CS 580, Fall 2015; Instructor: Elena Grigorescu
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Due: Solve but do not turn in.

Problem set 0

1 Bookeeping

1. Sign up on Piazza (see link on class website)

2 Solve but do not turn in (they might appear in exams/quizzes)

1. What is the running time of merge sort if the input array looks like

$$\langle 3, 2, 1, 6, 5, 4, 9, 8, 7, ..., 3n, 3n - 1, 3n - 2 \rangle?$$

   What is the running time of merge sort on an already sorted array? What is the runtime on an inversely sorted array?

2. Rank the following functions by increasing order of growth (i.e., the slowest-growing first, the fastest-growing last):

   $(\log n)^6$, $n!$, $\sqrt{n}$, n^{21}, $n(\log n)^4$, $n \log n$, 2^n, n^2, $\log n^n$, $(\log n)^{0.3}$

   where all the logarithms are to the base 2. If two functions have equal orders of growth then list them grouped together, e.g., between brackets {like this}.

3. Consider the recurrence $T(n) = T(n/2) + 1$ and $T(1) = 1$. Solve this recursion by any method you know. Can you use induction to give an alternative proof of the solution you found in the previous step?

4. Alice needs to climb n stairs to get to her room. In a single step Alice can climb 1 stair or 2 stairs. What is the number of ways in which Alice can climb the stairs if, say n = 12? Two walk sequences are considered different if the sequence of steps taken is not identical. (Hint: Do you recognize the sequence of integers described by the resulting recurrence?)

5. Given an array of n integers and a target integer D, describe the best (in terms of running time) algorithm you can, that finds two elements of the array whose difference is D or outputs that no such elements exist.

6. Suppose s and t are vertices of an undirected graph G with n vertices and m edges. How fast can you determine if there is a path from s to t? What algorithm do you use?

7. Bob picks a number from 1 to n, and asks Alice to guess it. The rule is that Alice can ask him questions about the number, but he can only answer 'yes' or 'no'. Describe a strategy that ensure that Alice finds out the number by asking $o(n)$ questions.

8. What is the maximum attainable height of a red-black tree with black-height k?

    A: $k + 1$

    B: 2^k

    C: $2k - 2$

    D: $2k$

9. Consider a red-black tree storing an arbitrary n-element array $[a_1, a_2, \ldots, a_n]$. What is the number of rotations needed in order to preserve the red-black trees properties, when we insert a new element a such that $a < \min\{a_1, a_2, \ldots, a_n\}$?

    A: $\Theta(n)$

    B: $\Theta(1)$

    C: $\Theta(\log n)$

    D: $\Theta(n \log n)$