CS 57100(Artificial Intelligence)

1 Probability

Jasmine and Sarah compete in two different events at the Olympics. Jasmine and Sarah win the race with probability p_1 and p_2 respectively. Assuming independence, are these scenarios possible?

i)P(nobody wins)=P(either one of them wins)

ii) P(nobody wins)=P(either one of them wins)=P(both win)

Soln Sketch:

J-> Event Jasmine wins. =>P(J)= p_1

S-> Event Sarah wins. =>P(S)= p_2

Per i), P(Jasmine doesn't win **and** Sarah doesn't win)=P(Jasmine wins **or** Sarah wins)

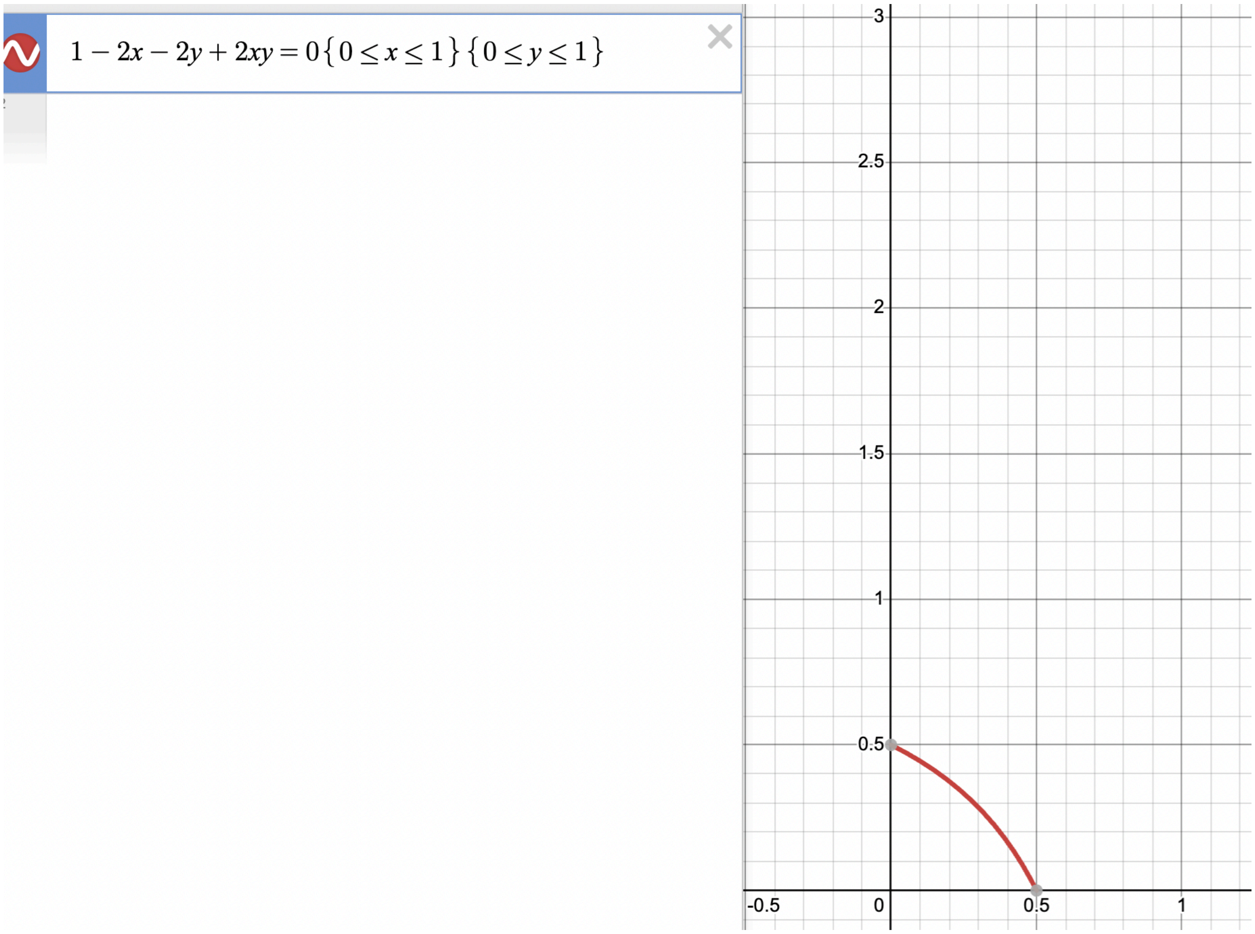
$$P\left(J^C \cap S^C\right) = P(J \cup S)$$

$$=> \left(1 - p_1\right)\left(1 - p_2\right) = p_1 + p_2 - p_1 p_2 \text{ (If events A, B are independent, } A^C, B^C \text{are independent too)}$$

$$=> 1 = 2p_1 + 2p_2 - 2p_1 p_2$$

$$p_1 = \frac{0.5 - p_2}{1 - p_2}\ with\ 0 \le p_1 \le 1; 0 \le p_2 \le 1$$

Possible!



Per ii), P(Jasmine doesn't win **and** Sarah doesn't win)=P(Jasmine wins **or** Sarah wins)=P(Jasmine wins **and** Sarah wins)

$P\left(J^C \cap S^C\right) = P(J \cup S) = P(J \cap S)$

When you try pairwise solving the equations, you will end up with $p_1(1-p_1) = \frac{1}{2}$.

But per this quadratic equation, the roots(i.e. p_1) *are not real. Discriminant* < 0. Hence this scenario is not possible.

2.

Show that for any random variables A and B,

$$If F_A(x) \le F_B(x), then P(A > x) \ge P(B > x)$$

for any real x in their domain

Proof sketch:

$$P(A > x) = 1 - P(A \le x) = 1 - F_A(x) \ge 1 - F_B(x) = 1 - P(Y \le x) = P(Y > x)$$

3. Linear Algebra

3.1. Determine whether the following matrix has an inverse without trying to compute the inverse. (Hint: Use determinant)

A =

3	-2	1	0
5	0	2	0
2	-8	-1	0
0	7	0	1

sol) det(A) = -10. A has an inverse.

3.2. Find eigenvalues and eigenvectors of the following matrix.

B =

3	-1
0	2

sol) $\det(B - \lambda I) = (\lambda-3)(\lambda-2) = 0$

$\lambda_1 = 3$, $\lambda_2 = 2$

For $\lambda_1 = 3$, we set eigenvector v_1 as $[x_1 \;\; y_1]^T$, then $(B-\lambda_1 I)v_1 = 0 \rightarrow y_1 = 0$

Thus, the eigenvector $v_1 = [1 \;\; 0]^T$

For $\lambda_2 = 2$, we set eigenvector v_2 as $[x_2 \;\; y_2]^T$, then $(B-\lambda_2 I)v_2 = 0 \rightarrow x_2 - y_2 = 0$

Thus, the eigenvector $v_2 = [1 \;\; 1]^T$

5. Complexity Theory

Arrange the following functions by increasing order of growth(base here is 2):

$$n^{0.5}, 2^{log(n)}, log(n!), log\left(log(n)\right), n!, 2^{2^n}, nlog(n), n^{-1}, \left(log(n)\right)^2, 10000$$

$$\frac{1}{n} < 10000 < \log(\log(n)) < (\log(n))^2 < n^{0.5} < 2^{\log(n)} < \log(n!) < nlog(n) < n! < 2^{2^n}$$

log(n!)< nlog(n) (see Stirling's Approximation)

6.

Sol) n as the length of a grid → solving the problem takes $O(c^n)$

Setting n as the number of tiles → we have sqrt(n+1) as the length of a grid → solving the problem takes $O(c^{sqrt(n+1)})$

Still exponential

7. Convexity and Concavity

Find the convex and concave intervals for the following functions

4.1. f(x) = 3x3 - 18x2 + 12x + 8

4.2. f(x) = ex

sol)
4.1. $f'(x) = 9x^2 - 36x + 12$
$f''(x) = 18x - 36$
We find f''(x) = 0 where x = 2. Since f(x) goes to +∞ as x goes to +∞. Thus the function is convex after the inflection point, and concave before the inflection point.
Convex interval: (2, +∞), Concave interval: (-∞, 2)

4.2. $f'(x) = f''(x) = e^x$
There's no inflection point since there's no x value that makes f''(x) = 0. Since f''(x) > 0 for any points in the domain, the function itself is convex.
Convex interval: (-∞, +∞), Concave interval: None