CS 334 Midterm

Problem 1. Fundamentals

(a) Derive the test for the 2D point p to lie in the triangle abc.

The triangle vertices are in counterclockwise order. Points abp, bcp, and cap are left turns. Points uvw are a left turn when $(w - v) \times (u - v) > 0$.

(b) Derive the test for the 2D line segments ab and cd to intersect.

One of cda and cdb is a left turn and the other is a right turn. Likewise for abc and abd.

Problem 2. Matrix operations

(a) Derive the 2x2 matrix that projects a 2D vector along the unit vector $n = (n_x, n_y)$.

The matrix is $I - n^t n$. It maps p to $p - (p \cdot n)n$.

(b) Derive the 3x3 matrix that reflects a 3D vector around the unit vector $n = (n_x, n_y, n_z)$.

The matrix is $2n^t n - I$. It maps p to $2(p \cdot n)n - p$.

Problem 3. Perspective projection

(a) Derive the projection, (u, v), of a 3D point, (x, y, z) with $z > 1$, onto the image plane $z = 1$ with center of projection $(0, 0, 0)$.

Using similar triangles, $(u, v) = (x/z, y/z)$.

(b) Use part (a) to project the line segment from (x_1, y_1, z_1) to (x_2, y_2, z_2). Show that the projection is a line segment.

The parametric form is $(x_1, y_1, z_1) + k(u, v, w)$ with $u = x_2 - x_1$, $v = y_2 - y_1$, $w = z_2 - z_1$. The projection is

$$\left(\frac{x_1 + ku}{z_1 + kw}, \frac{y_1 + kv}{z_1 + kw}\right) = \left(\frac{x_1 - uz_1/w}{z_1 + kw} + u/w, \frac{y_1 - vz_1/w}{z_1 + kw} + v/w\right)$$

using synthetic division. The righthand side is the parametric line

$$(u/w, v/w) + s(x_1 - uz_1/w, y_1 - vz_1/w)$$

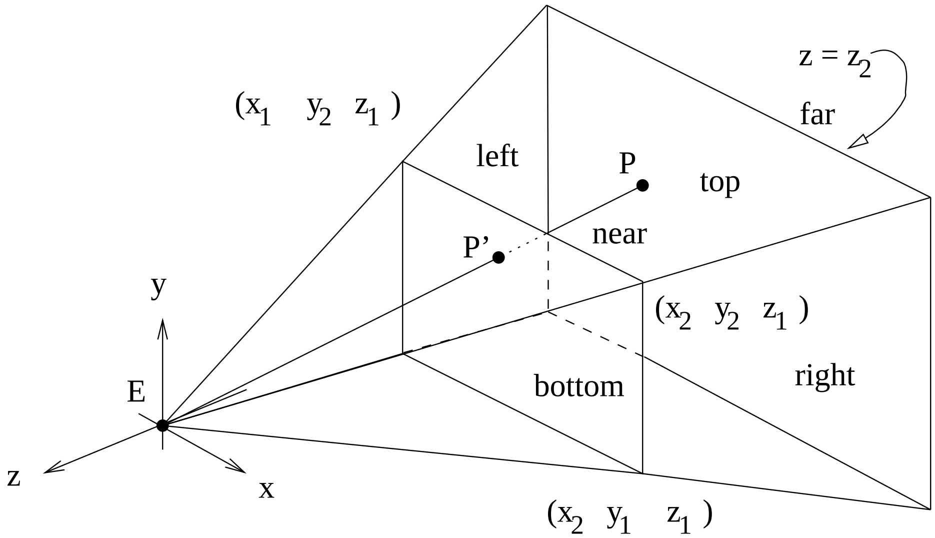
with parameter $s = (1/z_1 + kw)$.

Problem 4. OpenGL

(a) Describe and draw the OpenGL fixed pipeline.

World coordinates are transformed to camera coordinates by the modelview matrix. Camera coordinates are transformed to clip coordinates by the projection matrix. Geometry is clipped to the viewing volume. Clip coordinates are transformed to window coordinates by perspective division and the viewport matrix. The clipped projected geometry is rasterized. The geometry is converted to pixels, each pixel is assigned a color using local lighting or texture mapping, and the visible pixels are written to the frame buffer using z-buffering.

(b) Draw the OpenGL viewing volume and label the center of projection, image plane, near, far, left, right, bottom, and top planes.



Problem 5. Local lighting

(a) Using the Phong shading model, explain how to illuminate a red object using a white diffuse light with a blue specular highlight.

Assign the object a red diffuse material property $(1, 0, 0)$, a blue specular material property $(0, 0, 1)$, and a shininess coefficient of say 100. Assign the light white diffuse and specular intensities $(1, 1, 1)$.

(b) State two lighting phenomena that local lighting does *not* implement. Briefly explain why.

The local model assigns color to geometry independently of other geometry for efficiency. It cannot model one object that modifies the light that reaches another object, such as shadows and reflections.

Problem 6. Collision detection

(a) Explain how to test two 2D polygons for overlap.

Test if any A edge intersects a B edge, using solution 1b. If so, the polygons overlap. If not, test if the first A vertex is inside B or vice versa. Intersect an outward ray from the vertex with the edges of the polgyon. The vertex is inside when the number of intersections is odd. If either vertex is in the other polygon, the polygons ovelap; if not, they do not.

(b) Explain how to test for contact.

Contact occurs when there is no overlap and an A vertex is on a B edge or vice versa. Point p is on line segment ab when $(p - a) \times (b - a) = 0$ and $0 \leq (p - a) \cdot (b - a) \leq 1$.