Spectral graph theory

Network & Matrix Computations

CS 59000-NMC

David F. Gleich

WHAT IS IT?

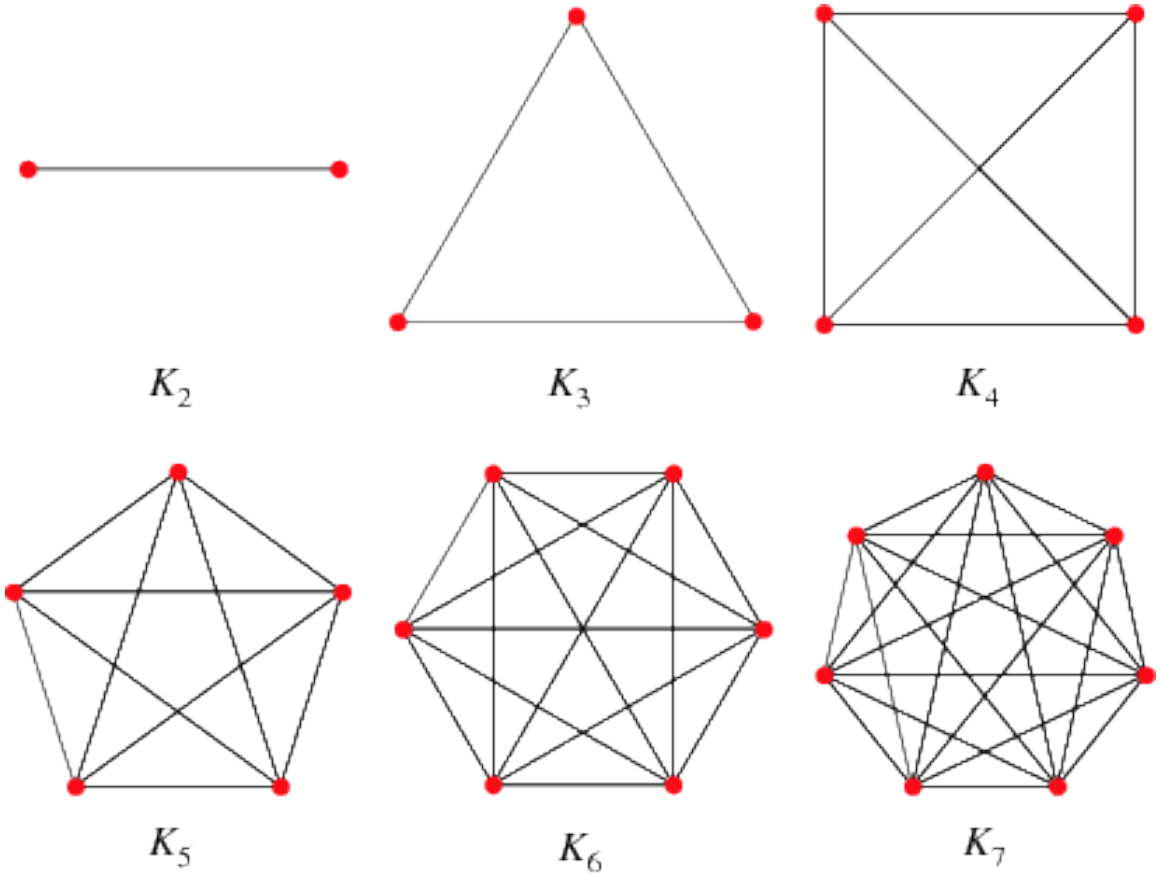
Spectral graph theory

From Wikipedia, the free encyclopedia

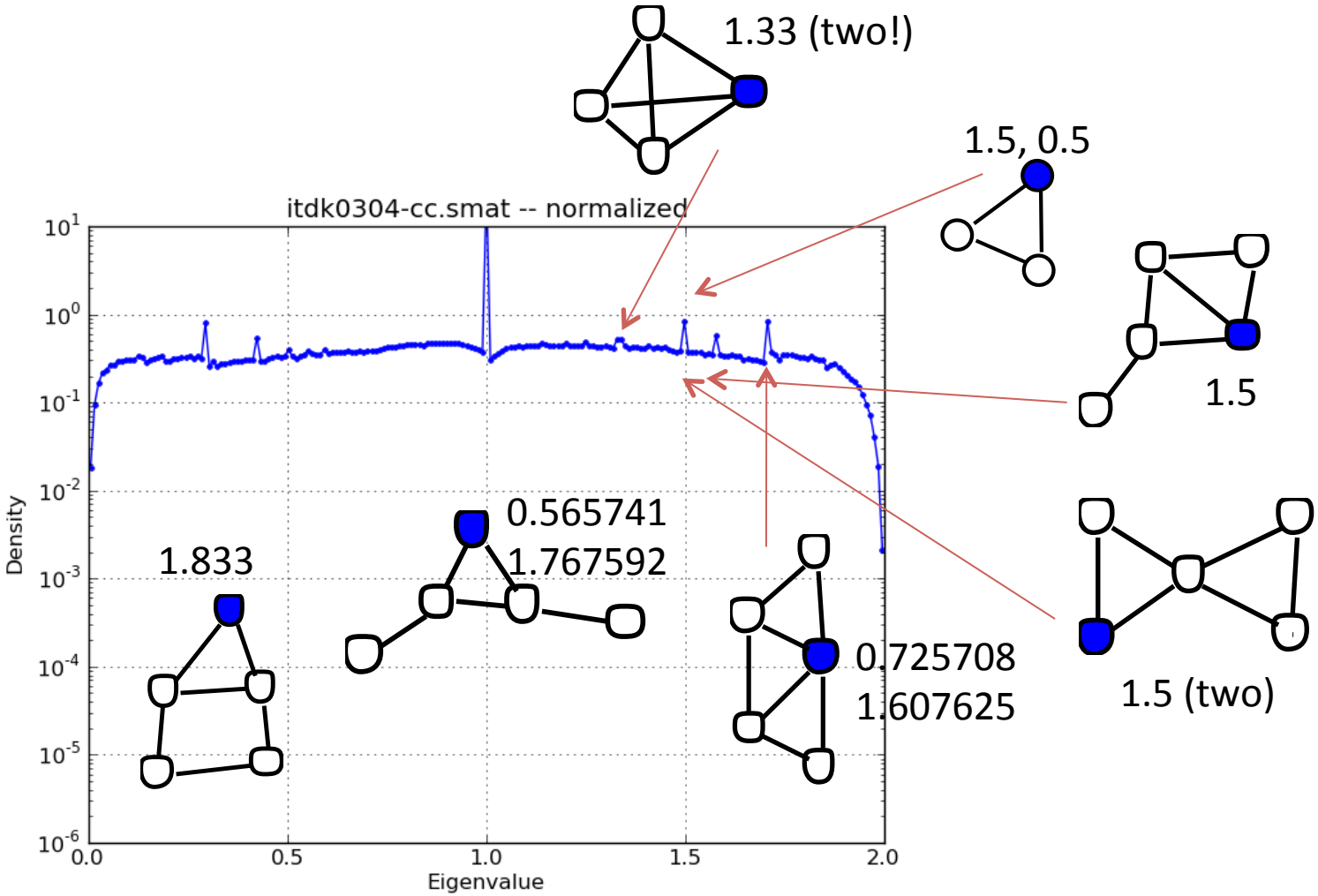
In mathematics, **spectral graph theory** is the study of properties of a graph in relationship to the characteristic polynomial, eigenvalues, and eigenvectors of matrices associated to the graph, such as its adjacency matrix or Laplacian matrix.

Aspects of spectral graph theory

Eigenvalues of graphs



Eigenvalues of graphs



Aspects of spectral graph theory

Combinatorial properties of graphs

the matrix-tree theorem (1850-ish)
the number of spanning trees of a graph is product of the eigenvalues of the Laplacian matrix

Aspects of spectral graph theory

Connection properties of graphs

the Cheeger inequality (????)

the relationship between the minimum conductance cut of a graph and the second smallest eigenvalue

Recent developments

support graph theory how to preserve the eigenvalues of a graph while removing edges

leads to “best” solver for Ax=b

1-norm Laplacians produce better cuts

semi-supervised learning how to interpolate functions on graph data

local spectral graph theory

Outline

Today the combinatorial Laplacian

Next time the Cheeger inequality

Next week SDP Approximation algorithms

Latex next week local spectral graph theory