Evaluation of Shape Grammar Rules for Urban Transport Network Design

Date of submission: 2012-11-15

Basil J. Vitins
Institute for Transport Planning and Systems (IVT), ETH Zurich, CH-8093 Zurich
phone: +41-44-633 27 02
fax: +41-44-633 10 57
vitins@ivt.baug.ethz.ch

Ignacio Garcia-Dorado
Dept. of Computer Science, Purdue University, West Lafayette, IN 47907-2066, USA
phone: +1-765-494-6010
fax: +1-765-494-0739
igarciad@purdue.edu

Carlos A. Vanegas
Institute of Urban and Regional Development, University of California, Berkeley, CA 94720-1870, USA
phone: +1-765-409-9226
fax: +1-510-642-1641
cavanegas@gmail.com

Daniel G. Aliaga
Dept. of Computer Science, Purdue University, West Lafayette, IN 47907-2066, USA
phone: +1-765-494-6010
fax: +1-765-494-0739
aliaga@purdue.edu

Kay W. Axhausen
Institute for Transport Planning and Systems (IVT), ETH Zurich, CH-8093 Zurich
phone: +41-44-633 39 43
fax: +41-44-633 10 57
axhausen@ivt.baug.ethz.ch

Words: 5487 words + 8 figures = 7487 word equivalents

ABSTRACT

Shape grammar rules are increasingly applied in urban simulation. Even though many network design standards propose shape grammar rules, little is known of the measurable impact of these rules on the performance of transport networks. This paper provides a general definition of shape grammar rules for transport network design. Different rules are evaluated regarding a comprehensive objective function. Networks are designed and simulated on featureless planes to avoid a bias due to history. Findings are compared with real-world case studies. Different network characteristics are evaluated in this paper.

The densities of network loops are high in all generated networks, and comparable with real-world grids and medieval fabrics. The average length of network loops decreases as an inverse function of road density, which is in line with graph theory. Intersection density is proportional to the network length. The average number or arms of an intersection depends on road density. A denser network has a disproportionately higher density of 4 arm intersections, compared to less denser networks.

Additionally, different road types are assigned to each road segment. Hierarchical road type distribution has a significant but low influence on network user costs. Terrain boundaries, as well as predefined roads (e.g. boulevards) increase average user costs. However, the average increase strongly depends on the number of bridges and on the boulevard capacity. The results show that shape grammar rules for transport network design can be evaluated to increase the understanding of their impacts, which supports future design standards.

INTRODUCTION

Urban network patterns have changed during the last centuries from medieval fabrics, to a grid layout, and finally to more dentritic fabrics *(1)*. Today, rapidly growing urban areas around the world require good transport systems and design recommendations. For planning purposes, transport institutions provide handbooks for network design *(*e.g. *2, 3, 4, 5)*. They propose patterns and rules that are based upon current experience, and are often rule of thumbs. However, no consistent sets of recommendations and no underlying research evidence can be found for road network design. Existing rules mostly lack a systematic evaluation, e.g. cost-benefit or statistical analyzes. Thus, research is needed to improve and refine planning guidelines and their standardization in design handbooks.

Shape grammars provide rules for how network elements of the same or different types may be added to each other. A major advantage of shape grammar rules is their straightforward application in network design *(6, 7, 8)*. Shape grammar rules are able to adapt to different network optimization and design scenarios, and even to spatial planning rules *(9, 10)*. The application of shape grammar rules has very low computational requirements *(7, 8)*. Therefore, rules are suitable for interactive planning tools *(*e.g. *6, 10, 11)* to incrementally build transport networks. They contrast for example with bi-level network optimizations, which are limited due to their computational requirements *(12, 13)*.

Network shape grammar rules can address topological characteristics. Characteristics include the numbers of arms per intersection and the densities of intersections and loops. Characteristics are also subject to design standards. However, they vary between the different network fabrics, e.g. grid and dentritic networks. We investigate these characteristics in different optimized networks.

Shape grammar rules influence infrastructure and user costs, both of which are relevant for network design. Practitioners often aim at optimizing user and infrastructure costs. Therefore, total infrastructure and user costs of a fabric are compared with the fabrics' characteristics. Road length is compared to accessibility, intersection and loop densities.

This paper describes the design of different networks on featureless planes to not bias the outcome due to history and politically driven solutions, similar to Eichler *et al. (14)* or van Nes *(15)*. For example, Yerra and Levinson *(16)* optimized network revenues to evaluate self-organization in network design. Additionally, a featureless plane allows a comparison between sets of networks designed with different rules. Therefore, the impact of the rules on network design can be evaluated for an improved understanding. The design of the networks is an optimization problem, subject to given infrastructure budgets. When networks are optimized according to an objective, e.g. generalized costs, they can be compared regarding their characteristics and properties.

The findings are compared with Cardillo *et al. (17)*. They showed in a graph-based evaluation the low performance of modern, dendritic transport networks, e.g. Irvine, Brasilia, Walnut Creek, and better performance in medieval (e.g. Ahmedabad, Cairo, London, Venice) and grid networks.

Definition of Shape Grammar Rules in Transportation

Shape grammar rules are defined differently in separate fields of science. Chomsky *(18)* and Stiny and Mitchell *(19)* provide definitions for linguistics and urban planning, respectively. The definition below focuses specifically on transport planning.

Shape grammars provide a finite number of rules of how network elements *e* of the same or different type are added to each other. *I* defines the initial stage where the network design process starts. *E* is the finite set of generic transport network elements *e*. *R* is a set of shape grammar rules *r* in the form of $\alpha \rightarrow \beta$, where $(\alpha,\beta) \in E$. $\alpha \neq \beta$, which means that an element *e* cannot be transformed into itself. *R* includes rules to stop the algorithm after initialization. Shape grammars allow the users to create a very large set of potentiala transport networks *N*. The large set is due to the high number of combinations of the different rules.

The rules *R* depict how an existing planning state and geometry can be extended , e.g. if a major arterial road can be crossed by a local access road, or if an intersection can have more than five arms. The elements *e* can further be subdivided for more details, to follow further rules, and to cover additional fields in urban planning, besides transportation. All rules *r* help to define useful networks and prevent impractical and overly expensive networks. They can be stated generically and independently of any case study, which makes a particular shape grammar even more valuable.

Example Shape Grammar Rules

The generation of an urban layout is arbitrarily complex. Numerous rules for urban and transport network design can be stated for a generic city layout *(*e.g. *7, 9, 20, 21)*. This paper focuses on transport networks and its elements; building blocks are not subdivided further. In the following, example rules are explained for illustration, which address road and intersection type hierarchies in network design, derived from Marshall *(9)*.

E is the set of defined, generic road and intersection elements *e*. The set *R* encompasses different rules such as: (r_1) network connectivity is obtained by requiring arterial roads to connect to other arterial roads, to simulate network growth; (r_2) an arterial can also be joined with an access road if a connected arterial network is maintained; and (r_3) connecting an access road to a local road requires using a right of way junction; therefore, r_3 refers to intersection type choice. r_1, r_2, r_3 are exemplarily listed below. An example *R* is visualized in Figure 1.

$R=\{r_1, r_2, r_3, ...\}$, with	$E=\{e_1, e_2, e_3, ...\}$, with
r_1: $e_1 \rightarrow e_1 + e_1$	e_1 = *arterial road*
r_2: $e_1 + e_1 \rightarrow e_1 + e_1 + e_2$	e_2 = *access road*
r_3: $e_2 + e_3 \rightarrow e_2 + e_3 + e_4/e_5$	e_3 = *local road*; e_4, e_5 = *right of way junctions*
r_4: ...	...

Research Question 1 and 2

Research question 1 aims at the evaluation of existing shape grammar rules, e.g. recommended number of arms *(20, 22)*, redundancy *(4)*, and their impact on infrastructure expenses. The question is whether existing rules can be determined for efficient urban transport networks, considering a given comprehensive objective function, and infrastructure budget constraints.

Research question 2 aims to describe the influence of shape grammar rules on network design. Only if the influences of existing or new rules are known, can recommendations for design standards be made for the future. The effect of the rules on network design should be quantitatively assessed in order to support any potential recommendation for future network design.

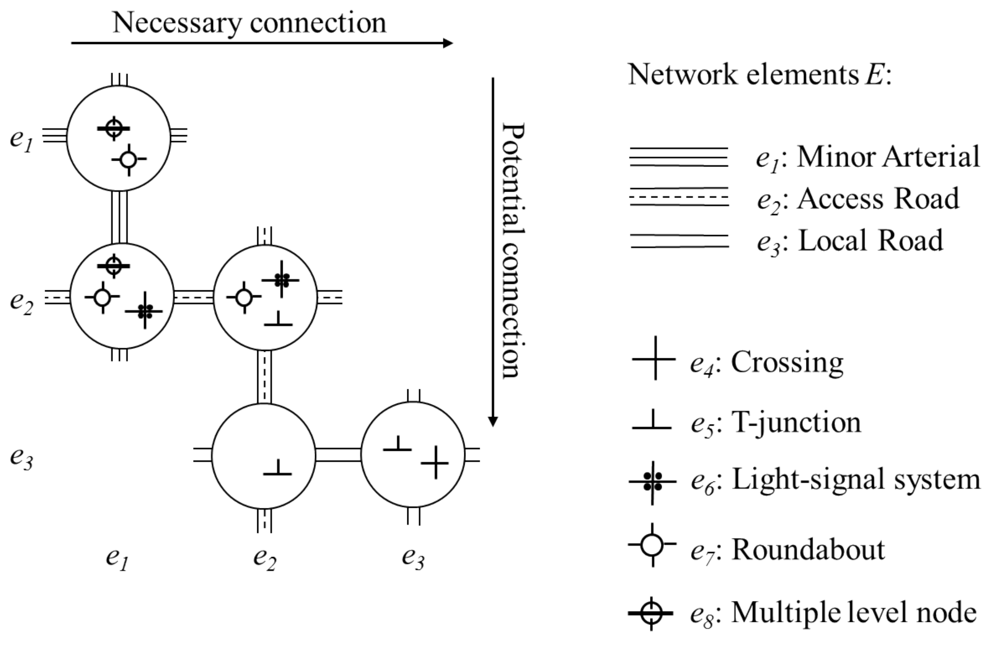

FIGURE 1 The example rules suggest hierarchical road and intersection type distribution (based on *9*).

Existing transportation networks and patterns are historically contingent, and, therefore, are only used for verification of the results. Instead, artificial transport networks are designed, similar to e.g. *(16, 23, 24)*. This approach is additionally suitable for the definition of new rules, and for comparison between different rules.

Loops, blocks, and their characteristics are essential in redundant transport networks and relevant for future design handbooks. They should be considered in this paper. Loops (graph theory: cycles) reduce congestion, lower travel times and improve redundancy in case of network failures. Blocks (bounded faces) are regions enclosed by a loop of links (edges) in a planar graph without any link from the loop going inside the region. Loops and blocks are elements of redundant networks, in contrast with tree networks. In tree networks, a network link failure causes two subtrees and therefore a separation of the originally covered network area in two separated subareas. In this study, by definition, one loop always refers to only one block and vice versa. Two adjacent loops are counted as two loops with two blocks.

METHODOLOGY

Network Design

Problem description

In literature, the network design problem has been studied in depth, some examples are *(25, 26)*. Networks are designed and evaluated according to an objective function, which is defined in advance, independent of the rules and the design method. The problem statement encompasses the candidate links $\mathbf{x}$ between nodes $(i, j) \in N$ of length $l_{i,j}$. Additionally, link type $t \in T$ is determined and the corresponding infrastructure costs w_t, which comprises construction cost *(27)*, but omit maintenance cost for simplicity here. Including T refines the problem definition, compared to a standard definition in literature. c defines the generalized user costs. A penalty factor p_+ penalizes budget B [Mio \$] violation. The total costs are due to minimization:

$$\begin{aligned}
\text{minimize} \quad & a + c + p_+ \\
\text{subject to} \quad & a = \sum_{(i,j)} x_{i,j,t} \cdot l_{i,j} \cdot w_t \\
& c = f(\mathbf{x}) \\
& p_+ = \begin{cases} 0.0 & \text{if } a < B, \\ 20.0 \cdot (a - B) & \text{else.} \end{cases} \\
\text{whereas} \quad & (i,j) \in N, t \in T, \mathbf{x} \in \{0,1\}^{|N| \times |N| \times |T|} \\
& \mathbf{c} > \mathbf{0}, B > \text{cost for minimum spanning tree network.}
\end{aligned}$$

Generalized User Costs c

The generalized user costs comprise demand weighted travel time according to travel distance *(28)*, wear and fuel cost. Calculation of total travel time is the computationally most expensive measure. Therefore, the function can be easily enriched with further quantitative or semi-quantitative variables, without adding additional computational time.

$$c = f_{gen.\,user\,costs} = \left(\sum_{o=1}^{O} \sum_{d=1}^{D} demand_{od} \cdot \Big(tt_{od} \cdot \gamma(l_{od}) + distancecost_{od} + fuelcost_{od} \Big) \right)$$

o, d: Origin and destination demand generating nodes.
tt_{od}: Travel time between o and d.
$\gamma(l_{od})$: Weighting factor (value of time as a resource), dependent on travel distance l_{od} extrapolated for a year.

This paper focuses on an economic perspective, therefore the function excludes aspects such as quality of urban life, safety issues, and environmental factors. However, we claim that from an economic perspective, it is crucial to optimize travelers' generalized costs, due to their considerable economic relevance (e.g. *29)*. We anticipate rules can be adapted in the future to implement those new criteria.

Network Design Algorithm

Our network design algorithm is able to generate many feasible transport networks that satisfy the aforementioned objective function. Network elements are exchanged between different candidate networks to generate more efficient networks as per our objective function. The design method is an integration of Ant Colony optimization with a Genetic Algorithm (IACGA). Both are applied for discrete optimizations and are suitable for network generation problems. They are merged in order to reduce computational times. Due to their heuristic nature, the IACGA does not guarantee to find the optimum solution. The full algorithm is described in Vitins *et al.* *(27)*. The algorithm can implement shape grammar rules.

The network design algorithm IACGA is capable of designing networks for different infrastructure budgets. Higher infrastructure budgets lead to denser networks, whereas lower budgets to less dense networks. The IACGA designs car networks, in contrast to other modes, like transit. However, car networks are considered here due to the fact that car is a major transport mode, also in multimodal networks.

Study Design

Evaluation of Shape Grammar Rules

Two separate subsets of rules are evaluated differently in this paper. The first subset ***A*** is extracted from most optimized networks. The rules of the second subset ***B*** are implemented during the design process:

***Shape grammar rule set A*:** A set of transportation networks are designed with the IACGA, but without any restrictions on topology and node design. The starting point is a plain grid with candidate links (Figure 2 below). Afterwards, the networks are evaluated regarding the following criteria:

- Average loop length and density
- Share of number of arms at the intersections
- Intersection density
- User costs
- Accessibility

This approach is similar to case study analyzes *(*e.g. *17)*, and to abstract network evaluations *(*e.g. *16, 23)*.

***Shape grammar rule set B*:** Subsets of networks generated with shape grammar rules can be compared with subsets of networks, which are generated with different rules *(*similar to *14, 15)*. Therefore, the comparison allows statistical testing between the subsets. The following rules are evaluated:

- Hierarchical link type distribution
- Block length and width ratios
- Inclusion of Boulevards
- Number of passages at linear terrain constraints (e.g. rivers, highways..)

Rules in ***B*** are unsuitable for evaluation of historical networks due to the fact that ***B*** compares subsets of artificial networks with different underlying shape grammars. However, the comparison between the subsets allows a quantitative evaluation of the effect of shape grammar rules, and of their combinations.

Configurational Background

The networks designed in this paper follow the configuration below:

- According to Cardillo *et al. (17)*, the average length of links in a network is between 30[m] and 130[m] in dense urban areas. A default value of 100[m] is assumed for each block size. However, this paper also evaluates increasing rectangle lengths.
- Strano *et al. (30)* evaluated historical network development and observed a transformation towards a rectangular and quadratic block shape. In their 20 case studies, Cardillo *et al. (17)* found very few 5 or 6 arm intersections. This paper assumes rectangular blocks.
- Travel demand is assigned to the network with the deterministic travel time user equilibrium, based on the BPR function *(31)*, Dijkstra *(32)*, and MSA due to the simple implementation and acceptable computational time in small networks. The weighting factor is set conservatively according to the previous results. Turn delays are disregarded except when stated explicitly.
- 10% of the trips are distributed on the generated networks *(33)*. 90% of the trips leave and enter the study area by default on the designated two through streets (Figure 2). Trip

distribution is equal in all networks. Routes outside the area are not considered in the design process. All trip purposes are included in the travel demand.

- Streets have to fulfill different functionalities. They serve not only for transportation, but also for shopping and as parking, leisure and recreation etc.. Regarding transportation, different modes share the same space. Streets are closed to return space for other modes like public transportation, bicycles or pedestrians. Alexander *et al.* *(20)* or Dutton *(34)* stated that streets can be pedestrianized for improved urban quality.

We generate new networks which are based on a grid structure, but not necessarily a full grid (see Vitins *et al.* *(27)* for a more relaxed example). Figure 2 shows a full grid and potential variation of the grid structure, subject to the condition that all demand generating points (centroids) are connected to the same network. Also, blocks can vary in length. As part of a regional network, two east–west through streets are given in advance on north and south end, respectively. The area considered for the network simulations is 900x900[m^2], and a smaller one of 600x600[m^2] to save computational time. The design of 900x900[m^2] networks takes about 36[h] on 30 parallel threads and 2.4[GHz] , indicating the complexity of network design.

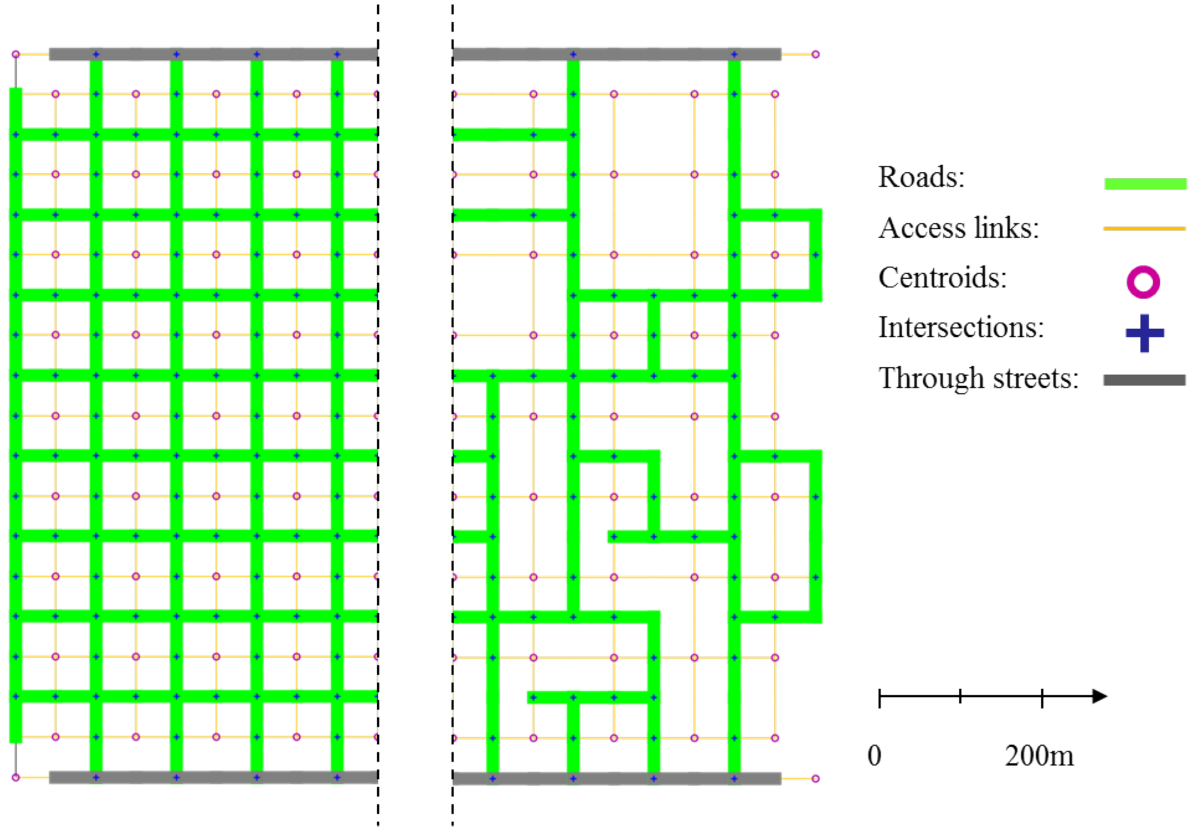

FIGURE 2 Base network layout of a full grid on the left side (half of a 900x900[m^2] network), and an reduced grid on the right side.

Quantities for Travel Demand Estimation

The data for travel demand estimation (listed below) refers to a medium dense neighborhood in Zurich *(35)*. The listed quantities are taken as default parameter values, if not stated differently.

• Population density:	15’068 $[pers/km^2]$
• Job density:	6’685 $[jobs/km^2]$
• Car trips per resident (as a driver):	1.32 $[trips/pers./day]$
• Car trips per employee:	0.47 [%]
• Average car trips:	26’172 $[trips/km^2/day]$
• Average lengths of car trips:	23.86 $[km]$

It can be assumed that the buildings are distributed evenly in the blocks, so the travel demand is generated evenly over the entire study area. For simplification, the buildings are not displayed in Figure 2.

EVALUATION OF THE SHAPE GRAMMAR RULES

Shape Grammar rule set A

Figure 3 and 4 summarize characteristics of transportation networks designed from which three network properties are emphasized in the following. Each data point refers to a network, which are designed without any geographical restrictions.

In both figures, the horizontal axis refers to the infrastructure budget. A high infrastructure budget leads automatically to a more grid like structure (Figure 2 left hand side). 100% infrastructure budget allows a full grid. Lowering the infrastructure budget reduces the total link length in the network proportionally. However, the network design algorithm, described above, suggests optimized networks under the given budget and objective function.

Network loops and blocks

The number and sizes of the loops are evaluated as a function of the infrastructure budget. Figure 3 shows the average lengths of the loops on the right hand vertical axis. The data is not evenly spread due to the discrete grid structure. Additionally, the applied algorithm IACGA optimizes infrastructure and user costs (see above), and adapts the final infrastructure cost during optimization.

The results show that average loop length of the network is not decreasing linearly with increasing budget. The loop length c decreases in inverse proportion to the total road density D: $c = f\left(\frac{1}{D}\right)$, which is also reasonable for general graphs.

The number of loops increases with higher budgets due to the fact that the loop length is reduced. This finding is inline with the general understanding of transport networks, with standards of network design, e.g. VSS *(36)* and Alexander *et al. (20)*, where redundant structures are proposed for network design.

Case Study Comparison of Network Loops and Blocks

The meshedness coefficient M *(37, 38)* considers the density of loops (cycles) and blocks (faces). M is the number of loops F divided by the maximum number of loops F_{max}, $F_{max} = 2N - 5$, with N nodes. $M = F/F_{max}$ can vary from 0 (tree structure) to 1 (maximally connected planar graph). The generated networks in Figure 3 have an average coefficient of M=0.28 (σ=0.052) for established 600x600[m^2] networks and M=0.25 (σ=0.075) for 900x900[m^2] networks, independent of their budget restrictions. These values are similar to values of cities with grid layouts (e.g. Barcelona, Richmond) as well as medieval fabrics (e.g. Ahmedabad, Cairo, London). This is interesting since both patterns like medieval fabrics, and grid patterns can have

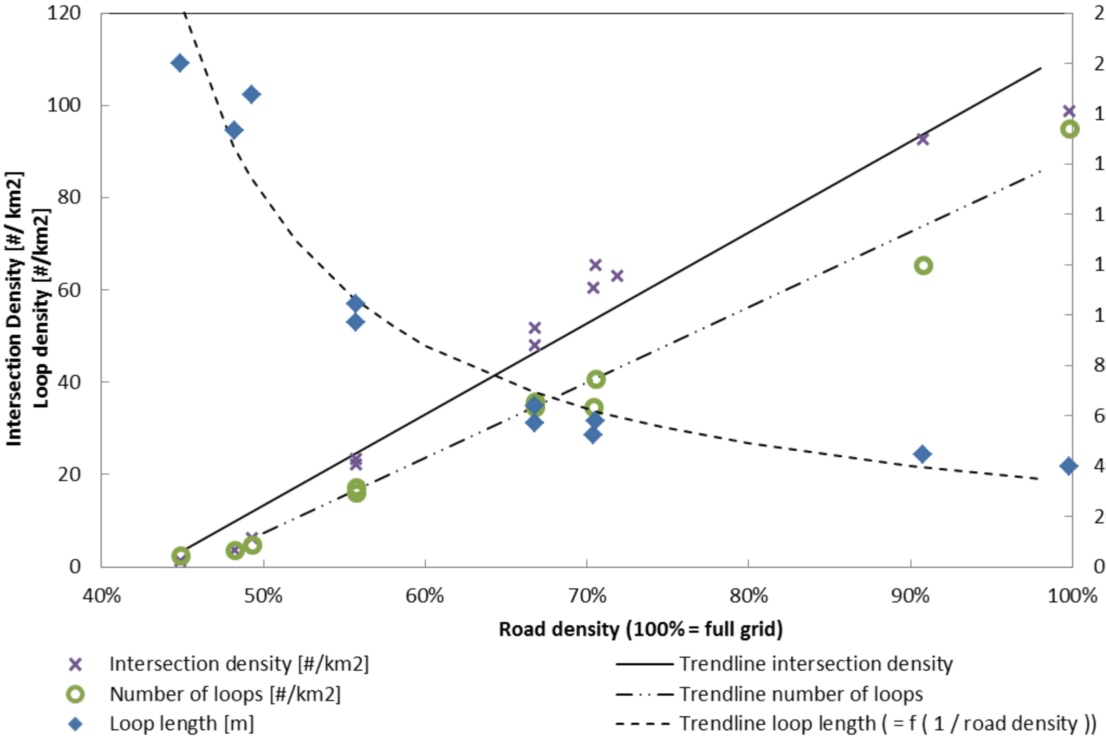

FIGURE 3 Intersection and loop densities of 900x900m^2 networks.

a high M value *(17)*. In contrast, Irvine and Walnut Creek have a coefficient M<0.1, due to their dendritic layout. However, high M values are achieved in the generated networks, and, after comparison with *(17)*, generally are more economically efficient networks.

Intersection Density and Types

Intersection density increases linearly with infrastructure investment (Figure 3). Intersections at through streets (Figure 2) are not counted due to boundary effects. This results in zero intersections at infrastructure budgets < 45% (Figure 3), as only intersections at the through streets remain.

Southworth and Ben-Joseph *(22)* as well as Alexander *et al. (20)* favor 3 arm intersections (T-junctions) instead of 4 arm intersections (crossings) for various reasons (safety, redundancy, avoidance of through traffic). T-junctions are favored in the United States *(22)*.

The share of 3 and 4 arm intersections are shown in Figure 4. Boundary effects can also occur on left and right borders, leading to a maximum share of 80% of 4 arm intersections. In Figure 4, 4 arm intersections are predominant when approaching a full grid (100%). However, lowering the budget below 85% leads to a predominance of 3 arm intersections. This effect is remarkable, and in line with Strano *et al. (30)* who observed that piecemeal urbanization and denser networks lead to an increasing share of 4 arm intersections.

Case Study Comparison of Intersection Density

Strano *et al. (30)* reported shares of 11% to 15% for 4 arm intersections and 87% to 84% for 3 arm intersections in their study area in northern Italy. These shares are similar to the results

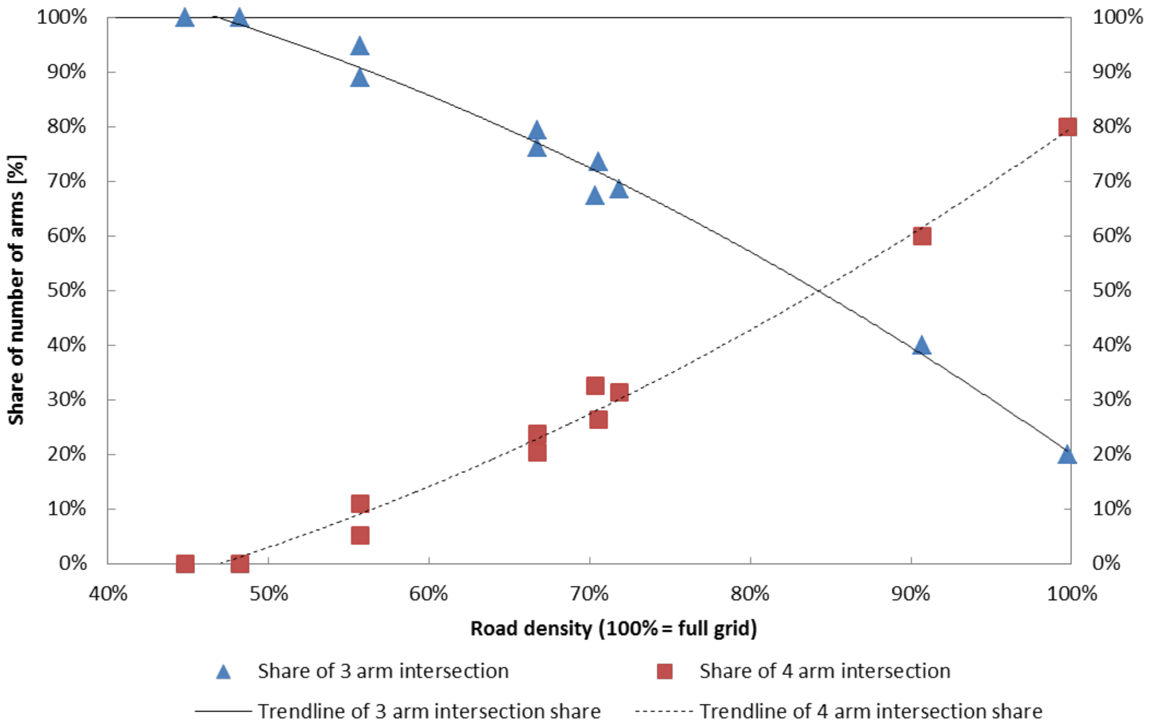

FIGURE 4 Share of number of arms as a function of infrastructure expenses

shown in Figure 4, especially since Strano *et al.* *(30)* results are based on a less dense study area. Strano *et al.* *(30)* observed that a higher density of 4 arm intersections does not have to result from a large-scale planning, but can also arise from a piecemeal urbanization.

Cardillo *et al.* *(17)* found that grid layouts as well as medieval fabrics (e.g. Ahmedabad, Cairo, London, Venice, etc.) can be efficient regarding the shortest paths between arbitrarily chosen origins and destinations. However, the average number of arms varies between the two classes of fabrics. The generated networks with budget <70% differ from classical grid structures, similar to medieval fabrics with a lower shares of 4 arm intersections. Future research on network design and turn delay (e.g. *39)* will give additional insights.

Infrastructure Costs and Accessibility

An advantage of the applied network design algorithm is its ability to adapt to different objective functions. Complementing Figure 3 and 4, Figure 5 refers to the objective function proposed above, as well as an additional accessibility measure, calculated separately. Accessibility is defined here as the logsum term giving the expected utility of all alternatives *(40)*. The accessibility is weighted with the number of residents benefiting from it.

$$\text{Total Accessibility} = \sum_{i=1}^{I} B_i \cdot \underbrace{ln\left(\sum_{j=1}^{I} A_j \cdot f(c_{ij})\right)}_{\text{Accessibility of location } i}$$

I: The set of locations i and j in consideration.
A_i: Attractiveness of location i (here: sum of workplaces and residents).
B_i: Weighting the accessibility (here: number of residents).
$f(c_{ij})$: Weighting function, dependent on the generalized costs of travel c_{ij} (here: $f(c_{ij}) = e^{-\beta c_{ij}}, \beta = 0.2, c_{ij}$ = travel time).

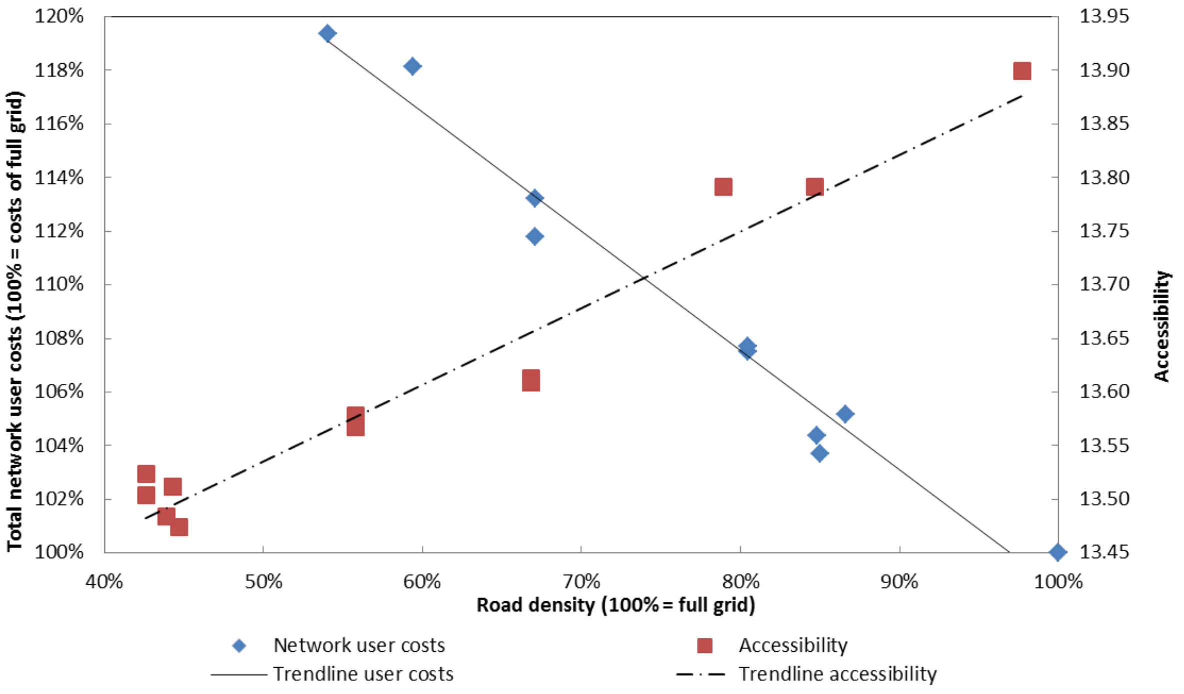

FIGURE 5 User costs and accessibility as a function of infrastructure expenses

Figure 5 shows the user costs of 900x900[m^2] networks and accessibilities of 600x600[m^2] networks as a function of infrastructure budget. Again, 100% budget refers to a full grid. Figure 5 shows a linear decrease of user costs when approaching a full grid. The decrease is mainly due to less detours in the origin destination paths. Total travel times decrease virtually with the same slope.

Accessibility increases linearly with an increasing infrastructure budget. The linearity is due to the fact that travel time c_{ij} is inserted in the exponent, and the accessibility of location i is logarithmized.

Case Study Comparison of Infrastructure Costs and Accessibility

Cardillo *et al. (17)* found that the different network patterns have different road densities. Grid networks are more dense, compared to medieval networks (e.g. Ahmedabad, Cairo). These findings can be confirmed with the results (Figure 5). As expected, grids perform best, compared to networks with lower road densities. However, the decrease of the user costs is small, compared to the decrease in road density. A decrease of 50% in infrastructure budget only causes an increase of about 20% in user costs. This finding is remarkable, and indicates, that not only 100% grid networks can perform well for transport purposes. This inelastic relationship is similar to the findings in Cardillo *et al. (17)*, where they compared non-grid networks, i.e. medieval networks, which cause less than 100% grid network costs, but which are almost as efficient. Additional research, e.g. on urban access roads, will provide more insights.

Shape Grammar rule set B

In the following, subsets of networks are compared among each other, differing in their implemented shape grammar rules.

Hierarchical Link Type Distribution

Hierarchical rules for link type distribution are proposed by many network design handbooks *(*e.g. *2, 3, 4)*. However, the economic effect on network performance was, to the authors' knowledge, never evaluated before. Vitins *et al. (27)* assessed hierarchical rules, focussing on regional scale networks. Here, hierarchical rules are evaluated for urban grid structures.

Link types are selected according to marginal generalized travel time and construction costs, and compared to alternative link types. Links with the highest link type additionally have to form a connected sub-network. The definition is given as r_1 and r_2 in Section *Example Shape Grammar Rules*. Networks generated with r_1 and r_2 are compared with networks following no hierarchical link type distribution.

Increasing user costs are expected due to the constraints given by the hierarchical rules, and resulted in +5.0% user costs ($n_{total} = 8$, $p = 0.020\%$), when considering hierarchical networks. However, the increase is moderate, and similar to previous results *(27)*. This finding supports a hierarchical network structure, when minor losses in performance are acceptable, in return for a more structured and safer network.

Variable Block Length

Strano *et al. (30)* found that the predominant block shape is a rectangle or square. Larger blocks are expected to increase route lengths and therefore travel time. Additionally, increasing travel distance reduce speed and increase user costs. The quantitative effect of the user cost changes are addressed in the following.

A set of networks with constant 9x9 blocks each are designed with same infrastructure budget per area (60% of a full grid network at a square block shape) for comparison reasons. The block length increases piecewise up to 500%; however, the block widths remain at the same. Due to the increasing total area, the total budget linearly increases for the networks with 9x9 blocks.

Independent of the variable block lengths, the densities of population and working places are increased to 200% and 300%, respectively, to verify the effect for higher traffic volumes. A density of 100% refers to the population default values in Section *Quantities for Travel Demand Estimation*. The resulting networks (n_{total}=15) are compared against each other.

The user costs increase disproportionately with increasing block length (Figure 6). This effect occurs especially for long block lengths and high densities, where user costs increase considerably. The disproportionate increase is (1) due to the increasing value of travel time savings at longer distances *(28)*, and (2) due to the increasing network loadings, causing delays due to the BPR function *(31)*. Additionally, optimized block spacing depends on the resident and job densities.

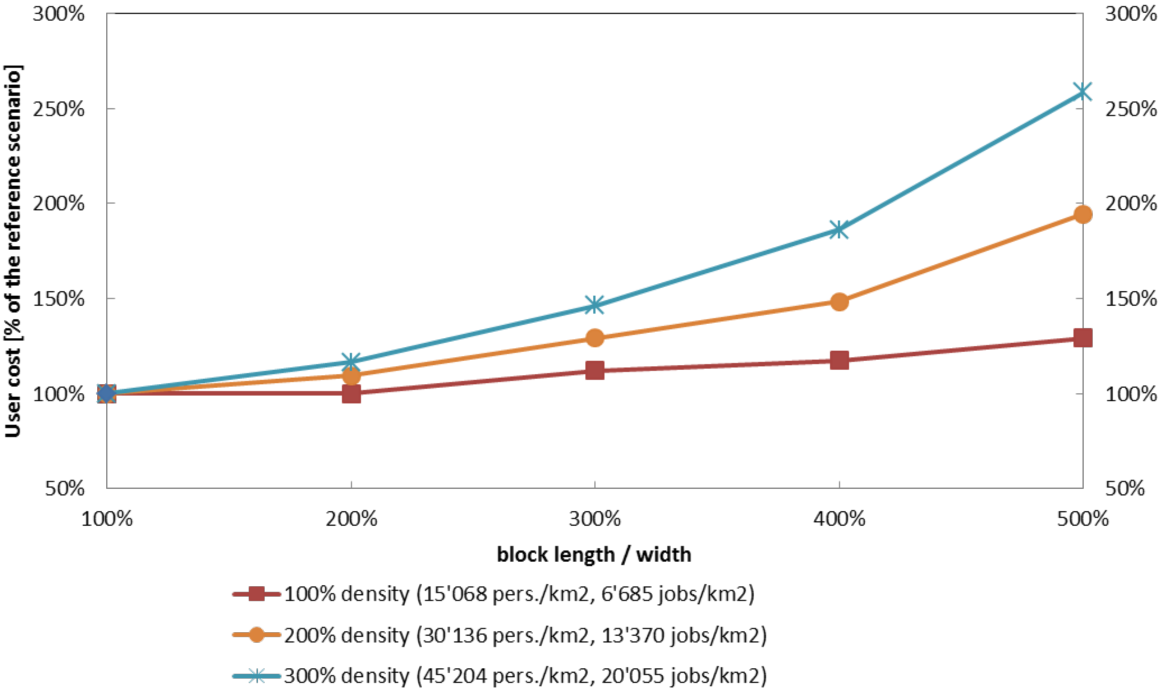

FIGURE 6 User cost sensitivity due to variable block length in a 600x600m^2 network.

Boulevards

Boulevards are fundamental in urban planning *(20, 41)*. Often, turn restrictions limit access on and off the boulevard *(41)*. Many boulevards allow only a right turn to get on and off the boulevard. Additionally, cars first access parallel one-way frontage roads. Access to the center through lanes is only provided occasionally *(41)*. The scenarios shown in Figure 7 (n_{total}=36) have the same infrastructure budget (60% of a full grid network). However, the boulevard type changes from a local road type to an arterial meaning that capacity and speed increases, which affects the user costs. Turn restrictions are taken into account on the right side of Figure 7, including a delay for the slower frontage road. The boulevard is located on a diagonal axis across the grid network. The boulevard's exact location is shown in Figure 8. A diagonal boulevard is simulated due to the fact that connected link type distribution in the grid is already evaluated with the *Hierarchical Shape Grammar Rules* above. Short links occur to a certain extend, and therefore potential spill backs. However, the remaining network is kept unmodified for comparison reason.

Diagonal boulevards increase overall travel times for a constant infrastructure budget. Especially turn restrictions increase travel time considerably. When increasing local traffic (in-perimeter traffic = 50% or 100%), user costs increase even more. Therefore, boulevards have a negative impact on transport user costs from a transportation perspective. Of course, boulevards have many other functionalities, e.g. pedestrian areas, city quality, shopping facilities. These functionalities are not taken into account and have to be considered in the future. Reduced

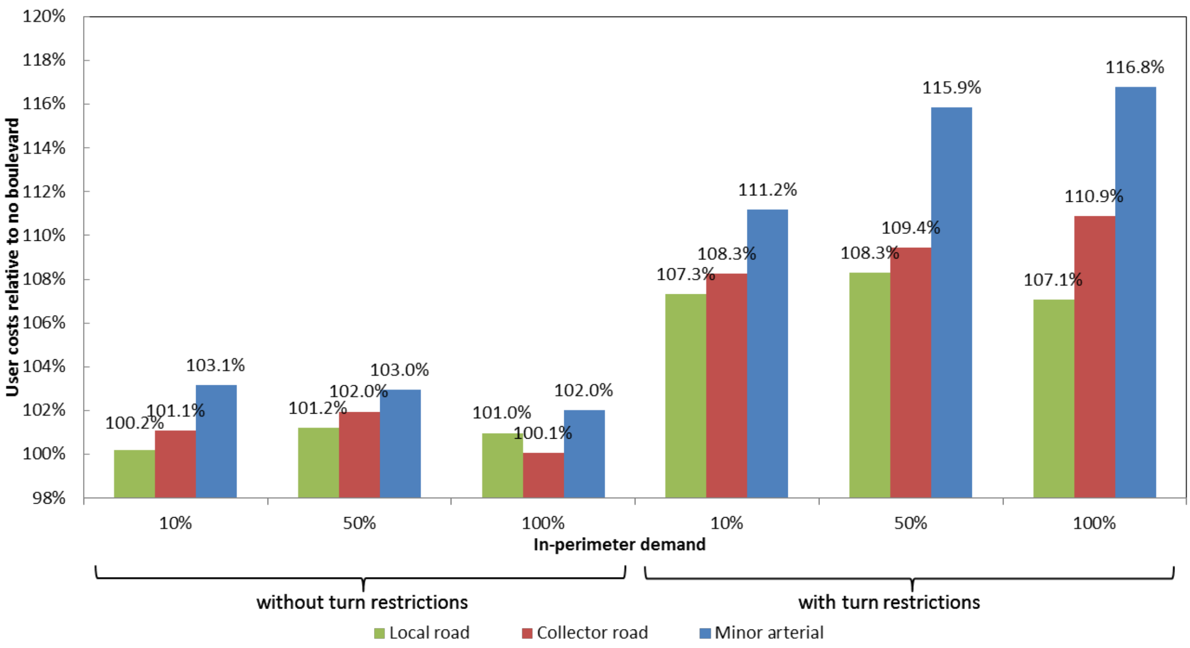

FIGURE 7 User cost sensitivity due to different boulevard types and in-perimeter demand.

capacity even reduces network user costs, due to the fact that the savings can be invested in other roads more efficiently, when assuming equal infrastructure budgets. More insights in turn restrictions (e.g. *14, 39)*, and variable through traffic on the boulevard will increase the understanding in the future.

Variable Number of Passages crossing Linear Terrain Boundaries

Linear terrain boundaries often occur in urban environments, e.g. highways, rivers, railways. The number of passages vary and effect the network performance. In this paper, the linear boundary crosses the network (Figure 2, Figure 8) from left to right. Therefore, 7 (600x600[m^2] area size) and 10 (900x900[m^2] area size) potential passages over the linear terrain boundary exist by default. However, the number of passages are reduced subsequently to only one passage. The link costs are equal for the passages and the remaining network, for improved interpretation.

The results show the increasing network user costs due to the reduced number of passages. Just one passage clearly increases user cost most (~+6%), due to route change and speed reduction. Surprisingly, the differences between 7 potential passages and 3 passages is very low (~+1.5%). This is due to the fact that performance losses are low when reducing road density in an optimal way.

Visualization of Urban Shape Grammar Rules

Visualizations of the rules are difficult. Schemes similar to Figure 1 help to understand the relationships between the network elements. However, they omit the larger picture of the entire urban area. New advances in computer graphics can improve the visualization of the shape grammar rules and their effect on the shape of the urban environment. New software tools account for rules in transport networks, building and architecture, urban planning and benefit of synergies. This is very valuable especially in an open planning process with authorities, other stakeholders, and the public. Interactive 3D renderings enable the planner to incrementally specify the design, and have the system complete the rest according to the recommended rules. Thus, an interactive planning framework can be used with adaptive control possibilities.

The open source software QtUrban based on Vanegas *et al. (11)* was adopted for visualization purposes. It combines enhancements, such as road networks with road types, building typology, terrain boundaries, control of the population and job densities. The final rendering in Figure 8 includes a boulevard, linear terrain boundary, slightly increased rectangle length, and hierarchical link type distribution. The floor space is set at 47.7[m^2/resident] and 40.9[m^2/workplace] *(35)*. Street widths are taken from AASHTO *(2)*.

Figure 8 allows a deeper interpretation in the shape grammar rules defined above. It visually shows the distribution of the road types and the connected arterial network. The generated parcels depend on the loops and blocks, they can be verified and adapted, if necessary. The block spacing, evaluated above, seems reasonable in the urban context. The network adapts to the linear terrain boundaries. The effect of the boulevard for urban planners is visible. The population and job densities relate to the building volumes in the 3D visualization. Further work on dependencies between network characteristics and population distributions can be integrated in such a framework.

DISCUSSION AND CONCLUSION

This paper investigates the complexity of transport network design in urban areas. Here, the effect of shape grammar rules are evaluated for user costs. The described novel approach bridges the gap between shape grammar rules and an independent objective function. This capability enables the estimation of the effect of rules, which so far were based mostly on intuition and little systematic testing.

The results are based on networks built with default traffic parameters, average urban densities, and on empty planes to avoid a bias due to history. The findings are compared and confirmed with empirical data from different network types worldwide (e.g. *17, 30)*, and prior results *(27)*. The performances of the emerging network designs were compared using two different utility functions. Notably, lowering the infrastructure budget and less grid-like patterns did not increase user costs as much as expected. Cardillo *et al. (17)* confirmed that also high performing network patterns exist beside complete 100% grids.

The number of arms per intersections, a long term debate in network design, depends on the road density, and does not interact with the performance of the network directly. The density of 3 arm intersections remains higher than 4 arm intersection density up to nearly (~ 85 – 90%) a full grid structure. Therefore, the number of arms depends on the road density. However, the optimal number of intersections increase linearly with the infrastructure budget.

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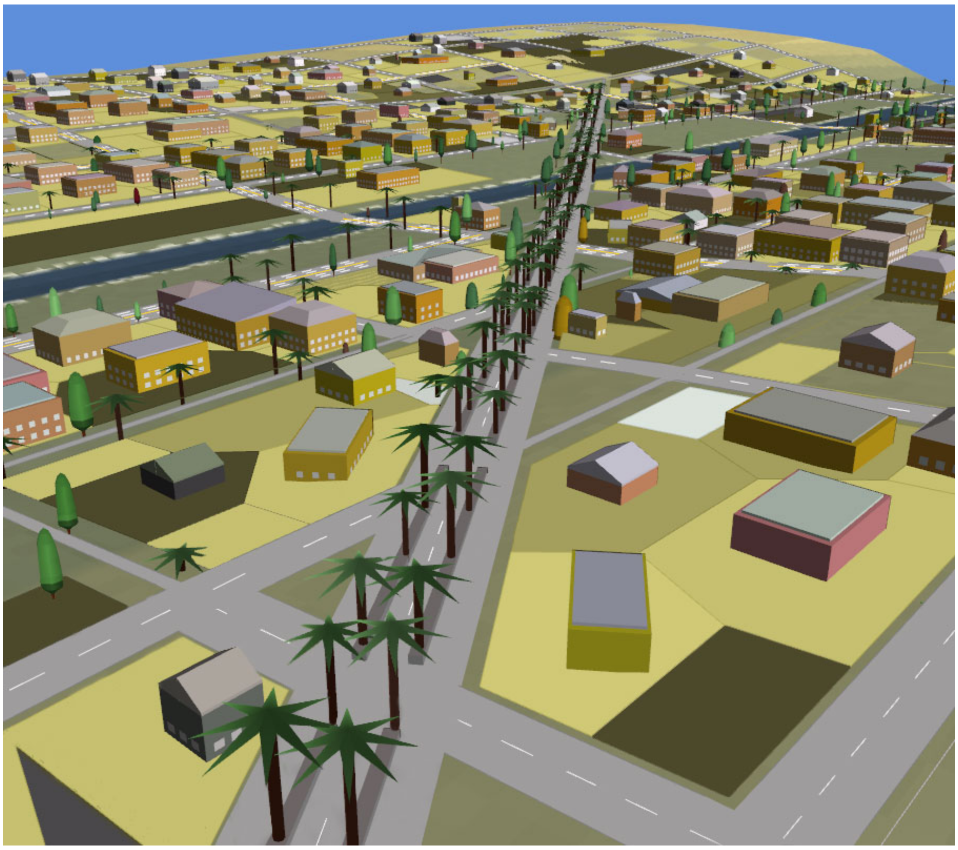
(a) A city example with all implemented shape grammar rules.



(b) A more detailed view of a boulevard with parallel frontage roads.

FIGURE 8 3D shape grammar implementation and visualization.

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