awards were Dean A. Zollman, a physicist at Kansas State University; Julio J. Ramirez, a neuroscientist from Davidson College in North Carolina; Walter C. Oechel, an earth systems scientist from San Diego State University; Kenneth G. Tobin, an urban educator from the City University of New York (CUNY) Graduate Center; Alice M. Agogino, of the University of California, Berkeley, Susan E. Powers, of Clarkson University in New York, and David F. Ollis of North Carolina State University.

On the Cover:

The 2004 USA Mathematical Olympiad Winners with Wendy Ravech, President, Akamai Foundation and Tom Leighton, Co-Founder and Chief Scientist, Akamai Technologies. Winners from left to right: Jae Min Bae, Tony Zhang, Janos Kramar, Jacob Tsimerman, Jongmin Baek, Oleg Golberg, Matthew Ince, Tiankai Liu, Aaron Pixton, Alison Miller, Brian Rice. Seated in foreground: Ameya Velingker. Photograph taken at the Einstein Memorial on the National Academy of Sciences grounds, Washington, DC. Sculptor: Robert Berks. Photograph by Sonny Odom.

The Noether Days at Texas Tech University

By Mara D. Neusel

This program rocked! – A lot of fun! Thanks a bunch! – Test was hard! Workshop was very, very interesting and fun. – The career panel was awesome. – The competition was challenging. I enjoy a challenge! – It was very good and I might even pursue a career in math thanks in big part to ya'll. - Today was cool.

Do you believe that these are student evaluations of a High School Mathematics Day? These are comments we received for our first *Emmy Noether High School Mathematics Day* (ENHD). This program started out in the Fall of 2002 as a modest but sincere effort of a group of faculty members hoping to expand the department's outreach efforts to enhance the educational and life experiences of high school students.

The first two ENHDs, on May 15, 2003 and a year later on May 4, 2004, were overwhelmingly successful. They have made it clear that our region needs this event. Feedback shows that although this is just a one-day event, it has a significant impact on the lives of our participants. It truly does make a difference. We are planning to offer the ENHD every spring and look forward to doing so for many more years. We believe that our concept is suitable for other universities and colleges around the country.

Approximately 160 female students from local and area high schools (grades 9–12) accept the invitation to participate, with their teachers, at the ENHD. Our goal is to help prepare high school graduates who are strong in the areas of science, mathematics, communication, and problem solving. We also hope to provide the opportunity for participants to discover and be enlightened about possible careers in mathematics.

More specifically, the missions of the program are to provide women students with a unique, high-quality experience designed to foster interest in mathematics and careers in mathematics, engineering, and science; to provide women students the opportunity to experience a university environment; to give insight into women professors' experiences and into the educational opportunities associated with mathematics; and to provide women students the opportunity to learn that careers in mathematics, science, and engineering are attainable.


Students participate in the Noether Day competition. Photo by Dustin Love.

Our diverse program addresses each of these objectives. After registration and an official welcome by Dr. Jane Winer, Dean of the College of Arts and Sciences, and Dr. Lawrence Schovanec, Head of the Department of Mathematics and Statistics, the students enter a competition. The problems are designed so that a general high school background suffices to solve them. Nevertheless, they are challenging and require serious logical thinking. For example:

What angle do the hands of a clock make at 7:38?

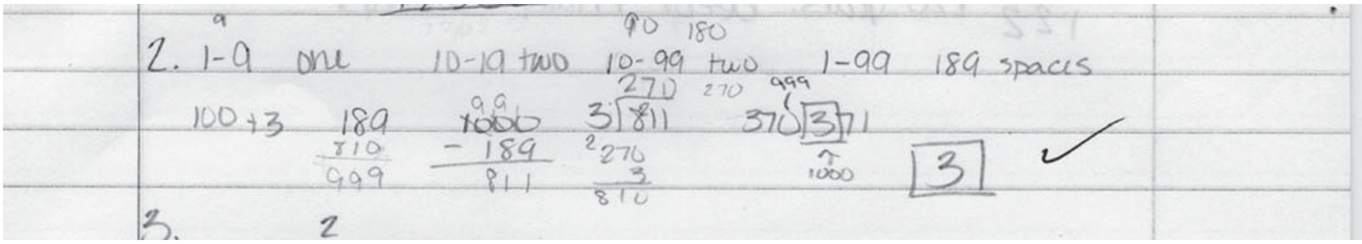
Or

A person intends to withdraw X dollars and Y cents from an account. By mistake, Y dollars and X cents are withdrawn. The mistake is realized only when, after spending exactly $10.00, the person observes that the remaining money is exactly twice the amount originally intended to be withdrawn. How much did the person intend to withdraw from the account and how much was withdrawn instead?

The competition is conducted in the morning so the rest of the day can be used for grading. As a result, we can announce the winners at the end of the day. We give prizes for the best exam in each grade, the best exam in each school, the best school in Divisions 1A and 2A, and the best school in Divisions 3A and 4A. Our awardees are highly gifted young women. One of our awardees in the first ENHD has been accepted to MIT, and two others received the highly prestigious Clark Scholarship.

After the exam, the students participate in workshops presented by faculty members. We try to present a broad variety of mathematics and its applications to diverse disciplines in a playful but challeng-

The natural numbers are printed one immediately following the other:123456789101112131415…Note that the numbers greater than 9 take up two digits (spaces) to print and the numbers greater than 99 take up three or more digits to print. What is the 1000th digit printed?



ing way. We do not teach mathematics in these workshops; rather, we want to advertise for mathematics and show that mathematics is used extensively in our lives, that it is interesting, important, and fun.

We prefer that research mathematicians discuss problems and questions arising in their research, which is, of course, not an easy task for the lecturers. But they rise to the occasion. For example, biomathematicians Linda Allen and Lih-Ing Roeger played the Disease Game with their group and geometer Dr. Magdalena Toda had them tile 2D and 3D dream houses. Other workshops focused on coding theory, cryptography, traffic accident statistics, and modeling of physical systems. Some were about fun things like the algebra of juggling, others about seemingly very dry things like binomial coefficients (but the girls loved it!). The success of these workshops depends partly on the personality of the lecturer.

These students absorb the mathematical material presented to them very quickly, and they are grateful for being exposed to mathematical ideas and applications they had never seen before. Surprisingly, these girls still feel awkward in their male-dominated mathematics and science classes. The ENHD helps by introducing them to many successful female mathematicians and to overcome a feeling of not fitting in or of being misplaced.

ENHD is not only meant to foster exceptionally talented students. We also want to help the less talented or weaker mathematics students. We hope to help them understand that even if they decide to major in something like English history, they will need some basic knowledge of mathematics and should strive to finish their college algebra courses with a decent grade. More importantly, we want them to overcome their fear of or hatred for mathematics, replacing them with a new appreciation of the subject.

Experiences in high school can have far-reaching social effects on students. Some of our students come from tiny little country schools. Lubbock is "the big city" to them, and Texas Tech is considerably larger than many of their communities. Some have never been on a university campus. Some girls were afraid of entering the large auditorium. Their teachers had to literally take them by the hand and guide them into the room.

Moreover, West Texas provides a unique opportunity for cultivating diversity. Our students have varied racial and socio-economic backgrounds and come from both the city and the country. These cultural clashes can indeed cause some tension, but it in the long run it is advantageous for the kids to be exposed to each other.

Accompanying teachers can also take advantage of workshops while their students are involved in ENHD. Topics range from classroom material to information that broadens their mathematical background.

A career panel follows the workshops, featuring successful young women with a degree in mathematics from our university. We believe it is important to invite our own graduates because the kids relate to them easily. We invite women from diverse professions, not only mathematics teachers or professors, to show the myriad of options available with a degree in mathematics. These women share their experiences as mathematics students and working mathematicians with the audience. Since they have already graduated from Texas Tech and have gone out into "the real world," they can convince the students that careers in mathematics and related fields are attainable, interesting, and fun.

At the end of the day we hand out prizes for the best exams. This usually turns the students into a loud and noisy cheering crowd. Each participant also receives a souvenir in the form of a booklet that contains the entire day's events, so that everybody will have a chance to look back on this wonderful day. Our ENHD is free of charge for everyone and includes lunch and refreshments. We do not select the students; that choice is left to the teachers.

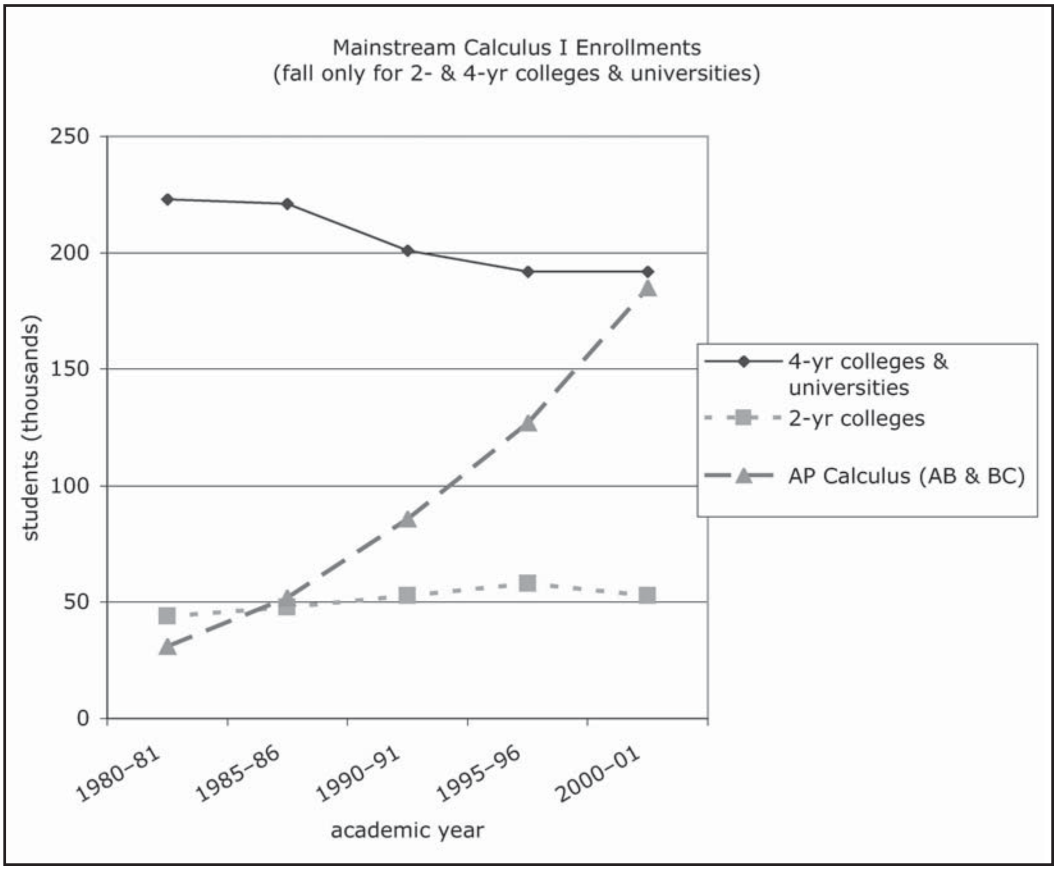
This event would not be possible without the many people who help in various ways to make it happen: the faculty members on the organizing committee, those who present workshops or grade the exam, the many faculty and student volunteers who help throughout the day, the moderator of the panel and the panelists, the photographer, the administrative support we obtain from our department and the entire College of Arts and Sciences, and, of course, the many sponsors, among others our local SIAM and MAA Student Chapters. It truly is an event which brings together the entire department, creating a strong bond between our university and the city of Lubbock and the surrounding areas.

Mara Neusel is associate professor of mathematics at Texas Tech University. You can read more about the Emmy Noether High School Mathematics Day at Texas Tech at http://www.math.ttu.edu/~mneusel/enhd2.htm.

The Changing Face of Calculus: First-Semester Calculus as a High School Course

By David M. Bressoud

Once upon a time, calculus was the first college-level mathematics course taken by mathematically talented students. The students in first-semester calculus were mathematically motivated, generally well prepared, and they were seeing these ideas for the very first time. This is no longer true. Most of our best-prepared mathematics students arrive in college with credit for at least the first semester of calculus, many of them with credit for both semesters. Despite steady growth in majors in science and engineering, enrollment in first-semester calculus has been flat or slightly declining at both two- and four-year undergraduate programs. It is the College Board's Advanced Placement Calculus Program that has been growing steadily at 7–8% per year (see figure 1).



Note 1: Mainstream Calculus I Enrollments. Fall two- and four-year college and university enrollments from [4][1]. AP Calculus enrollments from The College Board (most recent years available at [1]).

In 2004 over 225,000 high school students took the AP calculus exam. This number is far larger than the number of students who took mainstream first-semester calculus in all four-year undergraduate programs in the Fall of 2000. By the time of the next CBMS survey in 2005–06, we can expect that more students will take an AP Calculus exam than will take mainstream Calculus I in the Fall of 2005 in all 2-year and 4-year institutions combined. [2]

First-semester calculus has become a high school topic for most of our strongest students. This has several implications:

1. We should ensure that students who take calculus in high school are prepared for the further study of mathematics.

2. We should address the particular needs of those students who arrive in college with credit for calculus.

3. We should recognize that the students who take first-semester calculus in college may need more support and be less likely to continue with further mathematics than those of a generation ago.

This article will address the implications for calculus taught in high schools. A second article, "The Changing Face of Calculus: First- and Second-Semester Calculus as College Courses," will look at the implications for how we teach calculus in colleges and universities.

Recommendations for High School Calculus

The pressure to take calculus in high school is understandable. Competition for admission to the best colleges and universities is fierce. It has helped to create strong growth in AP programs across the board. Many mathematicians deplore this movement of calculus from the college to the high school curriculum, but the pressures are too strong to stop or even substantially slow it. What we can hope is to shape it.

With this in mind, the presidents of the MAA and NCTM issued a joint statement in 1986 [3] with two strong recommendations which I paraphrase here:

1. In spite of the pressures to take calculus while still in high school, students should never short-change their mathematical preparation in subjects such as algebra, geometry, or trigonometry. Solid mathematical preparation is far more important than exposure to calculus.

2. When calculus is taught in high school it should be a college-level course. This means that the goal of the course should be to give students the same breadth of topics and mastery of calculus obtained by students taking such a course in college. It means that the course should be taught with the expectation that students who perform satisfactorily will be able to place into the succeeding college calculus course.

I believe that these recommendations need to be repeated and re-emphasized. One of the inevitable weaknesses of the AP program is that student enrollment in an AP class appears on the transcript that is reviewed for college admission, but the test that evaluates whether or not the student has learned this material at a college level is not administered until after college acceptances have been sent out. This is why many students enroll in AP courses but do not take the examinations. Many schools are under pressure to offer a course that is nominally an AP Calculus course, even if they expect few students will be able to pass the AP exam. These recommendations are intended to back up the teachers who are trying to resist rushing students into calculus before they are properly prepared.

It is particularly important that the calculus taught in high school should be a substantive course that prepares students for further work in mathematics. A weak overview of calculus does little to reinforce student knowledge of algebra, geometry, or trigonometry. In fact, it may encourage slighting these subjects in order to get into the calculus course that will improve the appearance of one's transcript. On the other hand, a solid calculus course should require and help develop a level of mastery of these core subjects that is essential for any further work in mathematics.

Finally, these recommendations recognize that the students who take calculus in high school are among our best students. They must be prepared for college-level mathematics. Once they are ready for and are studying calculus, they should be learning it in a course that is comparable to what they would see in a mainstream college course.

The Responsibilities of Mathematicians

How calculus is taught is important. As I argued in 1992 [2], calculus is not only essential for building mathematical models of the world around us and thus informing disciplines such as physics, economics, and biology, its creation/discovery was the defining moment in the birth of modern mathematics. It has shaped our modern conception of and expectations for mathematics. Calculus should not be the only pillar supporting the undergraduate curriculum in mathematics. Discrete mathematics, geometry, and data analysis have equally important if very different roles to play. But calculus must remain one of those pillars. To ensure that it remains so, mathematicians must be concerned about how it is taught both in colleges and in high schools.

Calculus can be and is being taught well in high schools, but as the number of high school calculus courses expands, so does the number of high school teachers who must teach these courses without much more preparation than the undergraduate course they themselves took, often many years before. At many high schools, only one person teaches calculus, and so peer support may be lacking. The purpose of the AP Calculus examinations is to provide a common standard against which to measure students from all of these classes, but it can only accomplish so much. Ultimately, the way to ensure that what is taught in high school calculus really is a college-level course is through the preparation and support of the teachers who will lead these classes. The College Board runs many workshops for AP Calculus teachers. NCTM meetings include well-attended sessions that address their needs. The MAA is beginning to realize its own potential in this area. [3] But there still remain far too few university-level mathematicians who are willing to assist in the task of preparing and supporting high school teachers. At the very least, all mathematicians have a responsibility to be aware of the AP Calculus program: its course expectations and the nature of its examinations. Every department should encourage at least one individual to attend the annual AP Reading (the grading of the free response questions), to work with local AP Calculus teachers, or to help prepare and support those who will teach calculus in high school.

Calculus II as a High School Class

The same pressures that are pushing Calculus I into the high school curriculum are doing the same for Calculus II. Traditionally, it was a very elite group of students who took BC Calculus, covering the entire two-semester college syllabus. That group of students also grew by 6–8% per year until the mid-1990s. Over the period 1995–98, the rate of growth of BC calculus accelerated to 10–11% per year, a rate that has held up since then.[4] In 2004, the number of students taking the BC Calculus exam exceeded 50,000. It will likely exceed 60,000 by 2005–06, the year of the next CBMS survey.

In 2002, 23% of the students who took BC Calculus did so before their senior year [7]. These high school students are not necessarily well served by taking classes in linear algebra, several variable calculus, or differential equations at a local college. Picking up additional college credits is far less useful for them than deepening and broadening the mathematics they already think they know. These students need to be challenged, but they also need to be prepared for and enticed into a deep study of further mathematics in the company of their peers.

There are many local programs that recognize this. In Minnesota, we have the University of Minnesota Talented Youth

Math Program (UMTYMP). At the North Carolina School of Science and Mathematics, the mathematics department is developing courses that return to calculus, using several variables, differential equations, and modeling to explore its topics in greater depth. But not enough students have access to these kinds of programs. There is a need for a substantial national effort to create materials that can be used with these students and to help teachers learn how to use them.

The movement of calculus into the high schools is not necessarily bad, but it does require the efforts of the mathematical community—individuals, departments, and professional associations—to prepare and support those who will teach it and to resist the pressures that would weaken it.

Acknowledgement: Thanks to Ben Klein, Johnny Lott, Bernie Madison, Bob Megginson, Carol Miller, and Dan Teague for helpful comments.

Bibliography

[1] APCentral, *AP Research and Data*, http://apcentral.collegeboard.com/program/research/.

[2] Bressoud, David M., Why *do* we teach calculus?, *Amer. Math. Monthly*, vol. 99, no. 7 (1992), 615–617.

[3] Dossey, John A. and Lynn A. Steen, Calculus in the Secondary School, joint letter of the MAA and NCTM Presidents, 1986.

[4] Lutzer, David J., James W. Maxwell, and Stephen B. Rodi, *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States: Fall 2000 CBMS Survey*, American Mathematical Society, Providence, RI, 2002.

[5] Small, Don, Report of the CUPM Panel on Calculus Articulation: Problems in Transition from High School Calculus to College Calculus, *Amer. Math. Monthly*, vol. 94, no. 8 (1987), 776–785.

[6] Snyder, Thomas D. and Charlene M. Hoffman, *Digest of Education Statistics 2002*, NCES 2003-060, National Center for Education Statistics, U.S. Department of Education.

[7] *2002 AP Yearbook*, The College Board, New York.

David Bressoud is DeWitt Wallace Professor of Mathematics at Macalester College in St. Paul, Minnesota. He serves both as Chair of the MAA's Committee on the Undergraduate Program in Mathematics (CUPM) and as Chair of The College Board's AP Calculus Development Committee. He has been involved with AP Calculus since 1990–91 when he had the privilege of teaching an AB course at the State College Area High School and of learning how to teach calculus from some great teachers, especially Annalee Henderson.

Notes

1. For 1980 and 1985, the CBMS Survey only reports total numbers of students taking all calculus classes in the fall. The numbers of students taking Calculus I were estimated by taking 55% of this total for 4-year institutions and 60% for 2-year institutions (the approximate percentages in the years 1990, 1995, and 2000).

2. The total number of high school students taking calculus each year is unknown. Numbers range from 300,000, the NCES figure for 2000 ([6], table 141), to 500,000 or more. The larger number is based on the College Board estimate that 60% of AP Calculus students take the examination and the observation that many students take a high school calculus course that is not an AP course. This includes students in the International Baccalaureate program and students in joint programs between high schools and community colleges.

3. Daniel Teague, Benjamin Klein, and I are in the process of establishing a SIGMAA for high school teachers that will focus on support for teaching college-level mathematics courses in high schools.

4. The increased rate was almost certainly helped by the fact that an AB subscore was made available for the BC exam beginning in 1998.

Request for Information on College Algebra Courses

The MAA, in collaboration with AMATYC and NCTM, has initiated a major project to assess how well courses such as college algebra actually meet the needs of the students who take them. Anecdotal evidence abounds that indicates that these courses are not particularly effective. As a first step, CRAFTY (which is responsible for conducting the study) seeks some hard data on these courses:

Who are the students who take them and why?

How many are successful in the courses?

What subsequent courses do they take and how do they do in those courses?

Looking back from Calculus I, say, how many students came through "college algebra" courses?

If you or your department has conducted any detailed studies on these issues and are willing to share your results, please contact Sheldon Gordon (gordonsp@farmingdale.edu), Bill Haver (wehaver@vcu.edu), Jack Bookman (bookman@math.duke.edu) or Susan Ganter (sganter@clemson.edu). Any information that you provide will be kept in the strictest confidence.

MAA Prizes and Awards Announced at MathFest 2004

At the Providence MathFest, the MAA announced several prizes and awards for writing and teaching. We will include more details and photos in the November issue. Congratulations to all the winners!

Carl B. Allendoerfer Award for articles published in *Mathematics Magazine*

Charles I. Delman
and Gregory Galperin
"A Tale of Three Circles"
Mathematics Magazine,
February 2003, pp.15-32.

Trevor Evans Award for articles published in *Math Horizons*

Douglas Dunham
"A Tale Both Shocking
and Hyperbolic"
Math Horizons,
April, 2003, page 22.

Hugh McCague
"A Mathematical Look at
a Medieval Cathedral"
Math Horizons,
April, 2003, page 11.

Lester R. Ford Award for articles published in *The American Mathematical Monthly*

Noam Elkies
"On the Sums $\sum_{k=-\infty}^{\infty}(4k+1)^{-n}$"
The American Mathematical Monthly,
August-September 2003, pp. 561-573.

Charles Livingston
"Enhanced Linking Numbers"
The American Mathematical Monthly,
May 2003, pp. 361-385.

R. Michael Range
"Complex Analysis: A Brief Tour into
Higher Dimensions"
The American Mathematical Monthly,
February 2003, pp. 89-108.

Ruediger Thiele
"Hilbert's Twenty-Fourth Problem"
The American Mathematical Monthly,
January 2003, pp. 1-24.

George Pólya Award for articles published in the *College Mathematics Journal*

Greg N. Frederickson
"A New Wrinkle on an
Old Folding Problem"
College Mathematics Journal,
September, 2003, vol. 34(4),
pp. 258-263.

Chauvenet Prize for Expository Writing

Edward B. Burger
"Diophantine Olympics and World
Champions: Polynomials and Primes
Down Under"
The American Mathematical Monthly,
November, 2000, no. 9, pp. 822-829.

Henry L. Alder Award for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member

Francis Edward Su
Harvey Mudd College

Zvezdelina Stankova
Mills College

MAA Member Dennis Berkey becomes 15th President of WPI

Dennis D. Berkey assumed the Presidency of Worcester Polytechnic Institute on July 1, 2004. Dr. Berkey received a B.A. in mathematics from Muskingum College, an M.A. in mathematics from Miami University, and a Ph.D. in mathematics from the University of Cincinnati. He joined the faculty of Boston University in 1974, chaired the Department of Mathematics from 1978 to 1983, was dean of the College of Arts and Sciences from 1987 to 2002, and twice served as university provost at BU from 1987 to 1991 and from 1996 to 2004.

Dr. Berkey's published research is in applied mathematics, the theory of differential equations and optimal control. He is a member of the MAA, the AMS, and SIAM. He is the author of two textbooks, both in their third editions: *Calculus* and *Applied Calculus for Management, Social, and Life Sciences*. Professor Berkey was honored in 1978 with Boston University's Metcalf Cup and Prize for Excellence in Teaching.

In Memoriam

Elias Y. Deeba died unexpectedly on February 11 at the age of 54. Deeba, who was born in Lebanon and came to the University of Houston Downtown in 1983, had numerous publications in numerical systems of non-linear equations, mathematical analysis, general mathematical systems, mathematics education, and fuzzy logic. He had been a member of MAA since 1979.

Bert Yood died on March 17 at the age of 87, Yood was still active writing papers. The week before his death, he received offprints of a paper that had just appeared. He had been a member of the MAA since 1981.

POINT: A Call for Action

By Rick Norwood

Many talented mathematicians have devoted a great deal of time and effort to improving mathematics education in the public schools. There have been a few small successes here and there, but the problem remains as critical today as it was when "A Nation at Risk," the study that motivated much of this work, was published twenty years ago. A new book entitled *Who's Teaching Your Children,* by Vivian Troen and Katherine C. Boles, argues that a large part of the reason why so much money and effort have had so little effect is the lack of qualified teachers and the culture of incompetence that makes it hard to train and to retain good teachers.

It's all too easy to treat the problem in the abstract. Here are some concrete examples from my personal experience.

A second grade teacher asks students to divide a radially symmetric figure into four equal parts. A student divides the figure into four equal parts with a horizontal and vertical line through the center. The teacher marks that answer wrong. The student complains. The teacher shows the student the "right" answer in the book — the figure divided into four equal parts by two diagonal lines through the center. The teacher asks the student, in a scornful voice, "Do you think you are smarter than the book?" The book does not mention that there may be more than one correct answer.

A third grade teacher teaches students that the way to add fractions is to add the numerators and add the denominators. A college professor points out that this is incorrect, and gives the example "According to your rule, a half chicken plus another half chicken is two fourths of a chicken." The grade school teacher, defiant, announces, "I'm the teacher. I can teach any way I want and you can't stop me."

A grade school textbook teaches that 7 – 4 + 2 = 1, because, according to the rule "My Dear Aunt Sally" we must do the addition before the subtraction.

A principal sitting in on a class where a teacher fresh out of college is teaching hears the teacher say that a square is a rectangle with four equal sides. The principal stands up and tells the class that their teacher is stupid, because everybody knows that a square is not a rectangle.

These stories exemplify the problem: If mathematicians do not have the power to keep blatant errors out of the classroom, we certainly do not have the power to institute reforms.

Who's Teaching Your Children was written by authors who have broad personal experience as public school teachers. We had better listen to what they have to say. They provide compelling evidence that the trouble with our public schools is a culture of incompetence. According to this book, "…classroom teaching competency is lower now that it has been since the era of the one-room schoolhouse."

The book and movie *Serpico* tell the true story of a New York policeman who tried to be honest. He found it impossible. Even though he adhered to the code of the schoolyard, and never tattled on another cop, the other policemen found his honesty intolerable. *Who's Teaching Your Children* describes vividly an analogous situation: Incompetent teachers find competent teachers intolerable and drive them away. In one of the most moving stories in the book, the authors tell how an outstanding principal, together with a group of extraordinary teachers, tried to turn one school around. They made the mistake of letting the other teachers see what a low opinion they had of them. "[The other] teachers complained bitterly, some with shaking voices and tears in their eyes, about being snubbed, shut out, or just ignored." The other teachers were not villains. It is human nature to resent being told you are wrong. But any teacher who tries to do a good job had better be prepared. In her employment folder there will be numerous complaints from other teachers, complaints that she is "stuck up" and "does not get along."

Public school teachers and principals are largely autonomous in their own domain. Reform efforts fail because incompetent teachers and principals do not want, will not tolerate, reform.

The problem begins in the colleges and universities, where the weakest students are encouraged to be teachers and the stronger students are advised not to be teachers. It continues in the public schools. If a young teacher asks for guidance or advice the older teachers laugh at him behind his back. The effect is clear, and it is quantifiable. According to *Who's Teaching Your Children,* "The best teachers are leaving American classrooms at an accelerating rate." Society at large reinforces the problem because, despite the rhetoric one often hears, people seem to have little real respect for teachers. The common saying, "Those who can, do; those who can't, teach" suggests that we *expect* that teachers will be incompetent.

As long as the public school culture supports bad teachers and drives out good teachers, our schools will continue to get worse and worse. To attempt reform without addressing this problem cannot possibly work.

People are reluctant to even talk about the problem, both because we do not want to hurt people's feelings, and because we fear that criticism of incompetent teachers will reflect badly on the many good and dedicated teachers who struggle to survive in a broken system. But the incompetent teachers do not have any qualms about hurting the feelings of the good teachers or about hurting the feelings of inquisitive students.

continued on page 12

COUNTERPOINT: A Call for Reflection

By Bill Berlinghoff

Rick Norwood's article spotlights some truly troubling issues in mathematics education, issues that should concern all of us who do and/or teach mathematics at any level. I share his concern and his sense of urgency, but I have serious misgivings about his analysis of the problem. The "culture of incompetence" approach may be sensational enough to garner reviews and sell books, but it does not resonate well with my experience. I would like to suggest an alternative analysis and an alternative call for action. To do that, I feel obliged to begin by disagreeing with Prof. Norwood, not simply to be disagreeable, but to explain where his analysis and mine agree and where they diverge.

The concrete examples cited early in the article surely are troubling. But the unexamined assumption here is that they are typical of teacher behavior, rather than aberrations in a generally far more competent population. Prof. Norwood asks us to buy into this assumption based, apparently, on the claims of the authors of *Who's Teaching Your Children*. He uses that justification to claim, "Reform efforts fail because incompetent teachers and principals do not want, will not tolerate, reform."

I think not. This is far too sweeping an indictment. In my view, it is an oversimplification of a very complex and difficult problem. Incompetence may lie behind some resistance to improving mathematics education, but I suggest that the far more likely culprit is overwork. Unlike those of us whose professional specialty is a single subject, elementary teachers are expected to be experts in a broad spectrum of disciplines — mathematics, reading methodologies, children's literature, behavioral difficulties, social studies, general science, current events, and more. Such unrealistic expectations, coupled with the mounting demands of mandated testing, individual reporting, and the like, lead to a culture of simple survival. Unless and until we mathematics types recognize and accommodate these realities of elementary school teaching, we should expect efforts at change of any sort to be met with resistance. Change requires time and effort, two scarce commodities in an elementary teacher's world.

Prof. Norwood begins his article by saying, "Many talented mathematicians have devoted a great deal of time and effort to improving public school math education in the United States." The implication is that, if so many clearly competent people have labored so long with so little effect, there must be something wrong with the target audience. Let's examine that idea for a minute. First of all, I'm not at all sure what "many mathematicians" Rick Norwood has in mind. Perhaps in absolute numbers we're talking about scores or even a hundred or so mathematicians around the country, but in percentage terms I think the vast majority of working mathematicians do little more than ignore the problem of mathematics education unless it directly impacts their children or their students. Many of those who pay attention to elementary mathematics education at all do so by pontificating about how things ought to be, rather than by truly getting their hands dirty in the real world of the elementary teacher. (There are exceptions to this, of course. I know a few excellent counterexamples, but only a few.) Most of us who have spent our careers at the college and university level simply have no clear idea of the realities of teaching elementary school mathematics. Our expertise has a lot to offer, no doubt, but our delivery systems to date have been woefully primitive and misguided.

Later in his article, Prof. Norwood says, "The problem begins in the colleges and universities..." I agree. But it's not just a problem of who is encouraged to become a teacher and who is not. Our deficiencies run far deeper than that. We need to take a long, hard look in our professional mirror before labeling others as incompetent. The plain fact is that we, as a profession, do not place a very high value on the preparation or professional development of teachers. At most colleges and universities, mathematics faculty who devote significant time and energy to improving mathematics education courses and text materials do so at their own professional risk. Promotion and tenure are driven in large measure by original research and publication in peer-reviewed journals. Textbook writing and course development, particularly for non-major courses such as elementary education mathematics, are viewed at best as some sort of generalized service to the community. More often, they are regarded as a distraction from the "important business" of the department, activities to be avoided if one is serious about advancing one's career. Too often, the teaching of such courses is relegated to adjuncts on a catch-as-catch-can basis or to faculty who have been marginalized in one way or another.

The last thing we need is an MAA textbook review committee! We need to improve the expertise of the people who are using the books. If we do that, their growing intolerance for error and the realities of the marketplace will fix the textbook problem. If the MAA wants to do something, it should find a way to give more credibility and support to its members who are willing to devote at least part of their professional lives to working on the problem of how to make mathematical ideas a firmer part of the preparation of teachers, teachers for whom math is but one of many subjects, and often a distasteful one, to boot. That's a hard problem. It's a problem that has held my attention and challenged my efforts for 40+ years of teaching college mathematics, many of them at colleges that educate teachers. About 15 years ago I thought I had a good handle on it and co-authored a math textbook for future

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If we want to make a difference, we have to accept that we cannot drive out or change incompetent teachers. We must find ways to fight the system itself rather than getting bogged down fighting individuals within the system. We need to lobby our school boards to hire principals who support good teaching. Most important, we need to encourage good students to become teachers, while being honest about the problems they will face, and we need to support good teachers.

Here are a few things I believe would help. Citizens should start by demanding safe schools. To spend one cent on "Teacher Enrichment" when teachers (and children) fear for their safety is absurd. More than 100,000 teachers are victims of violent crimes *in school* every year. No wonder people leave teaching.

We should also enforce existing laws to stop incompetent teachers from entering the system. Every year more than 50,000 teachers are hired who do not meet already existing standards. School systems say that there is not enough money to hire competent teachers. It is ridiculous to spend billions testing students when their teachers cannot pass standardized tests. MADD has been successful. How about Mothers Against Terrible Teaching?

As mathematicians we should, when speaking publicly, make clear the difference between our personal opinion and mathematical fact. Say, "In my personal opinion, I do not think we should teach long division." But also say, "7 – 4 + 2 = 1 is wrong."

What can the MAA do? One idea is for the MAA to set up a committee to review elementary textbooks, and issue an MAA "seal of approval" for textbooks that are mathematically sound.

I am told that at some universities the least qualified professors are assigned the "Math for Teachers" course. I hope that is not true at my own school, because I teach that course. East Tennessee State University takes education seriously. The College of Education requires nine hours of mathematics for all Elementary Education majors. It is beginning to make a difference. I treasure a comment from one of the professors in the Department of Curriculum and Instruction. "I have seen a big change in students' mathematical abilities and understanding." Teachers use math on the job every day. Recommend to your College of Education that they require more mathematics, and let them know that you are willing to work with them to insure that their graduates have a profound understanding of fundamental mathematics.

Teachers are the most important people in the world. Without teachers we have no future.

References

National Committee on Excellence in Education, *A Nation at Risk*. USA Research, 1983.

Vivian Troen and Katherine C. Boles, *Who's Teaching Your Children: Why the Teacher Crisis Is Worse Than You Think and What Can Be Done About It.* Yale University Press, 2003.

Rick Norwood teaches at East Tennessee State University.

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elementary teachers. If I were to do that now, it would be a very different book. The more I work on this problem, the more I realize how little we in the collegiate mathematics community truly understand it. It's not simply a matter of incompetence, or of public vs. private education, or of limited resources, or of insufficient course hours, or of inadequate textbooks, although all these things affect it. It's a much deeper problem of communication and societal values, a problem that stubbornly resists any quick-fix proposals and demands far more patience, insight, and effort than we have been disposed to give it.

All this brings me to Prof. Norwood's final point, with which I wholeheartedly agree. Teachers — particularly elementary school teachers — hold the future of our children, our profession, and our world in their hands. Their needs must not be dismissed by some pejorative catch-phrase. If the MAA truly wants to contribute to solving this thorny problem, it needs to begin with some reflective humility. We may know what we want to tell teachers, but we don't yet know how to make them want to hear it. We don't know how to deliver our message in their language. We don't need more pronouncements or judgments; we need a clearer idea of the delivery question. If the MAA wants to form yet another committee, it might best be focused on figuring that out.

Bill Berlinghoff teaches at Colby College.

The Fifteenth ICMI Study: The Professional Education and Development of Teachers of Mathematics

The International Commission on Mathematical Instruction (ICMI) is pleased to announce its fifteenth Study, focused on the professional education and development of mathematics teachers around the world. A Discussion Document with full details of this ICMI Study can be found on the Study's web page at http://www-personal.umich.edu/~dball/icmistudy15.html. This announcement represents the first stage of the Study — the dissemination of the Discussion Document. The next two stages will be a Study Conference, to be held in Brazil on 15–21 May 2005; and the publication of the Study Volume, probably in 2007, containing a Report of the Study's achievements, products, and results.

The Study focuses on the initial and continuing education of teachers of mathematics and is organized in two main strands, each representing a critical cluster of challenges for teacher education and development:

Strand One: Teacher Preparation Programs and the Early Years of Teaching

This strand of the Study will examine a small set of important questions about the initial preparation and support of teachers in countries around the world, at the preservice stage and into the early years of teaching. These questions will center on aspects such as structure of teacher preparation, recruitment and retention, curriculum of teacher preparation (including issues related to diversity and subject matter preparation), early years of teaching, most pressing problems of teacher preparation, and history and change in teacher preparation.

Proposals for contributions to this strand should preferably come from a wide range of countries and should be based on systematically-gathered information and analyses.

Strand Two: Professional Learning for and in Practice

This strand of the Study adds substantive focus, complementing the first strand. It relates to teachers' learning across their life span: at the beginning of teachers' learning, during the early years of their work, and later, as they become more experienced. This strand's central focus is on two related and persistent challenges of teacher education: the role of experience in learning to teach and the divide between formal knowledge and practice. Both problems lead to the central question of Strand Two: How can teachers learn for practice, in and from practice?

Other questions asked in this strand include: What sorts of learning seem to emerge from the study of practice? In what ways are practices of teaching and learning mathematics made available for study? What kinds of collaborations are practiced in different countries? What kinds of leadership help support teachers' learning from the practice of mathematics teaching? What are the skills, practices, resources, and structures that support teachers' examination of practice? How does language play a role in learning from practice? What do teachers learn from different opportunities to work on practice?

Call for Contributions

The Study Conference will be a working meeting where every participant will be expected to be active. Participation will be by invitation only. Invitations will be extended based on submitted proposals, and total participation will be limited to approximately 120 people. Although research papers will be part of the program, substantial time will be set aside for direct engagement with artifacts and materials of practice, for critique and deliberation, and for collaborative work on significant problems in the field. With this in mind, proposals can be submitted which include demonstrations of practice; interactive work on common problems; program descriptions; and conceptual work. The Study explicitly welcomes contributions from individuals from a variety of backgrounds. Mathematicians and school practitioners are particularly encouraged to submit proposals for contributions. Participation from countries under-represented in mathematics education research meetings is also encouraged.

The Discussion Document gives full details for submission of proposals in one of three forms (papers, demonstrations, or interactive work sessions). Submissions should be sent before **October 15, 2004** to both of the Program Co-Chairs: Deborah Loewenberg Ball (dball@umich.edu) and Ruhama Even (ruhama.even@weizmann.ac.il). All inquiries concerning this Study should also be directed to both co-chairs.

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Profile: Kimberly Hopkins

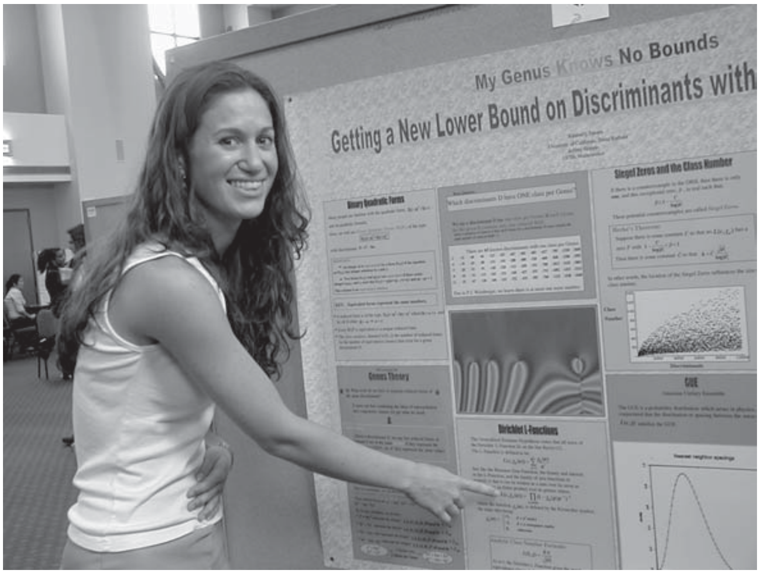
By Jeffrey Stopple

A couple of years ago, I received the following e-mail:

Professor Stopple,

My name is Kim Spears, and I am a sophomore here at UCSB. After you came to speak to our class, I began looking for an opportunity that would allow me to get involved in the "real world" of math. I came across a program called UC LEADS. It is a two-year program intended for those seeking a Ph.D. in math or science, and is formed around a core research project with a faculty mentor. I am interested in your field of study, number theory, and I would be delighted if you would serve as my mentor.

Kim began working with me in the summer of 2002, learning about positive definite binary quadratic forms. She later wrote about this time "We began fast," and I guess we did. I handed her Oesterlé's nice expository paper on the subject, and said to her "You *can* read French, can't you?". Despite her terror, she was game to try to do anything I asked. As we slowly went through the paper, she learned the way math is written is pretty much the same in French, and there are not that many new vocabulary words you need to know.



Kim presents her poster.

In the fall she began to learn genus theory from David Cox's book [5]. This is a beautiful subject with its roots in the work of Fermat, Euler, and Gauss. Binary quadratic forms $Q(x,y) = ax^2 + bxy + cy^2$ are in the same class if there is an invertible integer change of variables between them. Forms in the same class represent the same integers (i.e. have the same range as functions), share the same discriminant $-d = b^2 - 4ac$, and the classes form a finite abelian group. This last fact is quite deep. The order of this group is called the class number $h(-d)$.

Genus is a weaker equivalence relation than class; two forms of discriminant d are in the same genus if they represent the same integers modulo d. For example, the forms

$$x^2 + 14y^2 \text{ and } 2x^2 + 7y^2$$

both have discriminant –56. They must be in different classes since the first represents 1 via $(x,y) = (1,0)$, while the second clearly represents no nonzero integer smaller than 2. However they are in the same genus; the second form represents 57 via $2\cdot 5^2 + 7\cdot 1^2$, and $57 \equiv 1 \pmod{56}$. The other two classes of forms with discriminant –56 are given by $3x^2 \pm 2xy + 5y^2$. This discriminant has two genera, with two classes in each genus. In this example you can't tell whether a form represents a prime number simply by congruence conditions:

$$113 = 2\cdot 5^2 + 7\cdot 3^2$$
and
$$281 = 15^2 + 14\cdot 2^2,$$

but both primes are congruent to 1 modulo 56.

The special case in which there is only one class per genus is especially pretty. For example, there are two classes of forms of discriminant –20, namely

$$x^2 + 5y^2 \text{ and}$$
$$2x^2 + 2xy + 3y^2$$

An odd prime number $p \neq 5$ is represented by the first form if and only if it is congruent to 1 or 9 modulo 20. An odd prime number $p \neq 5$ is represented by the second form if and only if it is congruent to 3 or 7 modulo 20. Primes congruent to 11, 13, 17, or 19 are not represented by either form. (Try to work out some examples.) This is a nice generalization of the theorem of Fermat which says that an odd prime number p can be written as $x^2 + y^2$ if and only if it is congruent to 1 modulo 4. Discriminants $-d$ which have this property, one class per genus, and which also happen to be congruent to 0 modulo 4 are called "idoneal numbers," after Euler's work on the subject. Euler was interested in them because they gave him a simple test which let him produce (for his time) extremely large primes. See [5] or [6] for more on this.

During that fall and winter Kim and I met regularly, and I also gave her short explanations of the connections to analytic number theory described below. She took copious notes, and, at our next meeting the notes would be covered will little yellow Post-it stickies with questions which we went through and answered, one by one. Cox's book also got the Post-it treatment as Kim learned how one writes and reads mathematics. For instance, the example of the previous paragraph is introduced by Cox with the phrase "one easily computes that" This initially caused her a lot of grief. But here "easily" does not mean "without effort"; the example is a distillation of all the theorems covered up to that point, and eventually Kim was able to justify each of Cox's claims.

The UC LEADS program also included a lot of what Kim described as "Kumbayah events." At one such breakfast meeting, I showed up deliberately late to avoid the speeches. Unfortunately they were still going on so I went through the breakfast buffet line and filled my plate. As I sat down next to Kim I noticed that she had no food yet, and neither did any of the other hundred people in the room. I still get teased that I won't attend any meeting that does not include a meal. Later we discovered that if we sat in the back of the room, we could work on mathematics productively while the speeches went on over our heads.

A question inspired by Euler's work, and still open after 250 years, is to classify the discriminants $-d$ with one class per genus. There are 65 known examples which are fundamental discriminants, and it is known by work of Weinberger in 1973 that there is at most one more. Surprisingly, the existence of a 66th such would mean that the generalized Riemann hypothesis (GRH) is incorrect: a Dirichlet L-function would be forced to have a zero off the critical line, that is, with real part greater than 1/2.

The Riemann hypothesis is one of the Clay Mathematics Institute's million-dollar Millennium Prize problems, see [4]. The idea that the location of the zeros on a complex analytic function has a connection to subtle properties of prime numbers is one of Riemann's great contributions. More recently, number theorists have been looking at the distribution of the gaps between the zeros on the critical line. This is conjectured to be the same as the distribution of eigenvalues of random Hermitian matrices, the Grand Unitary Ensemble (GUE) probability distribution, which first arose in physics in modeling energy levels in quantum mechanics. The numerical evidence for the GUE distribution of the zeros is quite striking. It is important because it is the first evidence that the zeros might be spectral in nature. See [1], [2], [3], and [7] for more details.

Recently Brian Conrey and Henryk Iwaniec have proved a very nice theorem: assuming that sufficiently many gaps between zeros are smaller than the average (which is expected under GUE) they prove a very strong lower bound on the class number of positive definite binary quadratic forms.

I gave Kim the 1973 paper of Weinberger to read, as well as the Conrey-Iwaniec paper, and set Kim to working on the following problem: apply the Conrey-Iwaniec lower bound to the technique of Weinberger to get a theorem about discriminants with only one class per genus, assuming only GUE instead of GRH.

I expected this project would take a lot of hand-holding and fill up the rest of her career at UCSB. At that time we were meeting three times a week to discuss the project, and I wasn't saying much more than what's in the previous paragraph. After the third time, though, Kim just said to me, "I know enough to just go do this now, don't I?" And, to my amazement, a week later she was done. Assuming GUE, no discriminant greater than d_{66} (the smallest discriminant with 66 prime factors, approximately 10^{130}) has one class per genus.

By this time, Kim was learning to present her work in public. She gave a talk on her work in the abstract algebra course she was taking, and was surprised and a little disappointed to discover the class was not as interested in the subject as she was. She also gave a talk in our faculty seminar, and was surprised and pleased to learn that the faculty *were* interested. Poster sessions are a big part of the UC LEADS program, and Kim had to try to explain her work to an audience that knew no mathematics at all. Kim compensated for the stress by indulging in her sly sense of humor. The title of her poster was "My Genus Knows No Bounds."

Kim was selected by the Association for Women in Mathematics to receive the 2004 Alice T. Schafer prize (under her maiden name, Kimberly Spears). She also received UC Santa Barbara's own Chancellor's Award for Excellence in Undergraduate Research. This caused her to ask "How much do you want to pay me to say, "Thank you for choosing me as your next American Idol" at graduation while accepting the award?"

In the fall, Kim will begin a Ph.D. program in mathematics at the University of Texas, Austin as a Donald D. Harrington Fellow.

Bibliography

[1] B. Cipra, "Zeroing in on the Zeta Zeta Function" [sic], *Science*, **239**, 1988, pp. 1241–1242.

[2] B. Cipra, "Prime Formula Weds Number Theory and Quantum Physics," *Science*, **274**, 1996, pp. 2014–2015.

[3] B. Cipra, "A Prime Case of Chaos," *What's Happening in the Mathematical Sciences*, **4.**

[4] Clay Mathematics Institute web site, http://www.claymath.org/prizeproblems/.

[5] D. Cox, *Primes of the Form* $x^2 + ny^2$, John Wiley & Sons, 1989.

[6] G. Frei, "Leonhard Euler's convenient numbers," *Mathematical Intelligencer* **7**, 1985, no. 3, pp. 55–58, 64.

[7] A. Granville, "Prime possibilities and quantum chaos," at http://www.msri.org/ext/Emissary/EmissarySpring02.pdf.

[8] J. Oesterlé, "Le problème de Gauss sur le nombre de classes, *Enseignement Mathématique* (2) **34**, 1988, no. 1–2, pp. 43–67.

Business College Support of Mathematics for Business Decisions Motivates Students

By Richard Thompson

This has been an active and successful year for the MAA's new electronic texts, *Mathematics for Business Decisions*. We have showcased the program in nineteen workshops at national and section meetings and have finalized arrangements for translation of the e-texts into Spanish. Additionally, automated data and solution generators have been prepared to help instructors create interesting data sets for student projects.

Support From The Business College

In the midst of all this activity, one of the most significant aspects of the program has been an increase in student motivation arising from a continuing interdisciplinary relationship with our Business College at the University of Arizona. We have enjoyed some truly beneficial support from the Eller College of Business and Public Administration.

• While the program was under development, my co-author and head of the Finance Department, Christopher Lamoureux, walked across campus three times a week for an entire year to team-teach with me.

• A one-hour report on the program from four of our students was featured by the Business College at a presentation before their national board of business leaders.

• Members of the Mathematics Department are regularly invited to undergraduate Business College faculty meetings.

• We have been asked to set examination questions over *Mathematics for Business Decisions* that are used to screen business student candidates for upper division standing in their college.

• The Business College has made major, systemic changes in their statistics program and their upper division courses to use and build upon what students have learned in *Mathematics for Business Decisions.*

• Each semester the Business College organizes and runs a two-hour evening "kick-off" program for all *Mathematics for Business Decisions* students.

The last item merits some additional discussion. Last fall Pamela Perry, the Associate Dean of the Business College, welcomed our students.

Your math, analytical , and problem solving skills are the most significant take-away you'll get from our college.

Pamela Perry

Her theme was reinforced by comments from an upper division business student, Marisa Lucas, who had just completed a summer internship.

Marisa Lucas

I interned for Proctor and Gamble…Math for Business Decisions was very, very relevant.

Mathematical analysis is something we use literally every day.

Robert Lehner

The general meeting closed with comments from a Business College graduate and community leader, Robert Lehner. After the general kick-off program, students met with representatives from the Business College and their own instructors in the mathematics classrooms. The Business College leaders discussed the importance of working in teams and then conducted a training session on teamwork, using the students' own class teams.

Motivated Students

Given the high level of Business College support and the positive feedback that they receive from upperclassmen, most students are ready to "buy into" *Mathematics for Business Decisions.* The e-texts and the basic structure of the program provide the final steps in getting students to link mathematics and business.

Work is centered around two major business decision projects per semester. Mathematical and computer topics are presented as tools allowing student teams to formulate quantitative results that lead to sound business decisions. Knowing that the mathematics behind two of the four projects won Nobel prizes in Economics during the last ten years, students recognize that they are doing real world business. At the end of each project, teams present their conclusions in both oral and written reports to the class.

What do we see that indicates a high level of student involvement and belief in the value of the program?

• A tradition of dressing in suitable business attire for oral reports has evolved spontaneously from the students in our classes.

• While studying our *Loan Work Out Project*, students from several teams visited local banks and interviewed loan officers to see how our course work corresponds to the real world. (Happily, they found out that their project actually does represent what happens in the banking business.) This was done without any faculty suggestions or directions.

• Written comments on student evaluations are much more positive than was the case with our former traditional courses.

"Finally a math class has been created that has useful applications towards the future."
A student in Part 1

• Instructors often comment that, *"No one in Mathematics for Business Decisions* has ever asked, '*How will this be used in business?*'"

• A team of former *Mathematics For Business Decisions* students from the University of Arizona's nationally top ranked McGuire Entrepreneurship Program has selected our e-texts as the basis for a start-up business project. They plan to attract local venture capital to market our material for on-line corporate and executive training.

Students report in a Mathematics Decisions Class, in a scene from a video on the MBD program.

How Can You Make This Happen At Your School?

Support for your service courses from client departments or colleges, and the resulting energized and motivated students, lead to successful and rewarding teaching. Experience at the University of Arizona and at other institutions that are using *Mathematics for Business Decisions* has identified several key components of this achievement.

• **Find a few interested faculty members in the client unit and go to see them in their offices.** To make contacts, ask for recommendations from the unit's administrator, contact an undergraduate associate dean or head, or visit the chair of an undergraduate curriculum committee.

• **Ask about the mathematics that the majority of undergraduate students in the client area will actually need for their upper division work and in their future careers.** Do not tell your contacts what mathematics or technology they **should** be using. Ask them for real examples from their fields and show them that mathematics can add value for their students.

• **Use instructional materials and technology that model what students are seeing in their own field.** Do not rely on what mathematicians have traditionally believed are applied problems in the given area. Find out what problems they actually work on and what tools they use.

• **Learn and use the language and terminology of the client discipline with your students.** Using traditional mathematical topics, terminology, and methods that students do not encounter in their major departments will not make students into mathematicians. It will only cause them to disconnect from, and ignore mathematics.

The *Mathematics for Business Decisions* project is proof that interdisciplinary contacts can be established and that students can be motivated. The main stumbling block in achieving this goal is our own reluctance to venture into new territory and think in novel ways. Break this barrier and reach out to your client departments and colleges. The rewards will be huge.

New Jersey Section Hosts First Garden State Undergraduate Mathematics Conference

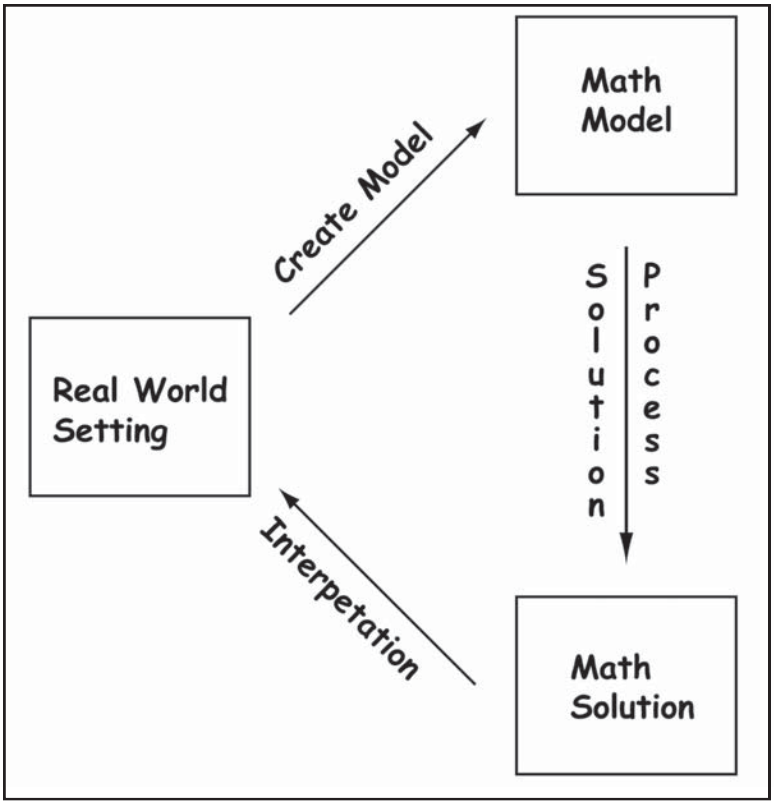
The New Jersey Section of MAA successfully hosted the first-annual Garden State Undergraduate Mathematics Conference (GSUMC) on March 27, 2004 at Rutgers University, New Brunswick, NJ, which was held jointly with its spring meeting and funded in part by MAA-NSF-RUMC Grant DMS-0241090. Approximately 100 students in attendance participated in many activities: student talks and posters presentations, New Jersey Undergraduate Mathematics Competition (NJUMC), workshops discussing graduate school and careers in industry and K-12 education, and the giving away of many door prizes. Keynote speaker Professor John H. Conway from Princeton University capped off the Conference with a marvelous talk enjoyed by all. Over 20 teams from 12 schools participated in NJUMC, with the University of Scranton, PA, taking home the prize for first-place. Complete results of the Competition, including photos and highlights from the Conference, are available at: http://www.rowan.edu/mars/depts/math/maa-nj/gsumc.html.

Watch out for GSUMC 2005!

Teaching Calculus via the Modeling and Inquiry Problem

By Mike Huber

For years, the differential calculus midterm exams given to freshmen students in our single-variable calculus course contained four pages. Two pages were devoted to assessing students' abilities to compute derivatives. The other two pages asked the students to solve an optimization problem and a related rates problem. Students were very successful in determining the various derivatives, as they were allowed to use their HP-48S or TI-89 calculators, but the majority struggled with the two "applied" problems. Overall, the strength of their rote differentiation skills enabled them to pass the exam. The same was true for the integral calculus midterm. Students could find antiderivatives and evaluate integrals using their calculators, but they could not always apply those skills and determine the work needed to lift an object with a variable force applied or accurately determine the distance a vehicle traveled, given a table of velocities and times.



The problem-solving/modeling process.

What to do? One of the missions of our department is to develop students into confident and competent problem solvers. Many textbooks give students the model and ask them to solve it. Students were comfortable with their "Plug and Chug" method of solving, as long as they knew what and where to plug. However, when confronted with a word problem and asked to develop the model, students struggled.

Two years ago, as an experiment, we gave a group of 200 students an optimization problem in the form of a long word problem and asked them to set up the model (define variables, state what is given, make a few reasonable assumptions, write down what they were asked to determine, and explain the technique they would use to solve it). We told them NOT to solve it, unless they had time. Afterwards, we asked the students to explain their problem-solving process. The Modeling and Inquiry Problem (MIP) was born.

The diagram describes a process in which students must transform a problem from a real world setting into a math mode, solve that model, and then interpret the results in the real world scenario. Instead of giving the students the model or algorithm and asking them to simply substitute numbers into the equation to find an answer, we began to ask students to develop the model itself.

Over the last four semesters, we have made a conscious effort to teach the students to become problem solvers. Creating the model and interpreting the solution are just as important — maybe more important — than simply finding a solution. Today's computer algebra systems can solve most traditional differential equations, take derivatives, and determine integrals. Knowing what to enter as the equation to be solved becomes the critical task. Making valid assumptions to simplify the model is stressed in the classroom. Asking, "So what?" or "Is my answer reasonable?" leads students into the interpretation stage. Writing is incorporated into the process by requiring students to explain their solutions in the context of the problem. The vertical side of the triangle (Solution Process) becomes the least important one. We provide the students a structure for setting up a model and assess the students with MIPs. An example of a MIP (related to applications of the integral) is:

An electric elevator with a motor at the top has been designed for a 15 story building. The elevator has a multi-strand cable weighing 4.5 lb/ft and the elevator car weighs 2000 lbs. It will take the elevator 70 seconds to go from the 1st floor to the 15th floor. Power is the rate of change of doing work per time. Electric elevator motors are rated using horsepower. Horsepower can be found using the following formula:

$$Horsepower = \frac{Power\left(\frac{ft-lb}{\min}\right)}{33000}$$

How much horsepower is required to lift the elevator from the first to the fifteenth *floor? The designers of the building planned to install a 20 horsepower motor to lift the elevator up the 15 stories. You feel the elevator should be able to hold at most 10 people. Do you think the designers provided a motor that has enough horsepower to lift the elevator with 10 people in it?*

In this problem, students have to make some critical assumptions, such as how much each person weighs, what the distance between floors might be, etc. The time needed to travel is irrelevant. This brings realism to the problem. The equation for horsepower is given, yet students must first determine the power. Finally, they must recognize that this problem contains fixed and variable forces and they must find the work for both before determining the power and horsepower.

Lessons are geared to solving groups of applied problems. Implicit differentiation is introduced as students try to solve related rates problems. Maximum/minimum ideas are learned while solving actual optimization problems. Eigenvalue/eigenvector and matrix algebra skills are introduced as students try to solve harmonic oscillation problems. Each block of material (related rates, optimization, etc.) culminates in a MIP. A possible drawback is that only one of each type of problem can be assessed. However, this drawback existed with the previous midterms, and we believe that many more possible applied problems can be introduced through the technique of "what if-ing". In addition, students are now writing about their mathematics. The discussion section of the MIP is not simply an answer that is double-underlined. It is the student's attempt at answering an applied problem and explaining the results to a "customer".

Student and instructor feedback has been very positive. Two years of experience has shown that students are more apt to really learn the mathematics when they can apply that math to solve problems. Surveys have revealed that once students sense that the lessons and assessments focus on problems which they might encounter in the real world, a metamorphosis occurs. The students feel that classes are more interesting. They also believe that their confidence levels rise when using mathematics to solve problems which surround them in life, because they have a feel as to whether the answer makes sense. They also are beginning to understand that their reasoning skills are as important as their analytic skills. If they are confident in their model, and they obtain an answer that just does not make sense, they receive partial credit for explaining what their intuition tells them should be the answer. For instance, "When I solved for the power, my answer was negative, which cannot be correct. The work performed, power, and horsepower should be positive."

We see more sessions at the annual mathematics meetings on teaching/assessment using modeling or problem-solving every year. Several other professors across the country are getting their students to model and solve word problems, and others are having great success in getting students to write about their mathematics. The MIP concept combines the best of both ideas.

Mike Huber teaches at the United States Military Academy. *He would like to acknowledge feedback from Professor Don Small on this article.*

2004 Award Winners for

Distinguished Teaching

Making Connections
To, From, and Within the Mathematical Community

By David Bressoud and Lynn Arthur Steen

Readers of FOCUS may know that the MAA is the world's largest professional society whose primary goal is, in the words of its mission statement, "to advance the mathematical sciences, especially at the collegiate level." Publications, both print and electronic, are a major means by which this mission is fulfilled. Ever since the end of WWII, when MAA established CUPM (the Committee on the Undergraduate Program in Mathematics), an important strand of the Association's publications has been a series of analyses and recommendations concerning curriculum and program options for collegiate mathematics.

This year MAA is publishing four reports dealing with the undergraduate curriculum, all of which emphasize the theme of "making connections:"

- *Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004.*
- *Curriculum Foundations Project: Voices of the Partner Disciplines.*
- *Achieving Quantitative Literacy: An Urgent Challenge for Higher Education.*
- *Math & Bio 2010: Linking Undergraduate Disciplines.*

The first three volumes were published early in 2004; the fourth will be available in the fall. (All are available for purchase on the web at http://www.maa.org or by phone at 1-800-741-9415.)

These reports emphasize three broad categories of *connections:*

- Connections to and from other disciplines;
- Connections within the mathematical sciences;
- Connections to the needs of students served by mathematics departments.

CUPM Curriculum Guide 2004

The *CUPM Curriculum Guide 2004 (Guide 2004)* is the sixth major curriculum report produced by CUPM during the last fifty years. *Guide 2004* is intended to apply to all of the mathematical sciences, including pure and applied mathematics, mathematics education, computational mathematics, operations research, and statistics. Unlike earlier CUPM reports, this guide addresses the entire college-level mathematics curriculum for *all* students, even those—the majority—who take just one course. It focuses less on prescribing what departments should teach and more on setting goals for student understanding and envisioning how departments can achieve those goals.

Curriculum suggestions and illustrative examples in *Guide 2004* are anchored in six core recommendations:

- Understand the student population and evaluate courses and programs in light of this understanding.
- Develop mathematical thinking and communication skills.
- Communicate the breadth and interconnections of the mathematical sciences.
- Promote interdisciplinary cooperation.
- Use computer technology to support problem solving and promote understanding.
- Provide faculty support for curricular and instructional improvement.

These recommendations are elaborated in terms of various student constituencies:

- Students taking general education or introductory collegiate courses in the mathematical sciences;
- Students majoring in partner disciplines, including those preparing to teach mathematics in elementary or middle school;
- Students majoring in the mathematical sciences (in general);
- Students majoring in the mathematical sciences with specific career goals such as secondary school teaching, the non-academic workforce, or post-baccalaureate study in the mathematical sciences and allied disciplines.

A sequel to *Guide 2004, CUPM Illustrative Resources*, is a web-based resource (still under development) containing examples, experiences, and resources. Contributions to the *Illustrative Resources* are welcome, and should be sent by e-mail to cupm@maa.org.

Guide 2004 is intended to be used as a tool for departments that want to find ways of improving their undergraduate curriculum. Its recommendations rest on the premise that excellence is achieved by focusing on the outcomes we want and tailoring the program to the specific needs of *our* students within the context of *our* institutions. *Guide 2004* intentionally avoids prescribing how to achieve the goals that it describes. Instead, it focuses on clarifying what is meant by these goals and illustrating how they are being met at a variety of colleges and universities.

The starting point at each institution must be connecting to students at that institution and helping them see how their interests are connected to what the mathematical sciences can offer. This is a theme that runs through all *Guide 2004* recommendations. It is explicit in the first recommendation—to understand the student population and assess the effectiveness of programs and courses in light of this understanding. It is implicit in the decision to tailor the recommendations to specific student audiences, recognizing that not all students come to mathematics with the same needs. The third and fourth recommendations—to communicate the breadth and interconnections of the mathematical sciences and to promote interdisciplinary cooperation— are really about enabling students to see mathematics as an exciting, dynamic field that lies at the core of the entire undergraduate curriculum.

Helping students see connections within mathematics is no less important than helping them understand connections to other disciplines. Mathematics is rich and varied and draws its strength precisely from the multiplicity of ways in which mathematical concepts can be viewed. *Guide 2004* emphasizes the need to balance and connect contrasting but complementary points of view such as continuous and discrete, algebraic and geometric, deterministic and stochastic, and theoretical and applied.

Connections to other disciplines are encouraged on many levels. Within individual classes, examples should be realistic and draw on a large variety of other disciplines. Interdisciplinary programs and team-taught courses are recommended. Fundamental to all these connections is regular, continuing dialog with other departments and disciplines, helping them to understand the mathematical needs of their majors and how these needs can be met by the mathematics department.

Both *Guide 2004* and its sequel are available for downloading from the web:

> *CUPM Curriculum Guide 2004:*
> http://www.maa.org/cupm
> *CUPM Illustrative Resources:*
> http://www.maa.org/cupm/illres_refs.html

Curriculum Foundations Project

As background for CUPM's curriculum study, its subcommittee on Curriculum Renewal Across the First Two Years (CRAFTY) conducted a series of workshops with numerous partner disciplines such as biology, chemistry, economics, and engineering. Each workshop produced a brief report directed to the mathematics community summarizing the workshop's recommendations and conclusions concerning the mathematical needs of these disciplines. (While mathematicians were present at each workshop to listen and seek clarifications, they did none of the writing.) A summary conference distilled a common set of recommendations, called "Collective Vision," that synthesizes these "voices of the partner disciplines."

The workshop reports and synthesis recommendations have been published in the volume *The Curriculum Foundations Project: Voices of the Partner Disciplines* which is available both on-line and for purchase:

The Curriculum Foundations Project:
http://www.maa.org/cupm/crafty/.

This report provides rich resources for multi-disciplinary discussions on individual campuses about connections between the undergraduate mathematical sciences and the many partner disciplines.

Achieving Quantitative Literacy

Quantitative literacy (QL) — the ability to use simple mathematics in the context of sophisticated problems — is very much about making all three types of connections. QL is inherently interdisciplinary. Although QL is anchored in mathematics, it does not comfortably fit within a mathematics department. QL is by nature contextual and increasingly pervades many other disciplines.

QL connections within the mathematical sciences are often overlooked, but nonetheless important. Quantitative literacy is dominated by the interplay of deterministic and stochastic models. Ideas that are opaque to most people when presented algebraically become transparent when communicated geometrically. Those who would teach quantitative literacy must be able to draw on the wealth of connections within mathematics to help their students learn how to tap into its power.

Above all, quantitative literacy is about connecting mathematics to the real needs of students. All aspects of students' lives are increasingly affected by quantitative skills, from personal finance to public policy, from course prerequisites to employment opportunities. Connecting quantitative literacy to a curriculum dominated by traditional disciplines is one of the major challenges facing undergraduate education.

In recent years, three reports prepared under the auspices of the National Council on Education and the Disciplines (NCED) have unfolded many issues associated with enhancing the role of quantitative literacy in the education of undergraduate students:

- *Mathematics and Democracy: The Case for Quantitative Literacy* (2001)
- *Quantitative Literacy: Why Numeracy Matters for Schools and Colleges* (2003)
- *Achieving Quantitative Literacy: An Urgent Challenge for Higher Education* (2004)

All three volumes are available from the MAA (at http://www.maa.org or 1-800-331-1622). Later in 2004 some of the chapters will be available on-line for browsing or downloading.

The case for QL in these volumes is argued on the grounds of informed citizenship, personal well-being, and career requirements. Among other things, contributors to these studies conclude that:

- Most students finish their education ill prepared for the quantitative demands of informed living.
- QL is largely absent from our current systems of assessment and accountability.

- Faculty in all disciplines need significant professional support in order for them to enhance the role of quantitative literacy in their courses.

Many institutions are addressing the challenge of quantitative literacy through a variety of innovative programs such as designated Q-courses, "mathematics-across-the-curriculum" programs, interdisciplinary core programs in science, or coordinated campus-wide programs in quantitative aspects of public policy. All these programs rely on building connections to and from mathematics and other disciplines.

Math & Bio 2010: Linking Undergraduate Disciplines

Of all undergraduate disciplines, the one whose connection to mathematics has been most radically transformed in the past decade is biology. The decoding of the genome has catapulted biology and much of medical research into a quantitative science whose fundamental understandings now rely on mathematical tools—many of which have not yet even been developed. This momentous shift is profiled in an influential report from the National Research Council *Bio 2010: Transforming Undergraduate Education for Future Research Biologists* (2002).

Following publication of *Bio 2010*, MAA helped organize a workshop on the theme *Meeting the Challenges: Education Across the Biological, Mathematical, and Computer Sciences* to explore how best to strengthen the relatively weak connection between mathematics and biology in students' education. Whereas formerly many students viewed biology as a safe harbor from the rigors of mathematics, today's biology requires that students be proficient in a wide range of mathematics including dimensions and scaling, rates of change, limited and unlimited growth, linear and nonlinear behavior, discrete and continuous systems, probability distributions, variance, statistical inference, and confidence intervals. Moreover, to comprehend and explore genetics, students need some familiarity with computer algorithms and tools of bioinformatics.

Bio2010 argues that biology majors need to be educated "in a more quantitative manner" and that this kind of quantitative education may require "new types of courses." Introductory courses especially play a major role in shaping student attitudes towards interdisciplinary concepts. Participants in the follow-up workshop identified numerous challenges, including:

- Biology student backgrounds and potential career options are becoming increasingly diverse.
- The kinds of mathematical knowledge biology students need for their future careers is not yet well understood.
- A major change in academic culture will be required to support necessary cross-disciplinary initiatives in faculty development.

Departmental boundaries need to be bridged with permanent institutional structures (not just isolated courses) if the needed connections are to thrive in both faculty and student culture.

The MAA has created a web page (http://www.maa.org/mtc) with links to a variety of papers dealing with these challenges. A printed report from the workshop entitled *Math & Bio 2010: Linking Undergraduate Disciplines*, including samples of current undergraduate projects connecting mathematics and biology, is being published by MAA in fall, 2004.

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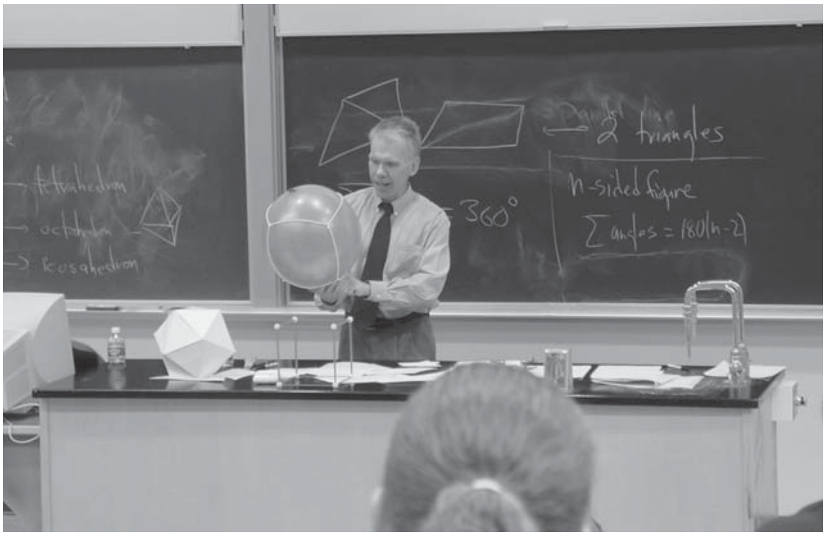
Lynn Arthur Steen is Professor of Mathematics and Special Assistant to the Provost at St. Olaf College. He is editor of the NCED reports on quantitative literacy and a former president of the MAA.

First Annual Pikes Peak Regional Undergraduate Mathematics Research Conference

By Hortensia Soto-Johnson and Mike Brilleslyper

The First annual Pikes Peak Regional Undergraduate Mathematics Research Conference, funded by MAA NSF Grant DMS-0241090 was held on February 28, 2004 at Colorado State University-Pueblo. With the beautiful front range of the Rocky Mountains as a backdrop and 76 people in attendance from 12 different institutions, the conference was extremely successful. The one-day program included a keynote speech, 18 student research presentations, and a panel presentation on graduate school experiences in mathematics.



Ron Tubbs illustrating *"Presidential Doodles and Geometric Perfections."*

The conference began with a keynote address, *"Presidential Doodles and Geometric Perfections,"* by Professor Rob Tubbs of the University of Colorado. The lively, entertaining, and accessible talk made connections between regular 3-D polyhedra and 2-D graphs. As one student said, "He had a neat presentation that I understood. He kept me engaged during the entire speech."

Two parallel sessions of student presentations ran in the morning and afternoon. There were a variety of topics, including history of mathematics, knot theory, geometry, and applied math. The breadth of the presentations was impressive, as was the backgrounds and diversity of the presenters, who came from a wide range of Colorado institutions.

The panel, called *"Planting the Graduate School Seed,"* was by far the most popular component of the conference. RaKissa Cribari (University of Northern Colorado), Trent Kull (Colorado State University), Sheila Miller (University of Colorado), and Leslie Varys (University of Colorado - Denver), all graduate students, served on the panel. They discussed why they chose to attend graduate school, their experiences so far, and how well they felt their undergraduate education had prepared them. Their personal stories provided a rare glimpse of the true rewards and struggles that go into pursuing a degree in higher mathematics. In addition, they discussed graduate school opportunities, the choice of an advisor, plans after graduate school, and other general advice for prospective mathematics graduate students. Several students commented how much they enjoyed the panel. "It was enlightening to hear from students who are actually doing what graduate students do as well as 'tips and tricks' to know beforehand."

A total of 76 people attended the conference: 18 presenters, 34 student spectators, 20 faculty, and 4 graduate students. Of the 57% of participants who completed a questionnaire, 90.7% of the participants were Caucasian, 2.3% were Asian, and 7% were Hispanic. According to the completed questionnaires, 7% of the students were freshman, 18.6% were sophomores, 23.3% were juniors, and 30.2% were seniors. Approximately 72% of the students were math majors. Many of the students were double majors in areas such as art, biology, business, computer science, physics, and engineering. Approximately 42% of the students were members of a math club and 23% were members of the MAA. Of the people who completed the questionnaire 47% said they planned on attending graduate school.

Overall the participants were very positive towards the organization of the conference and a majority of the students were interested in attending or presenting at future conferences. One hundred percent of the respondents felt the conference was well organized and 80% said they would be willing to present at future conferences.

The success of the first annual conference gives us confidence for continued success. Southern Colorado covers a large region of the state and contains a diverse group of colleges and universities. The participants made many suggestions for the next time around, and we plan to follow many of these suggestions for the second annual conference, which is tentatively scheduled at Colorado College in Colorado Springs. The continued support and enthusiasm of faculty and students should make the Second annual conference even more successful.

Hortensia Soto-Johnson teaches at the Colorado State University-Pueblo. Mike Brilleslyper teaches at the United States Air Force Academy.

Is Mathematics Now Throughout the Curriculum? Reflections on an MATC Project

By Daniel P. Maki, Marc Frantz, Bart S. Ng, and Ted Hodgson

During the late 1980s and early 90s, several National Science Foundation (NSF) initiatives sought to revitalize undergraduate mathematics education. One of these initiatives, Mathematical Sciences and Their Application Throughout the Curriculum (MATC), targeted non-mathematics majors and arose in response to two factors:

(1) The belief that success in a technological workforce requires an understanding of mathematics and an ability to apply it in practical situations;

(2) The concern that non-majors were unable to apply the mathematics they were learning in the classroom to settings outside the classroom.

Sponsored in 1994 by NSF's Division of Undergraduate Education, MATC-funded projects were to promote comprehensive improvements in undergraduate education that would integrate the mathematical sciences into other disciplines; improve mathematics instruction through the incorporation of multidisciplinary perspectives; and facilitate the collaboration of mathematics faculty with those in other disciplines.

Faculty members from the eight-campus Indiana University (IU) system responded to MATC by creating the Mathematics Throughout the Curriculum (MTC) project. Funded in 1996 by NSF and IU Strategic Directions Initiative, MTC supported the development of courses designed to connect mathematics to other disciplines, the real world, and students' ambitions and goals. Other MATC recipients included the University of Pennsylvania, Rensselaer Polytechnic Institute, Dartmouth College, a University of Nebraska/Oklahoma University consortium, the State University of New York, and the U.S. Military Academy. Although our five-year NSF grant was to expire in 2001, MTC continues to operate under a three-year, no-cost extension.

Students in the MTC course Catch the Wave to Calculus *display the results of their end-of-course project.*

The activities of the MTC project have been widely disseminated. Most of these dissemination efforts, however, simply describe activities or products of the MTC project. The initial funding impetus for MTC (NSF-MATC) no longer exists, yet there continues to be interest in interdisciplinary mathematics instruction (e.g., Bénéteau & Rohrbach, 2004) and the activities of MTC. As MTC funding draws to a close, therefore, we would like to share some of the lessons we have learned about interdisciplinary teaching and explore the larger question of whether or not mathematics is now throughout our undergraduate curriculum.

Accomplishments and Lessons

MTC supported the development of 24 interdisciplinary courses, of which half became institutionalized and are regularly offered on IU campuses. To ensure that MTC courses would be transportable, we promoted the creation of permanent course records — texts, CDs, workbooks, and teacher's materials — which have been disseminated through workshops and conferences and adopted by colleges and universities across the country.

One of our goals was to change students' beliefs about the importance and usefulness of mathematics. To determine whether this occurred, external evaluators conducted interviews, observed MTC classes, and administered pre- and post-course questionnaires to both MTC and traditional courses. In general, the evaluation data indicate that MTC courses positively affect students' beliefs. In contrast to students in traditional courses, for example, MTC students exhibited significant improvement in four critical areas, including perceptions of the personal usefulness of mathematics and its usefulness in other disciplines (Amarasinghe, 2000). Moreover, follow-up interviews indicate that students translated their beliefs into action. We received numerous reports of students pursuing additional studies in mathematics, for instance, even though they were not required to do so.

The number of courses that have been developed, widespread use of course materials, and documented impact on students' beliefs all attest to MTC's success. We have found, however, that interest in the project extends beyond typical measures of success. Below, we describe a course as "successful" if it was well received by students and faculty and was weaned of MTC support to become a self-sustaining, permanent addition to the IU catalog.

Given its focus on interdisciplinary courses for non-majors, there is tremendous interest in the logistics of MTC course and their underlying value. How does one start an interdisciplinary course? How does one keep it going? How does one overcome the obstacles that are present on every college campus? How does one manage the workload of an interdisciplinary course and is the work worth the effort? Although we won't address each of these questions, here are a few of the lessons that we've learned in our eight years with MTC.

Course Structures

The natural tendency is to focus on characteristics of the courses themselves. For instance, we are often asked if there is a "best" model for interdisciplinary instruction. The MTC project actually experienced success with several distinct models. The most common model, illustrated by our *Mathematics in Art* course, is the "blended" course. Concerned that art and design students' experience with university mathematics was disconnected from their world, Annalisa Crannell (Mathematics) and Marc Frantz (Art) infused mathematics into the study of standard artistic techniques. Their course is a mathematics course, yet students' understanding of mathematics develops through hands-on exploration of artistic content. The defining feature of the course, as with all blended courses, is balance between mathematics and another content area. Assessment focuses on students' achievement in both content areas and the teaching team shares instructional responsibilities.

Projects also play an important role in MTC courses. The prototype course, *Analytic Problem Solving* (APS), combines classroom instruction with out-of-class projects that require students to consult with local businesses to solve real problems. Developed by Dan Maki (Mathematics) and Wayne Winston (Business), APS focuses on developing the mathematical tools that are needed to complete each course project. Although APS tests students' ability to use these tools, the primary objectives are successful completion of the project and an understanding of the role that mathematics plays in real businesses.

Questions about a "best" instructional model, therefore, really don't address the heart of the matter. Many successful models for interdisciplinary instruction exist.

Institutional Settings

A second common question centers on the course setting. Specifically, are interdisciplinary courses best suited to liberal arts colleges, comprehensive universities, or research universities? The MTC project was active on six of the eight IU campuses, implementing courses at large and small universities, research universities and teaching colleges, and in urban and suburban settings. Although not an original MTC goal, the project served as an informal testing ground for questions about the "ideal" interdisciplinary setting.

While these were not the only important factors, we found success to be dependent on setting and content. As an example, Kochanowski and Shafii-Mousavi (2000) adapted the prototype APS course for use in an introductory finite mathematics course. Critical to the success of their course, which is project-based, and technology-rich, is the small class size — typically about 20 students. In many research universities, introductory courses are synonymous with large lectures and multiple sections. While MTC promoted reforms in IU's research universities and large lecture classes, our most successful introductory courses were implemented in settings with low student-to-teacher ratios and support for innovative teaching. An additional factor affecting the success of Kochanowski and Shafii-Mousavi's course is its urban/suburban setting, which provides easy access to businesses and a basis for course projects.

With regard to advanced or specialized courses, we have experienced the most success on IU's large and diverse research universities. *Mathematics and Statistics in Journalism* is well received on the Bloomington campus, for instance, in part because its active journalism program provides demand for the course and a steady source of students. Likewise, *Mathematics in the Speech and Hearing Sciences* (Kehle, Kewley-Port, and Eddins, in press) prepares upper division Speech and Hearing students for the mathematical rigors of graduate school in their field. The course succeeds because it satisfies the needs of a substantial number of students. Not surprisingly, courses involving undergraduate research prosper on research campuses. In general, the characteristics of a successful interdisciplinary course mirror the strengths and priorities of its institutional setting.

Costs and Benefits

The most common question we receive concerns the workload of an interdisciplinary course. All agree that interdisciplinary instruction requires more work than a traditional class. Is the work worth the effort? MTC course development teams spent countless hours preparing materials, testing them, and implementing them in an appropriate pedagogical setting. One of our findings, not surprisingly, is that successful interdisciplinary courses require a committed faculty. Convinced that all journalists should write clearly and accurately about quantitative information, for example, journalism professor Paul Voakes committed himself to our *Mathematics and Statistics in Journalism* offering. The success of the course is a testimony to Voakes' commitment and that of mathematics collaborator Chuck Livingston.

Some MTC faculty, on the other hand, found that interdisciplinary courses are not worth the effort. One professor concluded that his course required "too much work for too few students." Other faculty members observed that students didn't seem to appreciate the effort that was being expended on their behalf. Despite varying views of the value of interdisciplinary courses, however, nearly all faculty members speak positively about their MTC experience. The collaborative nature of MTC allowed mathematics faculty to build relationships with faculty in other disciplines, at other four-year institutions, and with members of the high school teaching community (who often attend MTC workshops). In interview after interview, faculty expressed newfound understanding and respect for colleagues in other disciplines.

Is Mathematics Now Throughout the Curriculum?

Our MTC proposal sought to change the culture of mathematics teaching on the eight participating campuses. While some changes have occurred, the culture is not radically different than it was when we began our project. To say that MTC had no impact on the culture, however, would also be misleading.

In a recent article on university mathematics reform, Steen (2000) identifies a growing trend in which mathematics is being unobtrusively embedded in routine courses in other subjects. With regard to our culture of mathematics teaching, we believe MTC has had the same effect. While the number of permanent courses is smaller than we had hoped, MTC resulted in several new and numerous revitalized courses. In addition, one course (*The Mathematics of Finance*) has prompted the development of an entirely new master's program. MTC materials continue to be used in departments across the IU campuses — and on campuses that "adopted and adapted" MTC materials.

The relationships fostered by MTC represent another permanent change. Faculty members in other disciplines, for instance, now know that they have colleagues in the mathematics department with whom they can talk. Likewise, mathematics faculty developed an appreciation for the extent that mathematics is used and taught in other disciplines. MTC facilitated subtle changes in the curriculum and thinking of our faculty so that the presence of mathematics on the IU system is greater now than it was in 1996.

Math in Art *students use window taping to explore the mathematics of perspective.*

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References

Rajee Amarasinghe. *A Study of Student Attitudes and Beliefs When Learning Introductory College Mathematics in Context*. Indiana University: Ph.D. Thesis (May, 2000).

Catherine Bénéteau and June Rohrbach. "Statistics, Technology, and the Social Sciences: A Successful Interdisciplinary Project." FOCUS, 24: 1 (January 2004) 9-10.

Paul Kehle, Diane Kewley-Port, and David Eddins. "An interdisciplinary project-based approach to teaching the mathematical foundations of speech and hearing sciences." *Journal of the American Academy of Audiology* (in press).

Paul Kochanowski and Morteza Shafii-Mousavi. "How to design and teach a project-based first-year finite mathematics course." *The UMAP Journal*, 21: 2 (June, 2000) 119-138.

Lynn Arthur Steen. "Revolution by Stealth: Redefining University Mathematics." In Derek Holton (Ed.), *Teaching and Learning of Mathematics at the University Level. Proceedings of the ICMI Study Conference, Singapore, 1998.* Dordrecht: Kluwer Academic Publishers (2000).

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MSEB Releases Report on Evaluating Mathematics Curricula

Choosing a school mathematics curriculum is a difficult task that continues to generate controversy. Inevitably, the question of how such curricula should be evaluated becomes very important. The Mathematical Sciences Education Board (MSEB) of the National Research Council has just released a report discussing how mathematics curricula should be evaluated. The report sets a very high standard for proper evaluation of curricula.

The need to evaluate curricula is a serious one. For the most part, curricula are chosen by local school boards, who often do not feel competent to accurately assess the quality and effectiveness of materials. Several years ago, the Department of Education released a report comparing the effectiveness of several middle and high school curricula. The report generated controversy (see the December 1999, January 2000, and February 2000 issues of FOCUS), much of which highlighted the "how do you know?" question. While the report was based on some evidence of the effectiveness of curricula, the quality of that evidence was called into question.

More recently, the "No Child Left Behind Act" has upped the ante by requiring that schools "use effective methods and instructional strategies that are based on scientifically based research." (The phrase "scientifically based research" is used repeatedly in the act, and is defined in Section 9101. It does not refer specifically to evaluating curricula.) The MSEB report attempts to *define* what "scientifically proved to be effective" means when applied to mathematics curricula.

The committee looked at 6 commercial mathematics curricula and 13 curricula that were funded by the National Science Foundation. It then tried to assess whether the available data was sufficient to evaluate the effectiveness of these materials. If not, the committee was asked to make recommendations on how it might be possible to obtain better data.

After analyzing 147 studies of curricular effectiveness, the committee concluded that they do not permit one to determine with a high degree of certainty the effectiveness of any of the 19 programs. Given these inconclusive findings, the committee went on to develop recommendations for future studies. In particular, they recommend that "a curricular program be designated as *scientifically established as effective* only when it includes a collection of scientifically valid evaluation studies addressing its effectiveness that establish that an implemented curricular program produces valid improvements in learning for students, and when it can convincingly demonstrate that these improvements are due to the curricular intervention." The report goes on to detail what such studies should look like.

The report sets a very high standard for evaluating curricula, recommending a multi-pronged approach that includes content analyses, comparative studies similar to those used in biomedical research, and case studies. Lynn Steen points out that "The standards that the committee set for future evaluations to meet the test of 'scientific validity' are generally sound, although taken together they pose hurdles that are both administratively and financially impossible to achieve." Ethical questions have also been raised. If one believes that a curriculum is inferior, is it ethical to subject children to that curriculum in order to produce a comparative study that demonstrates its inferiority? After all, children are not in a position to volunteer to participate.

Experts are certain to discuss the details of the report, and the Department of Education will need to decide whether the requirement that curricula be "based on scientifically based research" requires that schools undertake the kind of evaluations recommended in the report.

The report was produced by a committee chaired by Jere Confrey of Washington University in St. Louis. The study director was Carol Lacampagne. The MSEB report can be purchased from the National Academies Press or read online: http://www.nap.edu/catalog/11025.html. The No Child Left Behind Act is online at http://www.ed.gov/policy/elsec/leg/esea02/index.html.

"Tri-Section" Meeting

On November 5-6, 2004, the MAA sections from Indiana, Illinois, and Kentucky will hold a combined meeting at the University of Evansville, in Evansville, IN. The Midwest History of Mathematics Group is also helping to organize this meeting.

The featured speakers will be: Ron Graham, President of the MAA; Brian Conrey, Director of the American Institute of Mathematics; and Woody Dudley, Depauw University.

In keeping with the title of the meeting, Prof. Dudley will be reprising his well-known talk on trisectors.

Abstracts for submitted talks will be due on October 8. This is also the deadline for advanced registration.

Information on submitting an abstract, registration, local arrangements, Project NExT activities, etc., will be available on the website of the Indiana section, http://www.rose-hulman.edu/~rader/INMAA (or through the section links at MAA Online).

Philosophy of Mathematics SIGMAA at Recent and Upcoming Meetings

By Satish C. Bhatnagar

Philip Davis, author (with Reuben Hersh) of *The Mathematical Experience* and *Descartes' Dream,* was POMSIGMAA's keynote speaker at MathFest 2004 in Providence: he spoke on "The Decline, Fall, and Current Resurgence of Visual Geometry." POMSIGMAA is the Special Interest Group of the MAA for the Philosophy of Mathematics, whose purpose is to stimulate interest in the philosophy of mathematics in the wider mathematical community.

For the Atlanta Joint Mathematics Meeting in January, 2005, POMSIGMAA will again hold a contributed paper session and possibly have a speaker as well.

At the Phoenix Joint Meetings last January, POMSIGMAA sponsored a contributed paper session which was a great success as measured by the number of participants and the papers presented. The format of the session was original in that two breaks were provided between talks for refreshments, socializing, and continuing the discussions. The titles and abstracts of papers are on the web at http://www.wooster.edu/pom_sigmaa/activities.html. Roger Simons of Rhode Island College chaired the first part of the session and Satish Bhatnagar of UNLV the last part. At times, the room was overflowing, with about 100 people attending. Presentations were 10 minutes with 10 more minutes for questions, answers, and discussion.

During the break, light refreshments and drinks were available while people exchanged ideas and notes. As expected, some talks generated an enthusiastic acceptance, some an animated debate, and some a scathing rebuttal from the audience. To a certain degree the Philosophy of Mathematics is a fluid domain. Right at the inception of POMSIGMAA, it was decided that it be neither all traditional philosophy, nor foundations of mathematics. This remains subjective and its exploration is a purpose of the group.

The sessions and breaks went on from 2:00 to 6:30. They were followed by a POMSIGMAA business meeting, in which plans for activities at the summer MathFest and the Jan. 2005 Joint Math Meetings were made. After the business meeting, some members went out for dinner together.

It is not out of place to add a little background of POMSIGMAA. During the New Orleans Meeting in Jan 2001, Joe Auslander and Bonnie Gold organized a panel session on Philosophy of Mathematics (PoM). It attracted a large number of participants, partly because the panel included some well-known mathematicians and philosophers of mathematics such as Saunders MacLane and Reuben Hersh. Sensing an interest in PoM, Bonnie Gold asked the audience to sign up if interested in forming a special interest group. POMSIGMAA had just been approved by the MAA in time for the January 2003 Joint Math Meetings in Baltimore. The new organization held an informal meeting and the first paper session under the auspices of POMSIGMAA was held. (Abstracts of the talks are available at http://mathserv.monmouth.edu/coursenotes/gold/PhilOfMathCPS.htm.)

If you're interested, please join POMSIGMAA and start participating in our meetings (and, hopefully, soon our electronic discussion list): it's how you can have some impact on discussion within the mathematics community on the philosophy of mathematics, as well as what kinds of sessions are offered at MAA meetings.

Archives of American Mathematics Spotlight: NCTM Oral History Project Records

The National Council of Teachers of Mathematics (NCTM) Oral History Task Force established the NCTM Oral History Project in 2002. This group, chaired by Dr. David L. Roberts, has initiated a series of interviews with leaders in the mathematics education community. The taped interviews are transcribed and edited by the interviewee and the interviewer. All existing interviews have been conducted by Dr. Roberts. Completed interviews include conversations with: Stanley J. Bezuszka, F. Joe Crosswhite, Edgar L. Edwards, Jr., John C. Egsgard, Shirley A. Hill, Lola J. May, and Bruce E. Meserve.

The Archives of American Mathematics (AAM) began receiving this material in December 2003. We currently hold over two linear feet of audiocassette recordings of the interviews, typed transcripts, release forms, and related materials. We can provide access to these materials onsite, and answer questions and/or provide copies for off-site researchers.

These are well-researched, quality interviews that would be valuable resources for mathematics historians, mathematics educators, and other researchers. A complete inventory for the NCTM Oral History Project Records can be accessed from the Texas Archival Resources Online (TARO) website at http://www.lib.utexas.edu/taro/utcah/00319/cah-00319.html. More information on the oral history project and the NCTM in general can be found on the NCTM website: http://www.nctm.org.

The Archives of American Mathematics are located at the Center for American History on the University of Texas at Austin campus. (A longer article on the Archives appeared in the May 2004 issue of FOCUS.) The Archives' web site is at http://www.cah.utexas.edu/collectioncomponents/math.html. For more information on the archives, contact Kristy Sorensen, Archivist, k.sorensen@mail.utexas.edu, (512) 495-4539.

Letters to the Editor

A Call for Reform

Here's a reform I'd like to see in undergraduate and high school mathematics education. I wish that beginners didn't always have to start with calculus. I took calculus in high school, and it didn't make real, solid sense to me. I got good grades: I could do all the problems, but felt that something essential was lacking. In college I signed up for a class called Honors Mathematics, and it turned out to be more calculus. I was very disappointed. I felt that I had lost mathematics, that it no longer made sense. I became very depressed. I majored in philosophy and classics in college. I did some logic and liked it somewhat, but not as well as I had liked geometry in high school. The logic I studied seemed to lack content.

It wasn't until I had an M. A. in Latin that I realized that I missed mathematics so much that I wanted to study it again, even if it was just calculus and made no sense. I signed up for linear algebra. This turned out to be a proof class, and I loved it. I asked my professor what I would need to learn to go to graduate school in math. He told me to take real analysis and abstract algebra. I did this, went to graduate school, and got a Ph. D. I liked calculus once I had studied real analysis.

I am a reader and writer of fiction. I notice that not many modern writers of fiction in English know much mathematics. This is not always due to lack of interest on their part. I think that the sort of person who is more inclined to literature than to science or engineering likes and understands pure mathematics more than applied mathematics. We can appreciate applied mathematics and physics once we understand pure mathematics. But we do not like to apply rules we don't understand.

My point is this. Not everyone should begin the study of college mathematics with calculus. Some people (particularly, perhaps, those who are interested in language and literature) should begin by studying pure mathematics. I really wish that universities and high schools would offer two paths to mathematics: one that starts with calculus, and one that starts with an elementary course in pure mathematics. One advantage of this system is that a student who hates calculus would have another place to begin. Another is that pure mathematics may very well turn out to appeal to far more people than is generally supposed. I have often read that women are "more verbal" than men. Perhaps a course of study that began with pure mathematics would attract more female students.

It would do no harm to try it. I fervently hope that someone will do so. As matters stand, most university students are not exposed to pure mathematics at all, and do not even know that such a thing exists.

Amy Babich
Continuing Education Program
University of Texas at Austin

Mistakes in Standardized Tests

I read with interest your comment on my email, which you were so kind to print in the February2004 FOCUS.

This letter is to advise you that, as expected, there are errors in the 2004 multiple-choice math test I was obliged to administer. However, the Instruction Booklet now reads (p. 8): "No part of the test materials may be reproduced in any way prior to, during, or after testing, nor may any copies of the materials be kept in the school building or district office after the test administration is completed. Descriptions of individual test items MUST NOT be released to the general public. ANY BREACH OF THESE SECURITY MEASURES COULD RESULT IN LOSS OF LICENSURE" [emphasis in original].

So for the crime of bringing to public attention the absurdities of this year's test I would not only lose my job but also become unemployable as a teacher in the state of Oregon. That's how anxious the powers running this test are to improve the test.

Michael Meo
Benson Polytechnic High School

Well, this does seem rather harsh. But there are more reasons to keep test questions secret besides fear of mistakes being exposed. For example, they may want to re-use some questions next year. In any case, there may be other ways to help the test-makers improve their tests besides going public with their mistakes.

On the "Don't Span" Photo

I just saw it in FOCUS (May/June, p. 12). I do not recall claiming it had anything to do with a bridge. I am 95% sure it was to stop people walking on a nice lawn!!

Colm Mulcahy
Spelman College

My bad. I guess my mind created a bridge from the "span"…

Query

I am interested in finding out about the maximum number of decimal places to which *e* (base of the natural logarithm) has been generated, and the source of such data.

Tarald O. Kvålseth
Dept of Mechanical Engineering
University of Minnesota
kvals001@umn.edu

FOCUS Deadlines

	November	December	January
Editorial Copy	September 16	October 15	November 15
Display Ads	September 24	October 29	November 29
Employment Ads	September 10	October 11	November 10

Thirty-Third U.S.A. Mathematical Olympiad Winners Celebrate in Washington, DC

By Steve Dunbar

The 33rd U.S.A. Mathematical Olympiad Awards Ceremonies took place in Washington DC on Sunday and Monday, June 20 and 21. This event honors the twelve top winners of the annual USA Mathematical Olympiad exam, the premier high-school level mathematical problem solving competition in the United States. The two day celebration begins with a Sponsors' Reception at the MAA Headquarters. Representatives of the sponsoring organizations of the American Mathematics Competitions along with members of the MAA Executive Committee, are invited to meet and greet the winners and their families. On Monday morning, the National Science Foundation host their annual program for the winners which includes talks and a tour of the building. This year Dr. Joanna Kania-Bartoszynska, Program Director, Geometric Analysis, Topology and Foundations, spoke on *Tri-Coloring Knots* and Dr. Ken Shaw, Program Director, Applied Mathematics, spoke on *Photonic Crystals and Defects in Periodic Structures*. This program was arranged by Dr. Deborah Lockhart, Acting Director, of the Division of Mathematical Sciences.

Top winner Tiankai Liu with Zuming Feng, leader of the USA Team in Athens.

The USAMO winners (in alphabetical order), **Jae Min Bae**, Academy of Advancement Science and Technology, Leonia, NJ; **Jongmin Baek**, Cupertino High School, Cupertino, CA; **Oleg Golberg**, Bedford, MA; **Matt Ince**, home schooled, Arnold, MO; **Janos Kramar**, University of Toronto Schools, Toronto, Ontario; **Tiankai Liu**, Phillips Exeter Academy, NH; **Alison Miller**, Home Educators Enrichment Group, Niskayuna, NY; **Aaron Pixton**, Vestal Senior High School, Vestal NY; **Brian Rice,** Southwest Governor's School, Marion, VA; **Jacob Tsimerman**, University of Toronto Schools, Toronto, Ontario; **Ameya Velingker**, Parkland High School, Orefield, PA; and **Tony Zhang**, Phillips Exeter Academy; Exeter NH were guests of Dr. John H. Marburger, III, Director of the Office of Science and Technology Policy in the Executive Office of the President, at the celebratory reception and dinner held at the U.S. Department of State. The formal awards ceremony took place at the National Academy of Sciences where Ed Burger, professor of mathematics at Williams College, delighted the winners and the audience with his USAMO Address titled *Conjugate Coupling: The romantic adventures of the quintessential quadratic.* The winners received the USAMO Medal, named in honor of Gerhard C. Arenstorff, twice a winner of the USAMO and a member of the first USA team in the International Mathematical Olympiad.

Dr. Marburger welcomed everyone to the Diplomatic Reception Rooms of the State Department for the elegant dinner and read a letter of warm greetings from President Bush. The Clay Mathematics Institute designated Matt Ince as the sixth CMI Mathematics Olympiad Scholar. Matt best fulfilled the prize's criteria of elegance, beauty, imagination, and depth of insight.

The highlight of the evening came when the Akamai Foundation presented TianKai Liu, Oleg Golberg, and Tony Zhang with the first, second, and third place Akamai Scholarships in the amounts of $20,000, $15,000, and $10,000 respectively. By awarding these scholarships, the Akamai Foundation hopes to encourage these and other students to continue their pursuit of mathematics education.

The "road to the USAMO" begins with the American Mathematics Contest 10 (AMC 10) and American Mathematics Contest 12 (AMC 12) exams. In February, about 240,000 students from over 4,000 schools participated in these contests. The AMC 10 and AMC 12 have 25 questions from the high school mathematics curriculum to be answered in a timed 75 minute format. The problems range from easy to quite challenging. The top 5% of scorers on the AMC 12 and the top 1% of scorers on the AMC 10 are invited to take the American Invitational Mathematics Exam (AIME). The AIME is a challenging 15 question contest spanning 3 hours. The difficulty of the questions range from equivalent to the most difficult on the AMC 12 to extremely difficult. In March, nearly 12,000 students took the AIME. Based on a combination of scores from these two contests, 363 students were invited to take the USA Mathematical Olympaid (USAMO) exam which was held on April 27 and 28. The USAMO is a 6 question proof-essay contest, taking 9 hours over two days. The problems on the USAMO would be challenging even to professional mathematicians. This year's USAMO and solutions are available on the web by choosing "Students" on the MAA home page (www.maa.org) and following the links.The twelve winners and 42 other young students from the Olympiad are invited to the Mathematical Olympiad Summer Program (MOSP) for advanced training for the International Mathematical Olympiad (IMO). The 2004 Mathematical Olympiad Summer Program was held on the campus of the University of Nebraska-Lincoln from June 12 to July 3 with 54 students and 14 instructors and graders in attendance. The students received a mix of training on mathematical problem solving, proof-writing and deeper instruction on algebra, geometry, number theory, combinatorics, probability, and trigonometry in preparation for solving Olympiad-style problems. The summer program was funded in part with a grant from the Akamai Foundation.

The final US team for the IMO is selected from among the 12 winners at the MOSP. Each year since 1974, a small team of exceptionally talented high school students has represented the United States at the IMO. The IMO is a rigorous two day competition again including problems that would challenge most professional mathematicians. In addition to comprehensive mathematical knowledge, success on the IMO requires truly exceptional mathematical creativity and inventiveness. The 2004 IMO in Athens, Greece will be the 45th since Romania initiated the annual competition in 1959. United States teams have placed within the top countries in all IMOs in which they have participated. In ten of these, the United States was awarded first or second place. In 2001 the U.S. team tied for second place as the six members of the U.S. team won four gold and two silver medals. In 2001 Reid Barton of the U.S. team distinguished himself by becoming the first student ever to win four gold medals in the IMO. A noteworthy accomplishment of the US team in the IMO occurred in 1994, when all six members of the US team had perfect scores, and earned gold medals.

From left to right: Edward Burger, of Williams College, Olympiad lecturer, John Marburger, host for the evening and Science Advisor to the President of the USA., and Ron Graham, President of the MAA.

Photographs by Sonny Odom.

US Team at 45th International Mathematical Olympiad Has Best Finish in 10 years

By Harry Waldman

Before the Summer Olympics in Athens, Greece, there was another Olympiad. Competing against teams from 84 countries at the International Mathematical Olympiad, the US team, comprising six students, outdistanced all but the team from China to capture second place in the 2004 competition, held in Athens from July 4-18. That was the best showing by a U.S. IMO team in ten years. The team from Russia came in third.

The IMO is the preeminent mathematical competition for high school students around the world. This year there were 485 students in the competition. Most countries were represented by six students; smaller nations such as Luxembourg and Cuba sent only one or two students to the Olympiad. Each student had to solve six challenging problems in nine-hours over two days. Each problem was worth seven points. The maximum number of points any student could attain was 42; the team 252. The US team total was 212, only eight points shy of China's total.

Individually, competitors won medals for outstanding and elegant solutions. There were 45 gold medals awarded overall. The US team took six medals, the most it had won in any IMO since 1994, when it won 6 gold medals.

All US team members were winners: Oleg Goldberg (Bedford, MA) took a gold with 40 points; Tiankai Liu (Saratoga, CA) was awarded a gold medal with 38; Aaron Pixton (Vestal, NY) garnered a gold with 37; and Alison Miller (Niskayuna, NY) and Tony Zhang (Arcadia, CA) also won gold medals with 33 points each. Matt Ince (Arnold, MO) won a silver medal with a score of 31 points.

The MAA sponsors the US team through its American Mathematics Competitions program, which is headquartered at the University of Nebraska, Lincoln. Steven R. Dunbar is the director. Training for the team at the University of Nebraska was aided by a grant from the Akamei Foundation. Travel support was provided by a grant from the Army Research Office and additional support came from the National Council of Teachers of Mathematics, Society of Actuaries, Mu Alpha Theta, Casualty Actuarial Society, American Statistical Association, AMATYC, AMS, American Society of Pension Actuaries, Art of Problem Solving Inc., Pi Mu Epsilon, USA Math Talent Search, Clay Math Institute, and INFORMS.

The official IMO website is www.imo2004.gr

MAA Tour of England

Members (and guests) traveled to England in late May to participate in the second MAA Study Tour abroad. There was a wide variety of travelers from all across the United States. Several had participated on the MAA's inaugural study tour to Greece. All agreed that the trip met, and even exceeded, their expectations. Three of them agreed to write brief comments about their experiences. A photographic essay of the trip can be found at www.maa.org/england

The group at Teddy Hall, Oxford University.

From Herb Kasube
Bradley University, Peoria, IL

My interest in the history of mathematics first motivated me to participate in the MAA Tour of England. To walk in the footsteps of Hardy, Newton, and Turing was an opportunity not to be missed. The trip surpassed all of my expectations. I truly felt as if we had followed the footsteps of Newton since we visited his birthplace at Woolsthorpe, his elementary school in Grantham, his university in Cambridge, and his tomb at Westminster Abbey.

It would be difficult to pinpoint any single point of the tour that was the "best." The museums contained artifacts from Charles Dodgson's photographic chemicals to a blackboard with a portion of a lecture given by Albert Einstein to a lock of Isaac Newton's hair. I expected a lot from these exhibits and my expectations were far exceeded.

Perhaps the single most impressive (and enigmatic) site was Stonehenge. We visited early in the morning and the sun's low rays added to the impact. While I had seen pictures of Stonehenge before, they did not do the place justice. I was particularly surprised and overwhelmed by the ***size*** of the stones. You can't help but be impressed by this 5000 year old structure.

Herb Kasube at Stonehenge

Visiting Oxford and Cambridge offered us a chance to see where Newton, Hardy, Turing and so many others had walked and worked. It humbled me. I will do my best to share this feeling with my students.

On a personal note, my traveling companions made the trip one that I will never forget. Old friendships were renewed and new ones have been formed.

From Carol Dotseth and Gregory Dotseth, Emeritus, Mathematics Department University of Northern Iowa

More than 25 years had elapsed since either of us had been to England; this study tour was a chance for us to revisit some places and friends and to add new ones. We couldn't have chosen a better venue. The leaders (Paul Wolfson, Lisa Kolbe) were outstanding; the sites that we visited represented mathematics through the ages. Highlights include the British Library, The Royal Society, a walking tour of Wren architecture in London, the London Eye, The Royal Observatory in Greenwich, Stonehenge, Woolsthorpe Manor (the birthplace of Newton), the Wren Library at Trinity College in Cambridge, and Bletchley Park, to name just a few. In addition we were treated to outstanding talks and interesting tours in Oxford and Cambridge. Each day we had enjoyable scheduled learning experiences and we were also given enough free time to enjoy art museums, theater, and pubs.

A typical day started with breakfast between 7:30 and 8:30 am. Then a bus would take us to our first destination. We would spend about 2 hours viewing a variety of original works and taking pictures in order to be able to remember all of the exhibits. After lunch a talk would be presented and then at about 3 pm we would have time to explore on our own. We would take the underground to the theater area, buy evening tickets, visit some place of interest, eat supper, and then attend the theater. This made for busy days, but thoroughly enjoyable ones.

It was particularly interesting to us to be in England so near the time of the Transit of Venus and the 60th anniversary of D-Day. It made our discussions of the mathematics behind the transit and the breaking of the code at Bletchley Park even more meaningful.

Greg and Carol Dotseth at the Prime Meridian, Greenwich, England.

Because of our association with the MAA, we had opportunities that were not available to the regular tourist. In the Royal Society we were able to view portraits and documents that were not generally on display and spend much time with questions and photographing. In Oxford, we dined at High Table and were given private tours within several colleges, It was a great tour with great people. We would do it again in a heartbeat—there is much to learn.

From Jackie Dewar
Professor of Mathematics
Loyola Marymount University

When I read the article in last year's FOCUS reporting on the MAA's math history tour of Greece in summer of 2003, I was disappointed that I had somehow missed a great opportunity. I was determined to go on the next trip, the 2004 MAA Trip to England, and so I did. Traveling alone from Los Angeles, I met the most congenial group of mathematicians and spouses in London. Upon my return home, when people asked me how it was, I said fabulous! Not only did I make new friends and connections, I had access to places, rare books, and experiences that a typical tourist would never have.

Can you imagine walking through Stonehenge in the early morning with a group of mathematicians bent on matching each stone to those indicated on the maps we held in our hands? Or conversing with conservators who were removing graffiti carefully so as to preserve rare lichens in the stones? How about seeing books with Newton's handwritten notes in the margin? Touring Newton's family home which is (still!) a working sheep farm? Being able to photograph rare items such as John Harrison's H4 clock, the first commercial calculating machine (late 19th century), and Newton's letter to Hooke answering (incorrectly) Hooke's question — what path would a dropped object take if it could travel through the earth? Talking with scholars from Oxford, Cambridge, and the Royal Society? Dining at high table in Oxford? Experiencing all this while terrific leaders (Paul Wolfson and Lisa Kolbe) were expertly handling the details of arrangements? Let me say it again, "It was fabulous!" See you on next year's trip.

Jackie Dewar at Woolsthorpe (Newton's Birthplace).

Remembering David Fowler

By Eleanor Robson, with additional material by Steve Russ

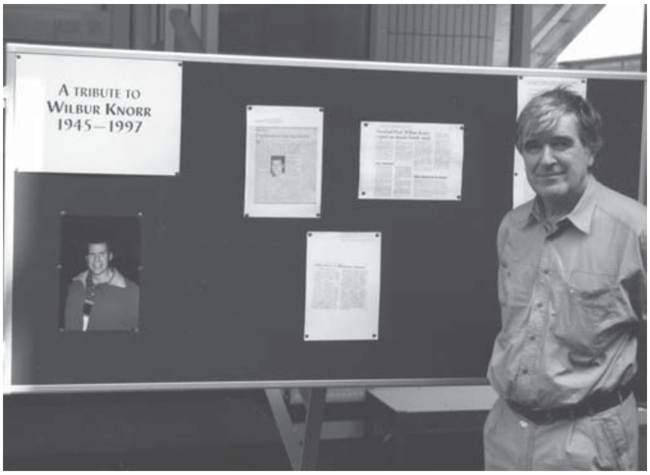
I like the way my career has run in reverse: start with administration, gradually shift over to research, then get a doctorate after I retire. You couldn't do that these days!
— David H. Fowler

David Herbert Fowler, DSc, historian of ancient Greek mathematics
Born Blackburn, UK, 28 April 1937; died Warwick, UK, 13 April 2004

David Fowler, who died of a brain tumour in April 2004 a few days short of his 67th birthday, was as vibrantly and fruitfully unconventional in his historical thinking as in his career trajectory. Following his lead, I will chart his life in reverse, beginning with his extraordinary approach to illness and death.

David died in Myton Hamlet Hospice in Warwick after several weeks of serious illness, surrounded by his wife Denise, their son Stephan and daughter Magali, and cards and presents from friends and colleagues all over the world. It marked the end of a ten-year relationship with an 'uninvited visitor' in the left frontal lobe of his brain — not a battle, because David chose not to fight it, except when it started to wrap itself round an optical nerve, but an accommodation, a path of least disruption. He and his family jointly decided that it was best to let the tumour run its course with minimal interference other than drugs to suppress the disorientating symptoms. He took early retirement in 1994 to focus on research and family and had several years of intensely lived activity before the visitor began to overwhelm him. David wrote up his experience for the *British Medical Journal* (volume 311, 1995, pages 1691–93).

David gave his last paper, appropriately enough, in Delphi, Greece, in July 2002, at a workshop on the history of ancient mathematics. His deeply held and articulately argued conviction was that the standard story of Greek mathematics was a myth of Late Antiquity, unsupported by any early evidence. The traditional modern account states that in fifth-century Athens, Greek mathematics was all about numbers, like the mathematics of ancient Egypt and Babylon from which it was derived. Then the Greeks discovered incommensurability: that some ratios of numbers or lines or areas could not be expressed rationally. This caused such a shock to the Greek mathematicians that they abandoned numbers altogether and instead invented the Euclidean geometrical tradition that describes and explores only the properties and relationships of mathematical objects, not their numerical values. The most famous of these de-arithmetised formulations is Euclid's *Elements* book I, proposition 47: The square on the hypotenuse of a right triangle is equal to the sum of the squares on its two shorter sides.



At a BSHM meeting to celebrate his 60th birthday, held at the University of Warwick in April 1997, David Fowler remembers his friend and colleague Wilbur Knorr. Photograph by John Fauvel, reproduced with the kind permission of Denise Fowler.

But, asked David, where is the evidence for this story? Early, pre-Euclidean mathematics suggests nothing of the sort. All of the evidence is in the works of later Greek commentators on mathematics and its history, who had no better access to the very ancient sources than we do. In fact, there is no direct evidence at all for the mathematics of the fifth century BC; the earliest extant source is Plato's dialogue *Meno*, from 385 BC. In it, Socrates teaches Meno's slave boy about the mathematics of ratio and proportion, not by lecturing but by questioning him in such a way that he allows the slave boy to discover the results for himself. Through treating these early sources seriously, as both textual and material evidence, David argued instead that Euclidean mathematics arose naturally and non-traumatically from a Greek tradition that was deeply concerned with ratio, proportion, and approximation in several different ways. Babylonian and Egyptian influence on pre-Hellenistic Greek mathematics was trivial at most.

The book that resulted, *The Mathematics of Plato's Academy* (Oxford University Press 1987, 2nd rev. ed. 1999) shone with humour, originality, and a concern for the reader that is as rare as hens' teeth in most academic writing. Historians of the written word are often at a loss to know exactly who and how many read and used particular books or manuscripts. But in David's case it seems that philosophers, classicists, papyrologists, and historians of early mathematics have all found value in his work: there are no less than fifteen copies of it in Oxford's university libraries alone. He also published many articles on this and associated topics. A full list can be found online at http://www.maths.warwick.ac.uk/maths/papers/dhf.html, but 'Inventive interpretations', (in *Revue d'Histoire des Mathématiques*, volume 5, 1999, pages 149–53) presents his argument at its most sparkling and succinct.

The inspiration for David's historical turn was Wilbur Knorr's magisterial book *The Evolution of the Euclidean Ele-*

ments: A Study of the Theory of Incommensurable Magnitudes and its Significance for Early Greek Geometry (Reidel, 1975), which, as he later described in 'The book that changed my life' (*BSHM Newsletter,* volume 33, 1997, pages 32–34), was erroneously sent to him for review. Intrigued that a book could be so incomprehensible and so expensive, he took it home out of sheer curiosity, fully intending to return it to the publisher the following week. But his weekend's reading was so absorbing and challenging that he kept the book, wrote the review, built his subsequent career on constructing a counter-argument, and became one of Wilbur Knorr's closest friends.

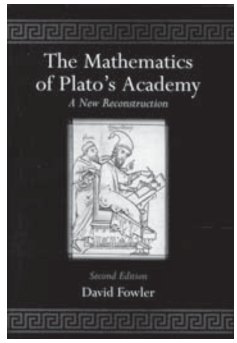
David was a naturally gregarious colleague, though never comfortable with leadership. He had a deep and benign influence on the British Society for the History of Mathematics. He never held office, but was a frequent participant at BSHM meetings. He gave ten lectures to the society over the years, but was equally valued for his supportive and stimulating reactions to graduates' presentations at the annual Research in Progress day. He became a profoundly influential mentor for several younger scholars of ancient mathematics: Reviel Netz, Serafina Cuomo, and I all benefited from his twinkly-eyed provocations, as did Jackie Stedall. For David had a second string to his historical bow, English mathematics of the era of Newton and Wallis, although he published little on this topic.

Socratic dialogue was as central to David's teaching as it was to his research and relations with colleagues. He taught analysis and later also history of mathematics at the University of Warwick, where he pioneered alternatives to traditional chalk-and-talk lecturing, which he saw as dull and ineffective. He enabled students to learn actively and collaboratively, discussing ideas, marking each other's work, and reflecting on their experience of learning. He was especially proud of one particular Friday-afternoon history of maths class in the late 80s, during which he fell asleep unnoticed by his students (myself amongst them) as we continued our vigorous debate. (See 'A third year university course in the history of mathematics: actively confronting the past,' in *The Mathematical Gazette,* volume 76, 1992, pages 46–48; and *BSHM Newsletter,* volume 28, 1995, pages 68–69.)

But before all this, as David reminds us above, he was an administrator. (Sir) Christopher Zeeman, his former tutor at Cambridge, appointed him in 1967 to manage the Mathematics Research Centre in his fledgling Mathematics Institute at the newly established University of Warwick. David brought many special and unusual abilities to the task. His great interest in people, and in mathematics, and his mastery of many practical issues in the maintenance of good living conditions, enabled him and his colleagues to provide conditions under which distinguished visitors created much new mathematics and proved many new theorems. As a result, there were, in almost every year during David's first 25 years at Warwick, more mathematicians visiting the University of Warwick mathematics department than there were mathematical visitors to all other English universities combined—a remarkable record for a new university.

But David had a busy and engaging life outside academia too. In 1961 he met Denise Stroh, from Strasbourg, at the Edinburgh Festival. They married soon after, forming an extraordinarily close and strong partnership. They provided hospitality to friends and colleagues from all over the world, first in Manchester where he held his earliest academic post, and then in an Elizabethan cottage in Warwick. Throughout his life, a natural expression of his compassion and humanity was the thoughtful and active support David gave to numerous social causes. These ranged from human rights and comprehensive education to more local issues of traffic congestion and other community concerns. In all these areas he found his preferred and most effective role to be working, perhaps little known, behind the scenes rather than in the public eye.

David grew up in Blackburn, Lancashire, and was educated at nearby Rossall School. His brother John recalls him as a post-war adolescent in grey flannel shorts, secretively constructing a monstrous apparatus which grew to occupy the entirety of a neighbour's front room. It turned out to be the first working television set in the street, built entirely out of parts bought with pocket money. But, inspired by his teacher R K Melluish, David's attention soon turned to mathematics. He took up a place to read maths at Gonville and Caius, Cambridge, where fatefully his young tutor was (Sir) Christopher Zeeman and fellow tutee the incomparable John Conway. Although he started research in analysis he never took a doctorate — or at least, not until 1999. Here is the occasion in his own words, from the same email to me with which I began:



Fowler's The Mathematics of Plato's Academy.

Denise collected her PhD a couple of weeks ago and I said I'd go with her to the ceremony, though I don't like that kind of thing. Quietly, to be able to accompany her properly, I fixed up a DSc more than a year ago, though almost nobody knew about it. Denise said that she only realized what was going on when I was on the stage getting it! She went as pink as my vulgar gown, and the children accused me of upstaging her on her day, but later they relented a bit. My father, aged 90, was moved to a degree that astonished me. Anyway, I have no intention of using it except where it may serve a proper purpose, e.g., in references. So I've put it about that anyone who Dr's me will get a glacial non-reaction, and that includes you.

As this incident suggests, he often seemed blissfully unconcerned with the exasperation he could provoke, and was expert at diffusing awkwardness or confrontation with mischievous good humour. David was an inspiration, challenge, and motivation to his family, students, colleagues and friends, much loved and much missed by many around the world.

A Meeting with Congress

By Tina Straley and Suzanne Lenhart

MAA joined mathematical, scientific, and engineering societies and universities across the country in the Coalition for National Science Funding (CNSF) 10th annual Science at Work exhibition on June 22 in the Rayburn Building, the large House of Representatives Office Building on Capital Hill. The purpose of the event is to exhibit to members of Congress and their staff the beneficial programs that National Science Foundation supports. Each year, the MAA hosts a project funded by NSF that fits the mission of MAA. During the day of the exhibit, the exhibitors visit the offices of members of their state's Congressional delegations. The program takes place from 5 pm to 7:30 pm, after normal work hours. A good number of the staffers and some of the Congressmen from both the House and the Senate stop by the exhibition and reception.

This year, the MAA sponsored an exhibit, led by Professor Suzanne Lenhart, accompanied by students Bill Holmes and Jeff Lowder, all of the University of Tennessee, on "Competition and Invasion: A Microcosmic View." Lenhart said, "Interdisciplinary research projects for undergraduates is an innovative idea from NSF and we are delighted to be funded for our work with biology and mathematics majors. We were glad to represent MAA at the CNSF exhibit. Discussing our work with Congressional staffers and seeing their genuine interest was gratifying."

Lenhart's presentation was based on an interdisciplinary study designed to demonstrate to biology and math students how the two areas can be mutually beneficial. "There's a big push in the math and science community to integrate the two, especially in biology," Holmes said. "Our project is the first of its kind, and we wanted to present it to show that we are trying to push (this issue) at the undergraduate level." Lenhart described the project as part of a new NSF program for undergraduate math and biology majors, the Undergraduate Mathematics and Biology Program, housed in the NSF/Division of Mathematical Sciences in collaboration with the Directorate of Education and Human Resources and the Biology Directorate. True to the spirit of the program, Holmes is a math and engineering physics major and Lowder is an ecology major. The other students involved with the project are Austin Faulkner, Jacob Kendrick, Elizabeth Martin, and Alex Perkins. In addition to Suzanne Lenhart, the faculty members involved are Louis Gross, Jake Weltzin, and Jim Drake. Rene Salinas, a postdoctoral fellow, is assisting with the project.

Prior to the program, Professor Lenhart, the students, and Tina Straley, MAA Executive Director, met with several staff members of the Tennessee Congressional delegations as well as with U.S. Representative Marsha Blackburn and Senator Bill Frist.

Commenting on the CNSF program, Holmes said, "It was good to see people from so many different areas come together to reach a common goal. It was kind of unexpected."

Tina Straley stated, "The most effective messages to members of Congress come from the people in their home states and districts. This program not only shows Congressmen the scientific and educational benefits of NSF funding but also the direct benefits derived from that funding in their own colleges and universities and home region."

Professor Suzanne Lenhart, Bill Holmes speak with Dr. Joye E. Purser, Legislative Assistant to Congressman Lincoln Davis at this year's CNSF . Photograph by Tina Straley.

Jeff Lowder, Bill Holmes, Senator William Frist, U.S. Senate Majority Leader, Professor Suzanne Lenhart, and Tina Straley, Executive Director, MAA. Photograph courtesy of William Frist's office.

Employment Opportunities

ALABAMA

University of Alabama in Huntsville
Department of Mathematical Sciences
The Department of Mathematical Sciences at the University of Alabama in Huntsville invites applications for a tenure track position at the rank of assistant professor, beginning spring semester 2005 or fall semester 2005. In exceptional cases, more advanced appointments may be considered. A Ph.D. degree in mathematics or applied mathematics is required. Applicants must have a strong commitment to teaching and show evidence of excellent teaching ability. Applicants should also show evidence of outstanding research potential in an area that matches the interests of the department. Preference will be given to applicants whose research areas are probability/stochastic processes or numerical analysis.

Applicants should send a curriculum vita with the AMS standard cover sheet, transcripts, and three letters of recommendation (with at least one letter addressing teaching) to:

Chairman
Department of Mathematical Sciences
University of Alabama in Huntsville,
Huntsville, AL 35899.

For more information about the department, visit our web site at http://www.math.uah.edu.

To ensure full consideration, all materials should be received by November 1, 2004. Late applications will be reviewed until the position is closed. Women and minorities are encouraged to apply. The University of Alabama in Huntsville is an Affirmative Action, Equal Opportunity Institution.