Fundamentals of information transmission and coding (a.k.a. communication theory)

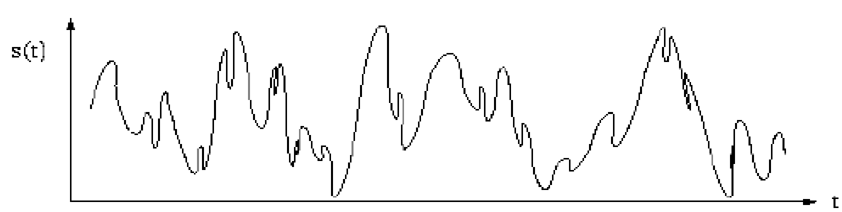
Signals and functions

Elementary operation of communication: send *signal* on medium from A to B.

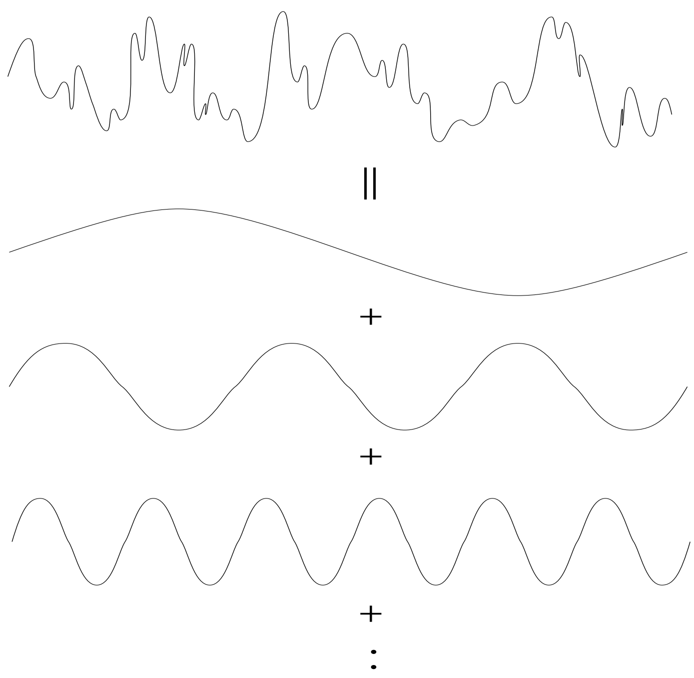
- media—copper wire, optical fiber, air/space, ...
- signals—voltage and currents, light pulses, radio waves, microwaves, ...

  $\rightarrow$ electromagnetic wave (let there be light!)

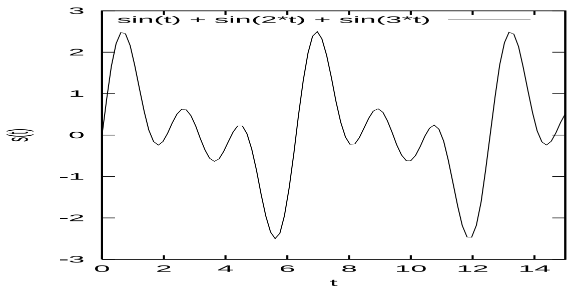
Signal can be viewed as a time-varying function $s(t)$.

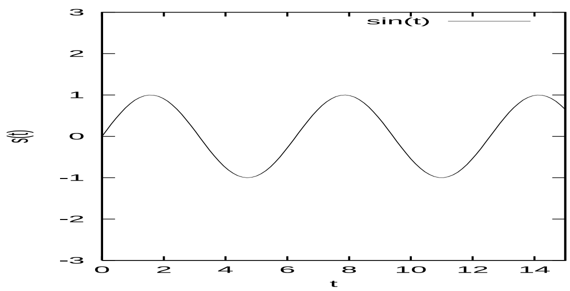


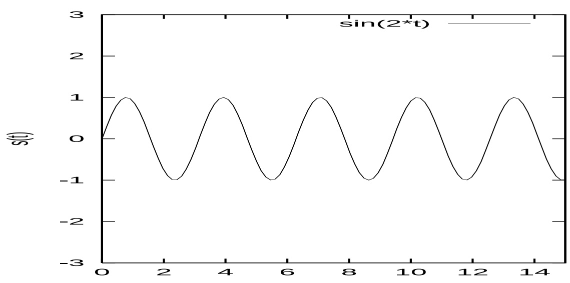
If $s(t)$ is "sufficiently nice" (Dirichlet conditions) then $s(t)$ can be represented as a linear combination of complex sinusoids:

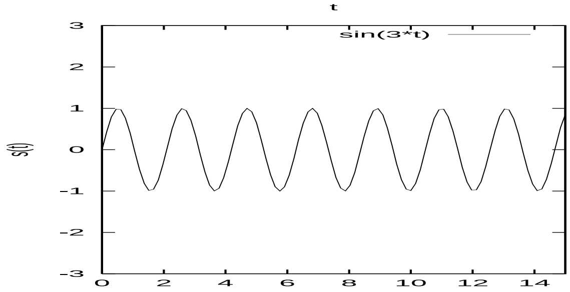


Simple example:









$\longrightarrow$ sinusoids form basis for other signals

Analogous to basis in linear algebra:

other elements can be expressed as linear combinations of "elementary" elements in the basis set

$\longrightarrow$ like atoms

Ex.: in 3-D, $\{(1, 0, 0),\ (0, 1, 0),\ (0, 0, 1)\}$ form a basis.

$\longrightarrow$ $(7, 2, 4) = 7 \cdot (1, 0, 0) + 2 \cdot (0, 1, 0) + 4 \cdot (0, 0, 1)$

$\longrightarrow$ coefficients: 7, 2, 4

$\longrightarrow$ spectrum

How many elements are there in a basis?

Vector spaces:

- finite dimensional
- infinite dimensional: signals

$\rightarrow$ infinite number of bases

$\rightarrow$ subject of functional analysis

Given an arbitrary element in the vector space, how to find the coefficient of basis elements?

$\longrightarrow$ e.g., given $(7, 2, 4)$, coefficient of $(0, 1, 0)$?

In linear algebra, matrix inversion:

$$A\boldsymbol{x} = \boldsymbol{y} \quad \Leftrightarrow \quad \boldsymbol{x} = A^{-1}\boldsymbol{y}$$

where A is $(n \times n)$ matrix, $\boldsymbol{x}$ and $\boldsymbol{y}$ are $(n \times 1)$ vectors.

$\longrightarrow$ solution techniques: e.g., Gaussian elimination

Note: the arbitrary vector $\boldsymbol{y}$ (our "signal") is represented as a linear combination

$$\boldsymbol{y} = A\boldsymbol{x} = x_1 A_1 + x_2 A_2 + \cdots + x_n A_n$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ and A_i is the ith column vector of A.

$\longrightarrow$ the A_i's are the bases!

$\longrightarrow$ correct viewpoint of the world (for us)

For continuous (i.e., infinite dimensional) signals . . .

Fourier expansion and transform:

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{i\omega t} d\omega,$$

$$S(\omega) = \int_{-\infty}^{\infty} s(t) e^{-i\omega t} dt.$$

$\longrightarrow$ recall: $e^{i\omega t} = \cos\omega t + i \sin\omega t$

$\longrightarrow$ signal $s(t)$ is a linear combination of the $e^{i\omega t}$'s

$\longrightarrow$ $S(\omega)$: coefficient of basis elements

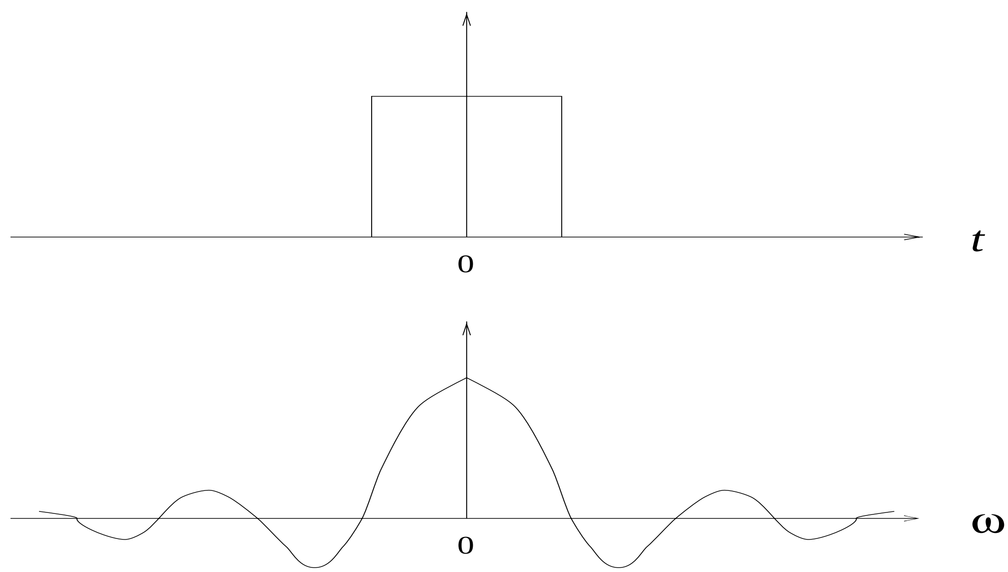
$\longrightarrow$ time domain vs. frequency domain

Frequency ω: cycles per second (Hz)

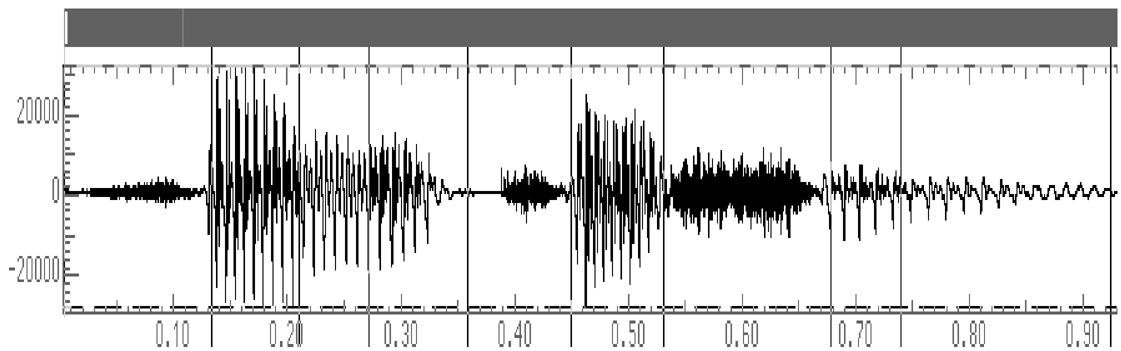
$\longrightarrow$ $\omega = 1/T$

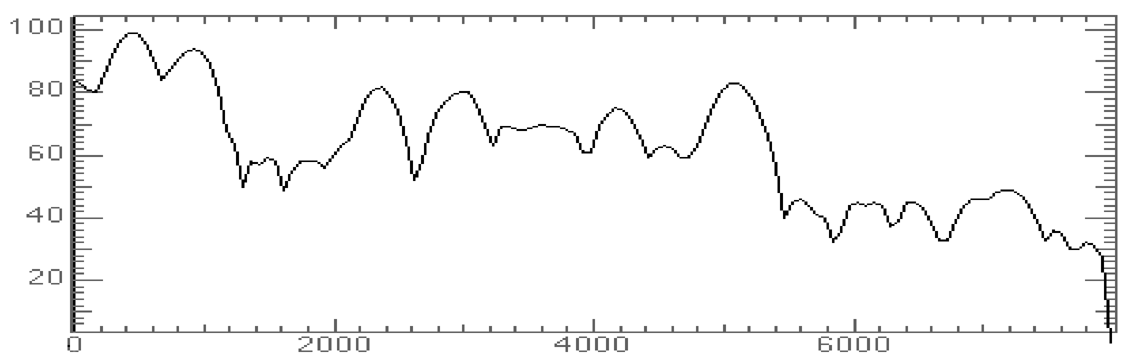
T: period of sinusoid

Example: square wave



Example: audio (e.g., speech) signal





Source: Dept. of Linguistics and Phonetics, Lund University

Random function (i.e., white noise) has “flat-looking” spectrum.

$\longrightarrow$ unbounded bandwidth

Why bother with frequency domain representation?

$\longrightarrow$ contains same information (invertible) ...

$\longrightarrow$ convenience

$\longrightarrow$ brings out “relevant” information

Luckily, most "interesting" functions arising in practice are "special":

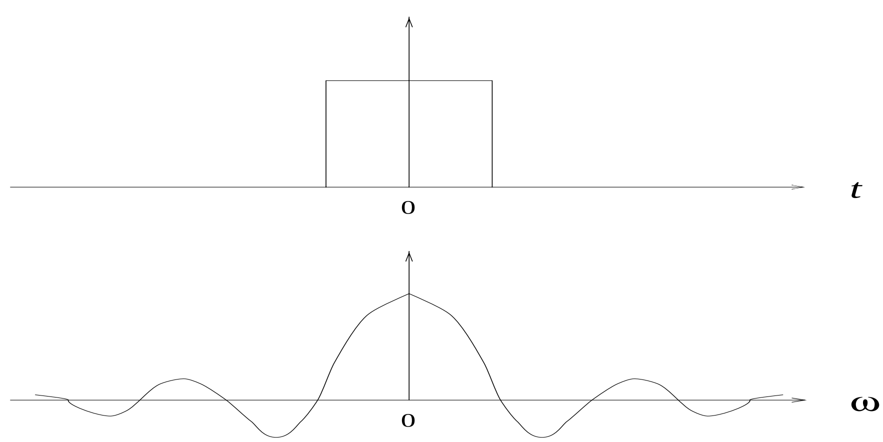
$\longrightarrow$ bandlimited

$\longrightarrow$ i.e., $S(\omega) = 0$ for $|\omega|$ sufficiently large

$\longrightarrow$ when $S(\omega) \approx 0$, can treat as $S(\omega) = 0$

$\longrightarrow$ let's approximate!

$\longrightarrow$ e.g., square wave



Ex.: human auditory system

$\longrightarrow$ 20 Hz–20 kHz

$\longrightarrow$ speech is intelligible at 300 Hz–3300 Hz

$\longrightarrow$ broadcast quality audio; CD quality audio

Telephone systems: engineered to exploit this property

$\longrightarrow$ bandwidth 3000 Hz

$\longrightarrow$ copper medium: various grades

$\longrightarrow$ no problem transmitting 3000 Hz signals

For communication:

Both absolute frequency and band**width** are relevant.

$\longrightarrow$ baseband vs. broadband

$\longrightarrow$ high-speed $\Leftrightarrow$ broadband

Manipulate shape of different frequency sinusoids to **simultaneously** carry information (i.e., bits).

$\longrightarrow$ multi-lane highway analogy

$\longrightarrow$ different lane $\Leftrightarrow$ different frequency

Manipulation of different frequencies can create complicated looking $s(t)$.

$\longrightarrow$ side effect of encoding

$\longrightarrow$ decoding: use Fourier transform